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The Unexpected Stability of the Sun and the Unexpected Instability of White Dwarfs

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The Unexpected Stability of the Sun and the Unexpected Instability of White Dwarfs

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The Sun does not explode like a hydrogen bomb, as one might expect. When a white dwarf accretes matter, it is shrinking, not getting bigger, as one might expect. I show that both effects are due to Newton's law of gravitation. I discuss the behavior of two different types of celestial bodies: the Sun (and other Sun-like stars) and white dwarfs.

I would like to briefly remind the reader what is typically said about this subject in a physics lecture, that is, a lecture that is not focused on astrophysical subjects. Regarding the Sun, one may discuss the fusion reaction, the heat transport to the surface of the Sun, and the emission of radiation. As for the white dwarf, one may mention that the temperature no longer appears in the equation of state (unlike that of stellar matter). The Pauli principle and the uncertainty relation are mentioned in order to explain why there is a maximum value for the mass of a white dwarf. The white dwarf, especially, seems to be a difficult topic. One gets the impression that one does not understand anything essential about it unless one invokes quantum mechanics and relativity. Thereby, one does not become aware of the fact that the reason that the matter of a white dwarf is extremely hard, and that in the equation of state the temperature no longer appears, is the same as for condensed matter here on Earth. If we wanted to derive the equation of state of solid matter on Earth from first principles, we would also need the Pauli principle and the uncertainty principle. But no one will doubt that solid matter is difficult to compress and that the pressure–density relationship is almost independent of temperature, even if these topics are not addressed. In the same way, important statements can be made about the behavior of white dwarfs without referring to quantum mechanics and relativity.

Moreover, in my opinion, this leaves some questions unanswered, questions, in fact, that can be answered only by using the tools of classical mechanics and thermodynamics.

Regarding the Sun: The fusion of hydrogen into helium in the solar core is exothermic. Then shouldn't the Sun explode like a hydrogen bomb?

And regarding white dwarfs: A white dwarf becomes smaller when its mass increases, for example, when it accretes matter as a partner of another star. This can reach the point that it collapses and a supernova explosion takes place. But why does the white dwarf get smaller? Shouldn't it get larger when matter is added?

So once again: The Sun does not explode, although it might be expected to do so, and the white dwarf can implode, although it might not be expected to do so. We will discover that both behaviors have essentially the same cause.

Since we are only interested in a qualitative understanding, we will refrain from a mathematical treatment. Such a treatment can of course be found in the technical literature. When teaching, I like to stick to the rule: first understand, then calculate. Our current concern is to understand.

The stability of the Sun

Once again, we will go over the (wrong) expectation, but now in more detail. Let us assume that the fusion reaction runs stationarily. The rate of heat production by the fusion reaction is equal to the energy flow to the Sun's surface, from which it is radiated away.

It is now inevitable that a slight fluctuation in the rate of heat production occurs. Let us assume it is upward; that is, for a short time, a little more heat is produced than flows away. This, we conclude, causes a small increase in temperature. However, since the reaction rate increases with temperature, the temperature increases even more, etc., etc. Such a process is known as positive feedback, and it would lead to an explosion.

We know that is not what happens. Something about our reasoning seems to be wrong. And indeed it is. It is the assumption that the temperature of the solar plasma increases when heat is added. In fact, the nature of the Sun is such that its temperature decreases when heat is supplied. The Sun is said to have a negative heat capacity.[1,2] So our process actually

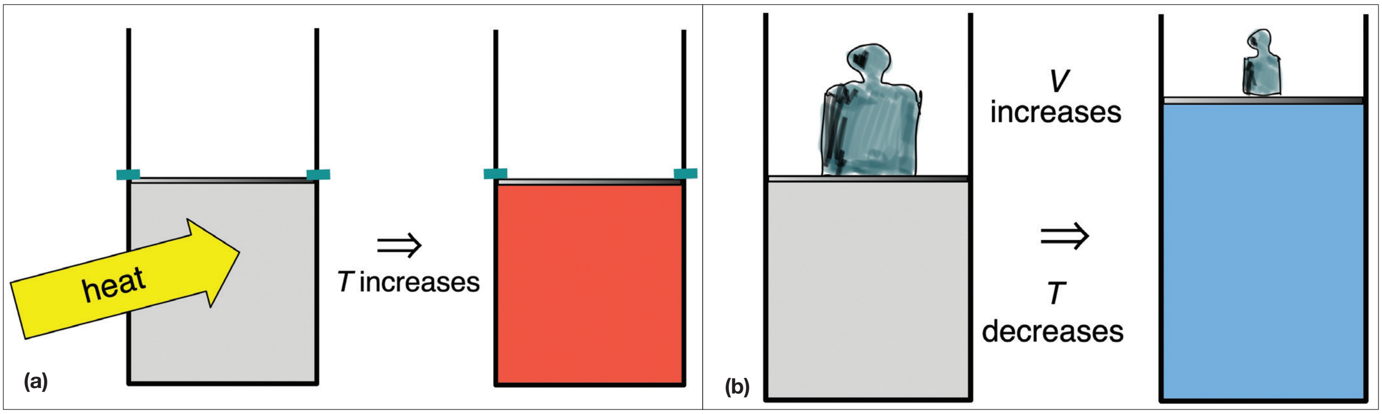

Fig. 1. (a) The temperature increases when heat is supplied; (b) it decreases when the volume is increased.

runs in a different way. When the heat production rate deviates upward from the steady-state value, the temperature does not increase, but rather decreases. This causes the production rate to decrease. We thus have a negative feedback loop, which leads to a stabilization of the process.

With this explanation, however, we have not solved our problem; we have only postponed it. For we are now faced with the question, What is the cause of the negative heat capacity?

We can find the answer by looking at a somewhat simpler system; a cylinder with a piston containing a gas (Fig. 1). We can supply heat to the gas, and we can influence its volume by placing a heavier or less heavy weight on the piston. If heat is supplied while the volume is held constant, the temperature of the gas increases [Fig. 1(a)]. If the volume is increased, its temperature decreases [Fig. 1(b)].

We now do both at the same time [Fig. 2(a)]:

- we supply heat and
- we increase the volume, albeit only slightly.

Now the temperature still increases, but not as much as in Fig. 1(a). We now can see what needs to be done to get a temperature decrease despite a supply of heat. We just have to make the volume increase sufficiently large, as in Fig. 2(b).

Back to the Sun: The material that makes up the Sun is not in a solid container. The Sun can expand or contract. The role of the piston with the weight is taken over by gravity. Now, the gravitational force has the peculiarity that it decreases strongly with an increasing radial position of the matter. This decrease is due to Newton's law of gravitation.

So how does the Sun react to an (additional) heat supply? It expands, and thereby the "weight" of each portion of matter becomes smaller—with the effect that the temperature decrease due to the volume increase is stronger than the increase due to the heat supply. Therefore, the solar matter becomes colder when heat is added, and we conclude that the negative feedback is due to Newton's law of gravitation.

The instability of white dwarfs

Again we begin by describing the (wrong) expectation in more detail. If the white dwarf behaved in a way we are accustomed to from other systems or situations, it should become larger when matter is added. If one pours more sand on a pile of sand, the pile becomes higher. But even if the material is compressible, the more matter, the larger the volume. Let's imagine that we could somehow add more air to Earth's atmosphere. The previously existing air would be compressed, but the height of the atmosphere (measured, for instance, by the height at which the air pressure has dropped to 10%) would still increase. So we would also expect the white dwarf to get bigger when matter is being added. So how is it that it gets smaller?

A white dwarf is made of matter that here on Earth we would call hard and incompressible. But this only means that it cannot be compressed by the pressures to which we are accustomed due to our terrestrial experience. With sufficiently high pressures, as prevail in a white dwarf, it is compressible and elastic.

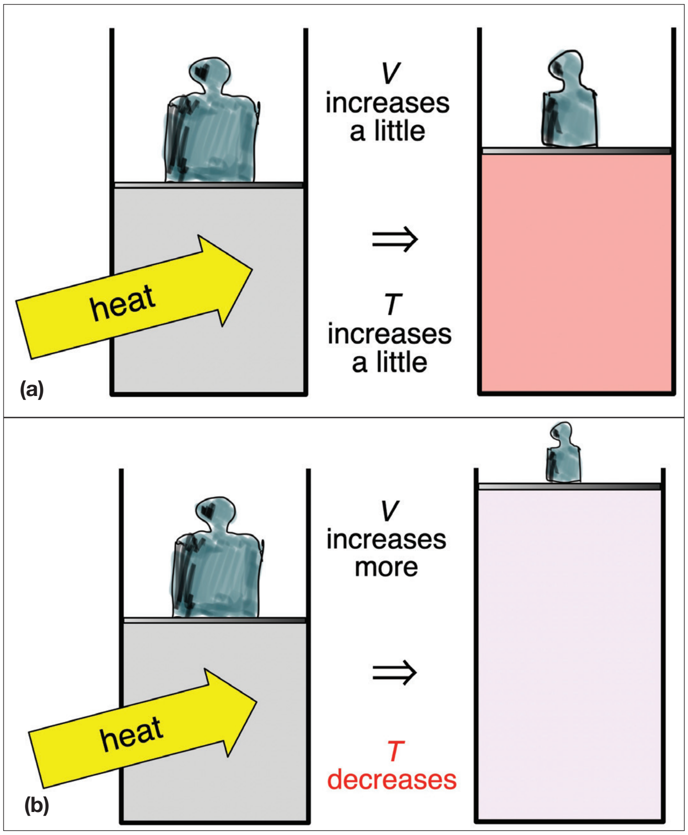

Fig. 2. Simultaneous supply of heat and increase of the volume. (a) The volume increase is small, so the temperature increases a little; (b) the volume increase is big, so the temperature decreases.

We now supply matter to the white dwarf from outside. This matter compresses the matter that is below. But this means that all of the matter of the white dwarf approaches the center, and every portion of matter gets to a place of higher gravitational force and therefore higher pressure. This results in an additional decrease in volume, and this decrease is great enough to overcompensate for the increase in volume that we might have expected. The white dwarf becomes smaller.

Thus, the radius of the white dwarf as a function of its mass depends on two factors[3]:

- the equation of state of the matter, i.e., the relation between density and pressure;
- the dependence of the gravitational force on the radius, i.e., on Newton's law of gravitation.

I would like to emphasize once again that we are concerned with a qualitative understanding. Of course, this is not sufficient if one wants to calculate the maximum mass a white dwarf can attain. Then one needs the constitutive equation of the matter of the white dwarf. On Earth, one gets equations of state preferably by measurement. But since one cannot realize the white-dwarf conditions in the laboratory, one has to derive the equation from first principles. However, I believe that in a first approach to the subject, one should leave this part of the problem aside.

Conclusion

The Sun does not explode like a hydrogen bomb, as one might expect. The fusion reaction that takes place in the Sun is incredibly stable. It is more stable than anything we know on Earth. It has been running for 5 billion years and will run for that long again.

When a white dwarf accretes matter, it does not get bigger as one might expect; instead, it gets smaller.

The discrepancy between the expected and actual behavior has the same cause for the two celestial bodies. The gravitational force decreases with increasing radius, or in other words, Newton's law of gravitation.

I recommend that teachers address these two phenomena in class, even if there is not much time for astrophysical topics.

References

1. R. Knight, "All about polytropic processes," *Phys. Teach*. **60**, 422–424 (2022).
2. F. Herrmann and H. Hauptmann, "Understanding the stability of stars by means of thought experiments with a model star," *Am. J. Phys.* **65**, 292–295 (1997).
3. From the equation of state and the law of gravitation, a differential equation can be derived, which allows one to specify the radius of a star as a function of its mass. Such a differential equation was introduced for simple equations of state in 1870. It is known as the Lane–Emden equation.

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