



The Karlsruhe Physics Course

for the secondary school A-level

The Teacher's Manual

Thermodynamics

The Karlsruhe Physics Cours – *The Teacher's Manual*

A textbook for the secondary school A-level

- Electrodynamics
- Thermodynamics**
- Oscillations, Waves, Data
- Mechanics
- Atomic Physics, Nuclear Physics, Particle Physics

Herrmann

The Karlsruhe Physics Course

Issue 2015

Edited by Prof. Dr. *Friedrich Herrmann* and Dr. *Holger Hauptmann*

Figures: *F. Herrmann*



Licensed under Creative Commons

<http://creativecommons.org/licenses/by-nc-sa/3.0/de/>



Physical Foundations

Just as in the Junior High School version of the Karlsruhe Physics Course, in the present text for the upper secondary school entropy is the central physical quantity. To those teachers who do not know our concept we recommend the Teacher's Manual of the secondary I course, as well as the literature at the end of this manual (Falk and Ruppel 1976, Job 1972, Fuchs 1996, Falk and Herrmann 1977–1982). Thus, in the following we will not discuss everything what a teacher should know about our approach, but mainly subjects that have to do with the teaching at the secondary II.

HERRMANN, F.: The Karlsruhe Physics Course – for the Junior High School.

http://www.physikdidaktik.uni-karlsruhe.de/publication/pub_fremdsprachen/englisch.html

FALK, G. und RUPPEL, W.: Energie und Entropie, Springer-Verlag, Berlin (1976).

JOB, G.: Neudarstellung der Wärmelehre – die Entropie als Wärme, Akademische Verlagsgesellschaft, Frankfurt am Main (1972).

FUCHS, H.: The dynamics of heat, Springer-Verlag, New York (1996).

FALK, G. und HERRMANN, F., Hrsg.: Schriftenreihe „Konzepte eines zeitgemäßen Physikunterrichts“, Schroedel Schulbuchverlag, Hannover (1977–1982).

1. Missing physical quantities

One of the most important tools of the physicist is the concept of a physical quantity. By means of physical quantities nature is described quantitatively. The physical quantities that are in use today have been introduced or constructed little by little. By doing so a structure became apparent that reflects a corresponding symmetry of the phenomena that are described with them.

Simple examples of such a symmetry is the appearance of pairs of physical quantities, such as

- electric field strength \mathbf{E} and magnetic field strength \mathbf{H} ;
- mass m of a body and spring constant D of an elastic spring;
- electric conductivity σ and thermal conductivity λ .

That the two partners of each couple belong together can be recognized in the fact that they appear in equations with the same mathematical structure.

There are also symmetries that refer to more than two variables. In the *Karlsruhe Physics Course* these symmetries play an important role, Table 1 (see also the Teacher's manual of the Junior Highschool version of the KPC). As a matter of course, all of the quantities of Table 1 have not been invented simultaneously. There were times when some of the quantities of Table 1 were still missing. Obviously, a scheme as that of Table 1 might have suggested to introduce the missing quantities. Such a procedure would be analogue to another one that everybody knows: When the periodic table of the elements was discovered it became obvious that there should exist chemical elements that had not yet been discovered.

The last missing quantity of Table 1 was the chemical potential. It was introduced by R. W. Gibbs who had become aware of the systematics.

extensive quantity	conjugated intensive quantity	current	energy current
electric charge Q	electric potential ϕ	electric current I	$P = U \cdot I$
momentum \mathbf{p}	velocity \mathbf{v}	force \mathbf{F}	$P = \mathbf{v} \cdot \mathbf{F}$
entropy S	absolute temperature T	entropy current I_S	$P = T \cdot I_S$
amount of substance n	chemical potential μ	substance current I_n	$P = \mu \cdot I_n$

Table 1

Although the table and the underlying structure is complete since more than 100 years, it is usually still represented in an incomplete way. Indeed, two of the quantities are so unpopular or out-of-favor that they even do not appear in many text books. These are

1. the entropy and
2. the chemical potential.

There are many situations where these quantities would be very helpful. Instead one has invented the most curious replacement constructions – only in order to avoid that one has to deal with the disliked quantities.

If they are finally introduced it is done in a way that one has the impression that they are not really indispensable.

However, precisely the entropy would be urgently needed in a beginner's course. This is the reason why there exist several replacement constructions for the entropy.

Regarding the chemical potential, in the beginner's course one simply does not ask the questions that could be answered easily by using the chemical potential.

2. Replacement constructions for the entropy

Some remarks about the history of the concept of heat

It began in the 18th century, thus it is younger than mechanics. The chemist and physician Joseph Black (1728-1799) got aware that one has to distinguish between a measure of an intensity, i.e. a measure for the „warmness“, and a measure for the amount of heat. The intensity measure or temperature was already known at Black's time; the amount of heat on the contrary was introduced by Black. It was given the name „heat“ (or in french „chaleur“). Black's heat was an extensive quantity and, using the language of modern thermodynamics, it was a state variable. In other words: It was a quite normal measure of the amount of something. From an actual point of view it should be interpreted or identified with what was later called entropy (Falk 1985).

The next decisive step was done by Sadi Carnot (1796-1832). Expressed in modern words, Carnot established the connection between entropy and energy. But how could he do so, since both quantities did not yet exist? Actually, the entropy was already there, if one identifies Black's heat with this quantity. Regarding the energy, it was introduced as a state variable only shortly after Carnot's work; but we must not forget that the concept of work was already an established concept.

With these contributions by Black and Carnot the basic concepts were there, and thermodynamics could have developed in a straightforward way, if there had not occurred a mishap in the course of the introduction of the energy. Of course, the mishap was not the introduction of the energy; it was rather the euphoria that accompanied the appearance of the new quantity, which resulted in the idea that, a part from the energy, no other extensive quantity is needed – at least as far as thermal phenomena are concerned. The name „heat“ was now attributed to something that had better not received a name at all, namely for a so-called process quantity.

In this way the heat of Black and Carnot, or in modern terms, the entropy, was banned from physics. In the following years it now and again appeared without a proper name in several publications, until in 1865 it was officially reintroduced by Clausius. However, Clausius' introduction was so complicated that the „new“ quantity got the reputation of being one of the most difficult concepts of physics. The name „heat“ could have helped to attenuate the difficulties, but this name was no longer available since it had been given to something else. Thus, the quantity got the name that it carries still today: entropy.

For several decades nobody noticed what had happened. Finally, it was noticed and published in 1911 by H. L. Callendar. However, Callendar's discovery remained almost without any reaction or consequence. The reason: it was too late. Entropy continues to exist in the form that it was given by Clausius: A function that is defined by an integral of which it is hard to get an intuitive idea. In particular it is almost impossible to see from Clausius' introduction that the entropy is a measure of a quantity, i.e. that it is an extensive or „substance-like“ quantity; that it can flow, and that one can establish a balance. Thus, it was impossible to recognize that entropy is indeed a very simple physical quantity. It was in the same year, i.e. in 1911, that Jaumann showed that a continuity equation can be formulated for the entropy.

A measure for the amount of heat

Based on our everyday experience one would expect that it is possible to define a physical quantity that corresponds to what in colloquial terms one would call heat. A „hot-water bottle“ with hot water contains more of it than one with cold water, and a big bottle contains more heat than a small one with water of the same temperature. That such a quantity can be introduced is well-known since the 18th century. As we have seen, this quantity was later banned or eliminated from physics.

Now, neither physics nor chemistry can do without an appropriate measure for the quantity of heat. That is why quantities have been constructed that could replace the old quantity of heat. One of them is the so-called energy form heat, a process quantity, as is usually said. Its use has the fatal consequences that every physics teacher knows. There are problems not only for the pupils. Many specialists in physics also stumble over the concept. The very name of the quantity misleads most learners to incorrect conclusions.

So, it is not incorrect to say, that heat is supplied to a body. However, if one relies on the logic that underlies our language, one would conclude that by supplying heat the amount of heat contained in the body increases. This statement, however, is not correct. Thereby we do not mean that the heat content does not increase. It is worse: The body does not contain any heat anyway. The concept heat content can simply not be applied. In other words: The new heat of physics is not the heat measure, that its name suggests, or that it pretends to be.

The jugglery attains its culmination in the calorimetric experiences that are often done in the beginners' courses. When these experiments are described the quantity Q appears and every student interprets it as the heat content of the liquid or solid body that is studied, whatever the effort of the teacher not to say a single incorrect word might be. And the quantity c is called the heat capacity, from the latin word *capacitas* for content, although there is no heat to be contained.

Now that a quantity of heat is not only expected by the learners but also needed by the scientists, more replacement constructions have emerged. In chemistry for instance enthalpy is to replace the „quantity of heat“. As a Legendre transform of the energy, enthalpy is such a difficult construction, that it cannot satisfy the artless requirement for a measure of an amount of heat. Indeed, it is a state variable (in contrast to Q), and it is allowed to call it an extensive quantity. To be an intuitive quantity, however, such as energy or electric charge, it should be possible to establish a balance of it, i.e. to write down a continuity equation, which is not the case. So, no proposition can be formulated about its conservation or non-conservation.

The efficiency

Machines are imperfect to the degree that they produce entropy. Energy is lost only because of entropy production. Any other „loss“ can be reversed. Usually the efficiency is formulated without referring to entropy production. Thereby, the actual cause of the losses is not mentioned.

The value of energy

Some authors propose to attribute a value to the energy in such a way that in many processes the value of the energy decreases. It can also gain value but only if some other energy loses value accordingly.

This way of treating a thermodynamical process is an example of attributing a property of the entropy to another physical quantity, i.e. the energy.

A technical process is bad if entropy is produced. In the present case this defect is expressed by a statement about the energy, with the strange consequence that the energy has, in addition to its known properties, one more property, namely a value.

Exergy

It is a construction of the same kind. It had been introduced to evaluate thermal and chemical facilities or installations. Which are the components that have to be improved? Obviously, these are those components where entropy is produced. The more entropy is produced, the greater the potential for an improvement. But what to do in view of the taboo on entropy? Construct a replacement quantity.

The second law

It can be formulated in simple terms:

Entropy can be created but not annihilated.

Obviously, the second law is a theorem of the same kind as many others: About each extensive quantity a statement about its conservation or non-conservation can be pronounced, for instance:

Energy cannot be created or annihilated.

Momentum cannot be created or annihilated.

Electric charge cannot be created or annihilated.

Amount of substance can be created and annihilated.

But how can one formulate the second law without mentioning the entropy?

One of Clausius' formulations was:

It is impossible to bring heat from a colder to a hotter reservoir without causing any changes in the environment.

Or Planck's formulation:

It is impossible to construct an engine that will work in a complete cycle, and produce no effect except the raising of a weight and cooling of a heat reservoir.

There is no doubt that these claims are correct. However, they refer to particular classes of processes, so that their importance can hardly be seen.

By the way: The situation is similar in mechanics. Newton's laws are no more than a hedged formulation of the conservation of momentum.

Thermal engines

Since Carnot's work their description is easy: Just as on a water wheel water moves top-down thereby driving something, in a heat engine entropy goes from a high to a low temperature thereby driving something. When doing so the amount of entropy does not change, just as the amount of water does not change when flowing over the water wheel.

The heat pump (that did not yet exist at Carnot's time) can be explained correspondingly: By means of the heat pump entropy is conveyed from cold to hot, just as water is brought from low to high by means of a water pump. Actually, a „heat pump“ is an entropy pump.

Such a simple explanation is not possible when using the energy form „heat“.

Convective heat transport

When employing entropy the explanation could be as follows: First, entropy is charged on the water. If now the water is put into movement in some way, it carries the entropy with it. So simple it is. But what an effort and what confusion if one tries to formulate these fact without saying that heat is stored in the water.

Adiabatic processes

To describe processes that are usually considered as thermodynamical, apart from energy four more variables are used: pressure, volume, temperature and entropy. Any two of them can be chosen to be independent; the values of the others are determined by the state equations of the considered system. To maintain an overview, one usually considers processes in which one of the variables is held constant. As a consequence only one independent variable remains. To express which of the variables is held constant one uses particular names for the processes:

isobaric ($\beta\acute{\alpha}\rho\omicron\varsigma$ = heavy) for $p = \text{const}$

isochoric ($\chi\omega\rho\omicron\varsigma$ = space) for $V = \text{const}$

isothermal ($\theta\epsilon\rho\mu\omicron\varsigma$ = warm) for $T = \text{const}$

And what is the name of a process with $S = \text{const}$? Not isentropic, but „adiabatic“. So one has not to pronounce the disliked word. It is not clear which of the variables is held constant, but what one has to do in order to keep it constant: The walls of the container have to be impenetrable ($\alpha\delta\iota\alpha\beta\alpha\tau\omicron\varsigma$ = impassable). By the way, the word is also used when no walls exist.

The temperature gradient within the troposphere

Why does the temperature decrease when going up in the troposphere? The answer to this question is simple if we dispose of the concept of entropy. When the air of the atmosphere is well-mixed, the entropy distributes regularly: Each mole contains the same amount of entropy. If a gas, whose entropy is held constant expands, its temperature decreases. Therefore, the air at a higher altitude must have a lower temperature than that at a lower altitude.

These simple facts cannot be formulated without referring to the entropy. One can now understand why this simple atmospheric phenomenon in many physics text books is even not mentioned.

Phase transitions

Imagine a pot of water standing on a hotplate is boiling. The description of this process by using the entropy is as follows: Entropy is flowing from the hot plate into the water. The water vaporizes. Since water vapor contains more entropy than liquid water at the same temperature, we have to supply entropy if we want the water to vaporize.

What is the description of the process when not speaking about entropy?

To the water energy is supplied in the form of heat. As a consequence the enthalpy increases by the same amount.

Students hardly understand why such a complicated wording should be necessary. The sentence: „We supply heat and thereby the heat content of the systems increases“ is not correct when using the term heat in the meaning of physics.

Might it be allowed at least to say: „We supply enthalpy and thereby the enthalpy contents of the system increases correspondingly“? No, also this sentence is not correct. So, let us try a statement about the energy, since we have more confidence in the energy: „We supply energy and thereby the energy content of the system increases correspondingly“. Unfortunately, also this sentence is not correct, since a part of the energy is used to „push the atmosphere upwards“.

CALLENDAR, H. L.: Proc. Phys. Soc. London 23, 153 (1911).

FALK, G.: Entropy, a resurrection of caloric – a look at the history of thermodynamics, Eur. J. Phys. 6, 108–115 (1985).

JAUMANN, G.: Geschlossenes System physikalischer und chemischer Differentialgesetze, Sitzungsber. Akad. Wiss. Wien, Nat.-Naturw. Klasse, Abt. IIA 120, 385–530 (1911)

3. Entropy and life

One can often read that the entropy is a quantity that allows us to characterize the particularity of living or organic matter. Life is a state of high order. It is suggested that a contradiction to the Second Law might be suspected. But then it is shown, with a certain effort, that the Second Law is not violated.

Such considerations are not incorrect, but they are simply inappropriate.

Living systems do not distinguish themselves by a low entropy content. A living person of say 80 kg has roughly the same entropy as 80 liters of water of the same temperature. The water has less entropy than the person if its temperature is lower, say by 5 °C; and it is much less if the water is frozen. The entropy is simply not the appropriate quantity to characterize what we want to characterize.

4. Statistical introduction of the entropy

Statistical physics is one of the most elegant subfields of physics. If the time for discussing it is available, one should do so. But let us be realistic. If we treat statistics it will be at the expense of the so called phenomenological thermodynamics. Someone who only learns statistical thermodynamics is hardly able to solve any of the practical problems of our natural and technical environment.

Introducing the entropy as a measure for the microscopic disorder is similar to introducing the temperature as a parameter in the Boltzmann distribution – instead of as a measure for hot and cold. It would be unworldly to introduce the temperature in this way. It is just as unworldly to begin the teaching of entropy with its microscopic interpretation.

5. The chemical potential

The situation is similar to that of the entropy, or even worse. Here, however, the common remedy is simpler than in the case of the entropy: When teaching physics, those questions that could be answered by using the chemical potential are simply not asked.

But it would be so simple to introduce the quantity in an intuitive manner. Just as the temperature or the electric potential, the chemical potential can be considered a measure of a driving force. It tells us from where to where a substance diffuses, in which direction a phase transition occurs or in which direction a chemical reaction runs.

From the point of view of statistical thermodynamics it is a parameter very similar to the temperature. The Boltzmann, Fermi-Dirac and Bose-Einstein distributions $\rho(E)$ contain two such parameters, namely the temperature T and the chemical potential μ .

$$\text{Bose-Einstein} \quad \rho(E) = \frac{1}{e^{\frac{E-\mu}{kT}} - 1}$$

$$\text{Fermi-Dirac} \quad \rho(E) = \frac{1}{e^{\frac{E-\mu}{kT}} + 1}$$

$$\text{Boltzmann} \quad \rho(E) = e^{-\frac{E-\mu}{kT}}$$

The temperature tells us, how steep the distribution is; the chemical potential defines its vertical extension.

Thus, temperature and chemical potential are two cognate quantities. They merit a similar treatment in the physics lessons. As is well-known, the chemical potential is not treated at all in the physics classes. Thus, when treating phase transitions, one simply does not ask for the driving force of the process. Just imagine a schoolbook in which temperature does not appear at all.



Remarks

1. Heat

1.7 The thermal conductivity

An entropy current is always accompanied by an energy current. The relation between the corresponding current strengths is

$$P = T \cdot I_S.$$

The strengths of the energy current and of the entropy current depend on the thermal resistance of the material in which they are flowing. To express this dependency one defines a conductivity that refers to the energy current or one that refers to the entropy current. Both these conductivities are equivalent – at least as far as mathematics is concerned. Traditionally, only the energetic conductivity is introduced and used. Its name is thermal conductivity, abbreviated λ . It is related to the entropic conductivity σ_S by

$$\lambda = \sigma_S \cdot T.$$

Both conductivity measures have pros and cons. The thermal conductivity has the advantage that the flowing quantity is a conserved quantity. In a conductor in which nowhere energy is accumulating, and which is not branched the energy flow is the same at every cross sectional area.

The entropy current has conceptual advantages. There is no intensive quantity corresponding to the energy, i.e. there is not „driving force“ for an energy current. When suggesting that the temperature represents such a measure, one is sinning in the sense addressed above. One attributes a property to the energy that it does not have. So, if one decides to use the driving force/resistance/current model, then the correct way is as follows: A temperature gradient is a driving force for an entropy current, and an energy current flows because to each entropy current an energy current is coupled. We also do not define the electric resistance as voltage divided by the energy current, but voltage divided by electric current.

There are other facts that support the idea that σ_S and not λ is the appropriate measure for thermal resistance. The Wiedemann-Franz law tells us that the „Lorenz number“ $\lambda/(\sigma_O \cdot T)$ has nearly the same value for many metals. Here, σ_O is the electric conductivity. One says that the ratio of the thermal and the electric conductivity is proportional to the temperature – what is not easy to understand. Expressed with the entropy conductivity, the law tells us that the ratio of the two conductivities, that of the entropy and that of electric charge, is constant, i.e. independent of the temperature. This is plausible, if one knows that the same particles are responsible for both transports, namely the free electrons.

In our course, we thus decided to use the entropy conductivity. The entropy production is in many cases a small effect: always when the temperature difference is small compared to the absolute value of the temperature. Then the produced entropy does not play an important role in the total entropy balance.

1.10 The relation between energy current and entropy current

In order to derive the equation

$$P = T \cdot I_S$$

we use the same procedure as in electricity when deriving the equation

$$P = U \cdot I$$

In both cases we have supposed as plausible that when connecting two „pumps“ in series (electricity pumps or entropy pumps) the corresponding „potential differences“ (electric potential difference or temperature difference respectively) add up. If this is accepted, the experiment is even no longer necessary. The result is obtained by pure logical deduction. However, who has a pronounced sense for axiomatic strength may not be satisfied by this way of proceeding.

Actually one should do the experiment, and the result would be: the temperature scale on which our thermometer is based, is such that when connecting the heat pumps in series the temperature differences add up. However, this argument is too sophisticated for the physics lessons at school.

Moreover, in order to realize the experiment one would need heat pumps that work reversibly. But this is far from being true for the Peltier elements that are normally used in school. We thus remain with our somewhat unfair argumentation and do not make an experiment.

By the way, if the experiment could be done its result would be equivalent to the statement: the absolute temperature, that displays our temperature meter is defined by the equation

$$T := \frac{P}{I_S}$$

That means that the equation

$$P = T \cdot I_S$$

could have been introduced with much less expenditure. The fact that P is proportional to I_S follows from the substance-like character of energy and entropy. The factor of proportionality is called temperature. Thus, the temperature is *defined* by the equation

$$P = T \cdot I_S$$

However, the students are familiar with the temperature from their everyday life, and they would perceive this way of proceeding as weird: Why define a quantity that I know already?

1.14 Energy loss and efficiency

1. Energy loss V and efficiency η are related by

$$\eta = 1 - V.$$

If an engine has to be assessed, the main interest is in the cause of the losses. We therefore believe that the focus should be on V , not on η .

Moreover, the expressions for the loss are slightly simpler than those for the efficiency.

Whereas the Carnot efficiency is

$$\eta = \frac{T_2 - T_1}{T_2}$$

the „Carnot loss“ $1 - \eta$ is simply the ratio of the low and the high temperature:

$$V = \frac{T_1}{T_2}$$

2. The efficiency is a quantity that allows us to characterize a machine with regard to energy losses. One tries to define it in a way that it has the following properties: It should be dimensionless and it should acquire values between zero and one. It should be zero if the machine is completely useless because of its energy losses, and it should be equal to one if it is impossible by principle to improve the machine or to replace it by a better one.

Customarily the efficiency is defined as

$$\eta = \frac{\text{in der gewünschten Form abgegebene Energie}}{\text{aufgenommene Energie}}$$

As long as neither the received energy (the energy input) nor the energy released in the desired form is heat, this definition meets the requirements formulated above. However, as soon as one of the „energy forms“ is heat, or in our way of speaking, as soon as one of the energy carriers is entropy, the definition yields values, that do not comply with our specifications. We consider three examples of machines or devices.

1. Heat pumps

It is well-known that a good heat pump supplies more energy „in the form of heat“ than it absorbs in „electric form“. Thus, its efficiency would be greater than one. To get rid of this inconsistency one simply gives a new name to η in this case. It is called *coefficient of performances*.

2. The Carnot engine

As it is customary, we assume the engine to work reversibly, i.e. without entropy production. The efficiency that results from the above definition is less than one, although the engine is, from a thermodynamic point of view, perfect. There are no losses due to entropy production. It is impossible as a matter of principle to improve its performance. Here, the fact that the efficiency is less than one appears as a particularity of thermodynamics that is difficult to understand.

3. Electric resistance heating

Consider an immersion heater. Since all the incoming energy is transformed into heat an efficiency of one results, although such a heating is a notorious energy waster. The heating can be improved by using a heat pump instead.

These inadequacies are home-made. They are due to an inappropriate definition of the efficiency. Actually, since a long time a better definition exists. However, it found its way only into the advanced specialized literature, and it carries names that are rather intimidating: second law efficiency or exergetic efficiency. Actually its definition is no more complicated than the definition given above. One way of writing it is:

$$\eta = \frac{P_{\text{ideal}}}{P_{\text{real}}}$$

Here, P_{real} is the energy consumption of the actual engine to which the efficiency refers. P_{ideal} is the energy consumption of an engine with the same performance as the real engine but in which no entropy is produced, and which therefore works without losses.

It is even more transparent to consider the loss instead of the efficiency:

$$V = 1 - \frac{P_{\text{ideal}}}{P_{\text{real}}} = \frac{P_{\text{real}} - P_{\text{ideal}}}{P_{\text{real}}}$$

The difference $P_{\text{real}} - P_{\text{ideal}}$ in the numerator is the rate of the lost energy, i.e. that part of the energy that is used (or misused) to produce entropy. We can thus formulate the loss yet in another way:

$$V = \frac{P_{\text{loss}}}{P}$$

Here, P_{loss} is that part of the energy current that is used to produce entropy, and P is the energy current that was called P_{real} above, i.e. the energy consumption of the engine.

With this definition all of the above-mentioned inconsistencies are eliminated. For the reversibly working heat pump as well as for the Carnot engine we get an efficiency of one. For the electric resistance heater, just as for any other heating system that is based on entropy production the numerator becomes:

$$P_{\text{loss}} = T \cdot I_{S, \text{produced}}$$

where T is that temperature at which the heater delivers the heat to the system to be heated. If a room with a temperature of 25 °C (298 K) is to be heated, we have $T = 298$ K and $I_{S, \text{produced}}$ is the entropy current released by the heater at this temperature.

1.16 Measuring Entropy

The corresponding section has been written quite exhaustively, more with regard to the teacher than to that of the pupils. We believe that it can be taught more succinctly and there are various ways to do so:

1. Leave it out completely.
2. Calculate the entropy from the average values of the entropy current at the beginning and at the end of the experiment.
3. Use a modeling software like COACH or STELLA. The software does the integration. The pupils do not need to know the concept of an integral.

1.17 The heatability

1. The word capacity tells us how much of something a container can accommodate. That is why it suggests something that is not true in two respects. The first deficiency can be better understood when referring to the electric capacitance. A capacity or capacitance should always be measured with the unit that is also used for the amount to be stored. The capacity of a gas tank is measured in Liters, that of a truck in tons, that of a data store in Megabytes. Correspondingly one would expect that the capacitance of a capacitor is measured in Coulomb. It would tell us how many Coulombs one of the capacitor's plates could store at most. Actually, the quantity C , i.e. the electric „capacitance“, tells us something different: by how much the charge of one of the plates changes when changing the voltage. Thus, it tells us how difficult it is to charge a capacitor, if we consider the voltage as a measure of the effort. Now, the name capacitance is so established that it is unrealistic to consider a change of the name. But we should advise to say that the meaning of the quantity is that of a content and avoid the students that the name capacitance is misleading.

The designation „heat capacity“ is inappropriate yet for another reason. A capacity or content for the quantity that quantity is called heat does not exist anyway, since the process quantity heat has not a certain value for any system in any state.

The expectation that one may have about c , but which this quantity does not fulfill, is indeed fulfilled by the specific entropy capacity c_S :

$$c_S = \frac{1}{n} \cdot \frac{\Delta S}{\Delta T}$$

It is related to the molar heat capacity c by

$$c = T \cdot c_S.$$

The two quantities are distinguished simply by the factor T .

In traditional physics the entropy capacity appears only marginally. In solid state physics it appears occasionally under the name Sommerfeld constant. For us the question of how to call it did even not come up, since we operate with the reciprocal of the specific entropy capacity.

2. Instead of the entropy capacity we introduce the reciprocal α of it:

$$\alpha = \frac{n \cdot \Delta T}{\Delta S}$$

α and c_S are equivalent in the same way as the electric resistivity and its reciprocal, the conductivity. In the case of the choice between the specific entropy capacity and its reciprocal there are some arguments in favor of the reciprocal.

On the one hand it has a simple intuitive meaning: It tells us how easy it is to increase the temperature of a body. That is why we call it heatability. The heatability is great when a small entropy supply results in a great temperature increase; it is small if the entropy supply causes only a small temperature increase.

However, the main reason for our choice is yet another.

If we want to represent graphically the reaction of a body on a supply of heat, it is better to represent T over S than S over T . Why? It is appropriate to choose as independent variable the one which can most easily be influenced by the experimentalist. And indeed, what we do when studying the dependence of the two variables is to supply heat or entropy and we look how the body reacts: how its temperature changes.

Incidentally, this is the usual method in thermodynamics: the intensive quantity as a function of the extensive quantity. (The best-known example is the p - V diagram.) Now, the heatability is simply the slope of the T - S curve whereas the specific entropy capacity would represent the reciprocal of the slope. Our choice turns out to be particularly appropriate when considering a phase transition. In a T - S diagram a (first order) phase change shows up as a plateau. The slope is zero. Thus, the body or the substance can not be heated as long as the phase change takes place. One is supplying heat but the temperature does not increase. On the contrary, the specific entropy capacity (just as the specific heat capacity) would become infinite. The impression results that we have to do with a singularity.

3. We relate the heatability to the entropy per amount of substance and not to the entropy per mass. If we had decided to use the heat capacity we also would have introduced this quantity via the entropy per amount of substance. Actually, it is customary to introduce the specific heat capacity as supplied heat energy per mass and temperature increase. The advantage of this way of proceeding is that the values can be found in the tables, and that it is easier to measure the mass of a portion of substance than to determine its amount of substance. However, the physical quantity mass has nothing to do with thermodynamics (neither with electrostatics). As long as gravity and inertia are irrelevant, mass is the improper measure of the amount of the substance.

When formulating the gas equation with the mass

$$p \cdot V = m \cdot R_i \cdot T$$

the „gas constant“ R_i becomes dependent on the nature of the material. That is no wonder since gravity and inertia have nothing to do with the phenomena described by the gas equation. Similar arguments hold regarding other thermal properties of materials, such as the specific heat capacity and the heat of evaporation. The molar specific heat of solid and liquid elements are at sufficiently high temperatures roughly equal. This is known under the name Dulong-Petit law. When the specific heat or the heatability are mass-referred this simple property is hidden.

4. In the traditional way of teaching thermodynamics calorimetric experiments play an important part. We propose to reduce the importance of these experiments, for several reasons.

These experiments and the common interpretation foster an incorrect idea that later can only hardly be corrected: the idea that the quantity Q represents a heat content. Thus, one is creating a problem that otherwise would not exist.

Moreover, the importance of the difference of the values of the specific heat capacities for various substances is overemphasized. Actually, the heat capacities differ only slightly, in particular when they refer to a mole of each substance. There is hardly any effect that is due to a particularly great or small value of the specific heat capacity. Sometimes it is said that the central house heating system uses water because the specific heat capacity of water is rather great. But which other liquid would be cheap enough to compete with water? Another argument is often given to underline the importance of the subject: The claim that the difference between the continental and the marine climate is due to the difference of the heat capacities of water and the material of the solid soil. However, this explanation is not correct. The higher heat storage capacity of the water is due to the convection of the water: it allows to store the entropy in a greater depth than in the solid soil.

5. For the conceptual clearness we have to pay with a concession to comfort. Both, the traditional specific heat capacity and our entropy capacity (as well as its reciprocal, the heatability) are temperature dependent. Now, in a large temperature range above the Debye temperature the specific heat capacity is virtually independent of the temperature. A priori, such a behavior could not be expected from a physical coefficient that characterizes a material. Resistivity, magnetic permeability, viscosity, compressibility and others all change upon a temperature change.

In this respect the entropy capacity and the heatability are more similar to these other physical coefficients.

However, as long as one has to do with small temperature changes this temperature dependence is not important. One can operate with and average value of the heatability.

2. Gases

2.3 Quantitative relations between S , T , V and p

1. A thermodynamical system with the variables S , T , V and p has two degrees of freedom. We therefore can dispose freely over the values of two of them; the remaining two follow from the constitutive equations that characterize the gas.

These relations can be described in various ways, for instance by means of the thermal and the caloric state equations. Such state equations always contain three variable. Two of them can be considered as independent and one as dependent.

Now, we are usually interested in the dependency of one variable on one other variable. Therefore, one of the independent variables is kept constant. However, the possible combinations of two variables (there are twelve of them) are not independent of one another. The question of which of them should be treated in the physics lessons is not easy to answer.

Our choice is not very inventive. We treat the ideal gas law, that contains three of the above-mentioned relations between two variables, as well as the equations for adiabatic processes.

From these all the other two-variable dependencies could be derived. However, these derivations are too difficult for the school.

Thus, we do not discuss for instance the entropy as a function of the volume at constant temperature.

2. The so-called adiabatic exponent κ is usually introduced as the quotient of the specific heat capacity at constant pressure and at constant volume. Since we do not introduce the specific heat capacities it would be more reasonable to consider κ as a material constant of its own right. If, however, one chooses $\beta = \kappa - 1$ instead of κ to characterize the material, we have to deal with a quantity that has a simple intuitive meaning. It is plausible that the temperature increases when a gas is compressed isentropically. The material constant β tells us how great this effect is.

3. Equations are often named after distinguished researchers. In this way one can on the one hand honor a person for his or her merits, and on the other hand it becomes easier to refer to the equation by calling it by its name: „According to Maxwell’s second equation...“

Now, in the context of this custom some problems arise.

The first: Whom should we honor? Some persons may become famous, although their merits are not as high as those of other persons who were not so lucky as to become the eponym of a much used equation.

Another problem: The fact that an equation has a name may induce us teachers to misunderstand the answer to certain questions about physics as an insight. „What is Ohm’s law?“ – „ U is equal to R times I .“

A downright grotesque example of the misuse of names is the ideal gas law.

This equation it is one of the various equations of state of the ideal gas. There are several equivalent formulations of this law, but none of them got a name, with one exception: that equation that is called the „ideal gas law“, but it has yet several other names: „universal gas law“, „general gas law“, „general gas equation“... Since the equation links four variables with one another, one can, by holding two variables constant, transform it in several two-variable relationships. These partial laws in turn carry the names of several scientists: One of them is Boyle’s law (in England) or Loi de Mariotte (in France) or Boyle-Mariottesches Gesetz (in Germany); another one is named after Gay-Lussac, a third one after Amontons and a fourth one after Avogadro. Why does one state equation merit so many names whereas another one has none at all?

3. Substance-like quantities

3.4 Mass and gravitational potential

The process sketched in Fig. 3.8 runs only in the way described in the text, if two conditions are fulfilled:

1. The frictional resistance must be so high that energy is „dissipated“ only within the tube. In other words: All the entropy that is produced must be produced within the tube and not by the swirling of the water that leaves the tube with too great a velocity.
2. The tube must have a constant cross sectional area.

Only if both these conditions are fulfilled, the pressure is the same over the whole length of the tube, so that the gravitational potential gradient is the only „driving force“.

4. Amount of substance and chemical potential

4.5 The relationship between chemical potential and pressure

The most important equation of chapter 4 describes the logarithmic relation between chemical potential and pressure. Although it expresses a rather simple fact its derivation is somewhat problematic. In principle two ways may be envisaged.

The equation follows from and is equivalent to the ideal gas equation.

The first possibility to get our equation would be to derive it from the gas equation. However, for the school this way is not feasible, although it would be the most straight forward possibility. The calculation is too difficult for the school since it requires partial derivatives. One can avoid this difficulties by using a trick – for instance by discussing the energy balance of a pressure engine (Herrmann 1978). In addition one has to integrate the function $1/V$. To avoid this integration another trick was employed in our text: We connected several pressure engines in series and in parallel. In this way we obtain the logarithm, but we do not obtain the factor of proportionality RT . Moreover, the arguments are somewhat cumbersome so that the pupils' ability to concentrate may not be sufficient.

The second way would be to obtain the equation from an experiment. This way is somewhat inelegant, since it results in the impression that the equation represents a new law that is independent of the other equations that are already known to the students. (this objection is valid also for several other equations.) Another problem is that the corresponding experiments are not easy to carry out and are not really transparent. One possibility would be given by the concentration cell. However, to explain the working principle of the concentration cell is so complicated that one may lose sight of the original objective of the experiment. Remember that the concentration cell works irreversibly by principle.

A candidate for an experiment that is conceptually simpler would be the lambda sensor as employed in automobile catalysts. Such a sensor is indeed an instrument for the direct measurement of a chemical potential difference. The electric signal at the output of the sensor is proportional to the difference of the chemical potentials of the oxygen in the exhaust gas and in the ambient air. The disadvantage for a use in school is that it works only at high temperature which makes the experiment rather elaborate.

Finally, there is a third possibility. It is neither elegant nor conceptually satisfying. The advantage is that it is short, which is a great advantage: One writes the equation on the blackboard and operates with it, thereby providing confidence in its reliability.

5. Phase transitions

5.2 The driving force for a phase transition

If an extensive quantity is a conserved quantity the zero of the corresponding energy-conjugated intensive quantity can be freely chosen. If the extensive quantity is not conserved one cannot freely dispose of the zero point.

In the case that an extensive quantity is conserved only under certain conditions, one can choose freely the zero point as long as one limits to the description of processes in which the quantity is conserved.

Actually, for any quantity that is generally not conserved there exist processes in which they behave like a conserved quantity. For the entropy this is the set of all the reversible processes.

The quantity „amount of substance“ is even more interesting in this respect. There are branches of science that are defined by the requirement that the amounts of certain classes of substances are conserved. Thus, chemistry is that part of science where the amounts of substance of the so-called elements obey a conservation law. When establishing a chemical equation one extensively takes profit of this property: the amount of substance of each chemical element must be the same at both sides of the equation. If they are not conserved we no longer have to do with a reaction that belongs to chemistry, but with a nuclear reaction.

Thus, as long as one envisages only chemical reactions, the zero points of the chemical potentials of all the chemical elements can be chosen freely. When nuclear reactions come along, this possibility no longer exists.

There are reactions or conversions of materials where not only the amounts of the chemical elements are conserved, but also the amounts of all the compounds. If in a chemical equations at the left-hand side there is 1 mol of water, then at the right-hand side there is also 1 mol of water. According to the scientific tradition such processes or reactions are said to belong to physics, not to chemistry. They are called phase transitions and are the subject of our chapter 5.

Just as in chemistry the zero's of the roughly 100 elements can be chosen freely, one can, when limiting to phase transitions, choose an origin of the chemical potential for each compound. In table 5.1 it has been taken profit of this possibility.

6. Coupled currents

6.1 One substance – several energy carriers

We did not discuss the energy carrier momentum in this section. This would have obliged us to discuss the velocity dependence of the chemical potential. (The chemical potential depends indeed on the velocity.) This subject is not appropriate for the school.

7. Thermal radiation

7.2 The light gas – thermal radiation

For the graphical representation of a spectrum various abscissas can be chosen: frequency, wavelength, wave number or photon energy. Different conventions have established in the various contexts.

Throughout the area of TV and radio broadcasting one uses frequency. In the domain from microwaves up to the ultraviolet spectral range the established measure is the wavelength, and from X-rays upwards (on the frequency scale) the photon energy, which to within a universal factor of proportionality is equal to the frequency, is employed.

The origin of the unsystematic use of different scales is the difference in the measuring methods. Where, as it is the case for radio and TV technology, frequency can directly be measured with the oscilloscope: in the optical domain interference experiments are used to determine the wavelength. For nuclear and particle reactions one uses accelerators that allow to deduce the energy of the created particles.

Physics at school should not necessarily adopt this habit. In particular when treating a subject where various radiations appear, for instance radio waves and X-rays and gamma rays, one should choose one common measure.

When it is to be decided between frequency and wavelength our choice would clearly be the frequency. Since it is proportional to both the energy and the momentum of the photons it seems to be the more fundamental quantity.

It should be borne in mind, that the maximum of the Planck curve is not at the same position when taking the wavelength or the frequency as abscissa.

8. Entropy currents in the gas layers of planets and stars

8.1 The troposphere

1. The atmosphere is a complex system. To establish its energy balance under the conditions of school physics one has to simplify. A good understanding does not result if all the contributions to the total energy flow are presented as equally important. Energy flow diagrams as we get them from the meteorologists area unnecessarily complicated and inappropriate for the teaching at school.

Who knows this kind of diagrams will notice which simplifications we have introduced. Those who don't know them, will not miss anything.

2. The greenhouse gases influence the heat balance of the Earth by the absorption of infrared radiation. If one wants to understand the influence of these gases a thorough balance of the energy transported by this radiation has to be done.

Often this balance is realized in a somewhat mistakable way. So, sometimes it is said that the Earth emits on the average 393 W/m^2 with electromagnetic radiation and 106 W/m^2 as „sensible heat“ and „latent heat“. Thereby inevitably results the impression that the radiation is particularly important for the heat removal. The importance of the radiation is further emphasized by noting that a radiative energy current of 342 W/m^2 is flowing back from the atmosphere to the Earth. This kind of presentation exaggerates the relevance unnecessarily. It would be more convenient, to consider only the net radiational energy flow from the beginning.

Of course it is possible to decompose the net radiational current in two contributions of opposite directions, but such a decomposition causes more confusion than clarity. In the same way we could decompose all of the infrared radiation around us. What insight would we gain by saying that in every point much energy is flowing to the right and just as much to the left? We could also decompose other than energy currents in this way. We would come to the conclusion that in a wire, even when not connected to an electric energy source there would be a huge electric current in one direction and an equal current in the other. Similar to the decomposition of the (infrared) photon current in the atmosphere would also be a decomposition of the phonon current in a solid material in which a heat current is flowing. In this case we would have to describe a heat current, in a glass rod for instance, in the following way: There is a phonon current in one direction and another one that is almost, but not exactly equal to the first one flowing in the opposite direction.

In our balances only the net energy current that goes with the radiation matters. This is 51 W/m^2 , i.e. much less than the energy current due to convection.

3. Another conceptual simplification results, when the difference between „sensible“ and „latent“ heat is not too much emphasized (in our approach one would call it „sensible and latent entropy), but if instead one would accentuate what they have in common. Both transports have in common that they do not need an temperature gradient as a driving force. Thus, both are convective currents. At the surface of the Earth entropy is supplied to the air and the water that it contains together and this entropy is transported upwards convectively.

8.3 Stratifications in gases

The state of an indifferent stratification is a state of equilibrium. However, this equilibrium is of a particular kind. Obviously it is not a thermal equilibrium: the temperature is not constant as a function of the altitude. Furthermore, it is not a gravito-chemical equilibrium, since the gravito-chemical potential is also not constant. Actually, we have to do here with a „gravito-thermo-chemical“ equilibrium: the sum of the three quantities gravitational potential, temperature and chemical potential (with appropriate constant factors) is constant, i.e. independent of the altitude.

for the purpose of the teaching at school this explanation is probably too complicated. We therefore have tried to make the result of the stirring or mixing plausible by using another argument, that we have not proven. We believe that it appears reasonable, that a state will establish in which the entropy per amount of substance is constant.



Solutions to problems

1. Heat

1.1 Entropy and temperature

1. In room A there is more entropy than in room B because the mass and the temperature of the air is greater.

2. In each of the cups 1/6 of the coffee is poured; 3/6 of it remain in the pot. Correspondingly each cup contains

$$S_{\text{cup}} = 3900/6 \text{ Ct} = 650 \text{ Ct}$$

and in the pot remains

$$S_{\text{pot}} = 3900/2 \text{ Ct} = 1950 \text{ Ct.}$$

1.2 Temperature difference as a driving force for an entropy current

1. (a) Because the temperature of the hotplate is higher than that of the pot; (b) because the temperature of the coaster is lower than that of the pot. (c) Initially, the temperature of the table is higher than that of the bottle. That is why entropy goes from the table to the bottle. Thereby the temperature of the table decreases.

2. Entropy flows from the big block to the small one. The final temperature is nearer to 120 °C than to 10°C.

3. (a) No, we would need more information about the sizes of the blocks and about the temperatures. (b) The temperature of the small block decreases, that of the big one increases. Entropy flows from the small to the big one. (c) At the end the big block has more entropy than the small one.

1.3 The heat pump

2. The heat pump conveys entropy from the interior of the fridge outside. The same amount is coming back again by the open door. (The analogue electric situation would be a short circuit.)

1.4 The absolute temperature

1. 273.15 K; 298.15 K; 373.15 K; 90.15 K; 77.35 K; 4.25 K; 0 K.

2. - 259.2 °C; - 252.8 °C; - 218.8 °C; - 210 °C.

1.5 Entropy production

1. Light enters into the lamp, which comes from the surrounding objects. Little by little, the battery is charged.

2. Water vapor and carbon dioxide enter the exhaust pipe. The radiator cools the air that passes by, whereby the cooling water gets warmer. The motor emits a fuel-air mixture which is separated into air and fuel in the carburetor. Fresh air leaves the motor at the air filter. Little by little the fuel pump fills the fuel tank.

3. Warm air gets to the brakes. The brakes cool down and the bicycle speeds up in the reverse direction.

1.7 Entropy conductivity

1. (a) The walls should be thick. (b) The total surface of the exterior walls should be small, i.e. the house should not have a contorted exterior surface. (c) The walls should be made of a material with a high thermal resistance.

2. (a) The wall of the radiator is thin. (b) The surface (i.e. the cross sectional area of the heat conductor) is large. (c) The material, the radiator is made of, is a good heat conductor.

Other devices where good heat conduction is required: die radiator of a car, the cylinder head of air cooled combustion engines, the heat exchanger at the back-side of the fridge.

3.

$$I_S = \sigma_s \frac{A}{d} \Delta T$$

$$\Delta T = 10 \text{ K}$$

extensive quantity	window	walls
$\sigma_s \left(\frac{\text{Ct}}{\text{s} \cdot \text{m} \cdot \text{K}} \right)$	0.0027	0.00013
A (m ²)	0.75	120
d (m)	1.00	120
I _S (Ct/s)	1.50	120

It is worth installing windows that are well insulating and not too large.

4.

(a)

$$\frac{I_S}{A} = \sigma_s \cdot \frac{\Delta T}{d} = 0.003 \frac{\text{Ct}}{\text{s} \cdot \text{m} \cdot \text{K}} \cdot \frac{20 \text{ K}}{0.12 \text{ m}} = 0.5 \frac{\text{Ct}}{\text{m}^2 \cdot \text{s}}$$

(b)

$$\sigma_{S, \text{brick}} \cdot \frac{A}{d_{\text{brick}}} \cdot \Delta T_{\text{brick}} = \sigma_{S, \text{styr}} \cdot \frac{A}{d_{\text{styr}}} \cdot \Delta T_{\text{styr}}$$

$$\frac{0.003}{0.12} \cdot \Delta T_{\text{brick}} = \frac{0.00013}{0.05} \cdot \Delta T_{\text{styr}}$$

$$\Delta T_{\text{styr}} = \frac{0.05 \cdot 0.003}{0.00013 \cdot 0.12} \cdot \Delta T_{\text{brick}}$$

$$= 9.62 \cdot \Delta T_{\text{brick}}$$

With

$$\Delta T_{\text{brick}} = 20 \text{ K} - \Delta T_{\text{styr}}$$

we get

$$\Delta T_{\text{brick}} = 20 \text{ K} - 9.62 \cdot \Delta T_{\text{brick}}$$

$$\Delta T_{\text{brick}} (1 + 9.62) = 20 \text{ K}$$

$$\Delta T_{\text{brick}} = 1.88 \text{ K}$$

(c)

$$\frac{I_S}{A} = 0.003 \cdot \frac{1.88 \text{ Ct}}{0.12 \text{ m}^2 \cdot \text{s}} = 0.047 \frac{\text{Ct}}{\text{m}^2 \cdot \text{s}}$$

≈ 1/10 f the value without insulation

1.8 Entropy transfer by convection

1. Through the walls, closed windows and closed doors losses by conduction; through gaps under the doors and untight windows by convection.

2. From the flame of the combustion of the gasoline convectively to the inner side of the wall of the cylinder; from there conductively to the channels for the cooling water; with the cooling water convectively from the motor to the radiator; through the wall of the pipes of the radiator conductively outside; convectively away with the ambient air.

3. From the cooling water for the motor to a kind of radiator. From here to air that is blown into the cabin.

1.9 Entropy as an energy carrier

1. See Fig. 1

2. See Fig. 2

3. The entropy is produced when the building blocks hurt the ground. The energy comes from the gravitational field.

4. See Fig. 3. The inverse is a thermal power plant (heated with coal, gas or nuclear fuel).

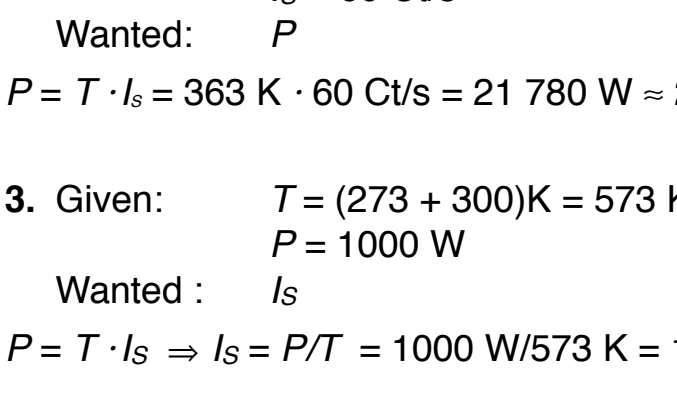


Fig. 1
Section 1.9, exercise 1

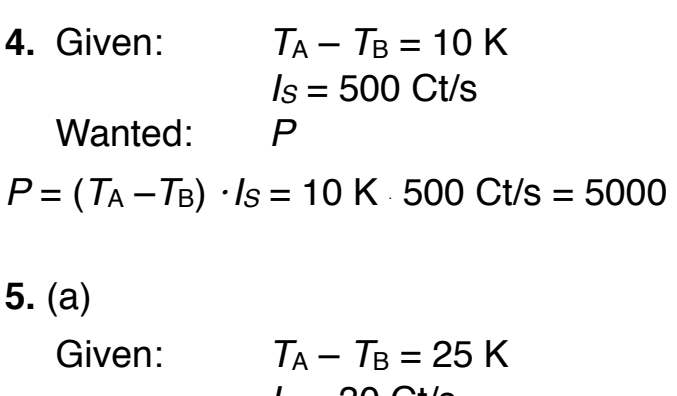


Fig. 2
Section 1.9, exercise 2

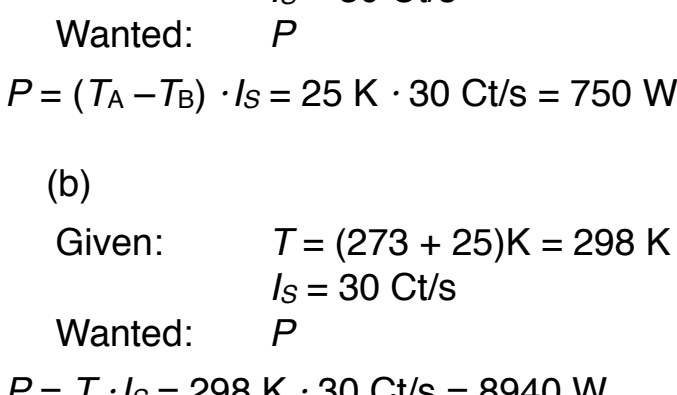


Fig. 3
Section 1.9, exercise 4

1.10 The relationship between energy current and entropy current

1. Given: $T = (273 + 20)\text{K} = 293 \text{ K}$

$I_S = 35 \text{ Ct/s}$

Wanted: P

$$P = T \cdot I_S = 293 \text{ K} \cdot 35 \text{ Ct/s} = 10\,255 \text{ W} \approx 10 \text{ kW}$$

2. Given: $T = (273 + 90)\text{K} = 363 \text{ K}$

$I_S = 60 \text{ Ct/s}$

Wanted: P

$$P = T \cdot I_S = 363 \text{ K} \cdot 60 \text{ Ct/s} = 21\,780 \text{ W} \approx 22 \text{ kW}$$

3. Given: $T = (273 + 300)\text{K} = 573 \text{ K}$

$P = 1000 \text{ W}$

Wanted: I_S

$$P = T \cdot I_S \Rightarrow I_S = P/T = 1000 \text{ W}/573 \text{ K} = 1,7 \text{ Ct/s}$$

4. Given: $T_A - T_B = 10 \text{ K}$

$I_S = 500 \text{ Ct/s}$

Wanted: P

$$P = (T_A - T_B) \cdot I_S = 10 \text{ K} \cdot 500 \text{ Ct/s} = 5000 \text{ W}$$

5. (a)

Given: $T_A - T_B = 25 \text{ K}$

$I_S = 30 \text{ Ct/s}$

Wanted: P

$$P = (T_A - T_B) \cdot I_S = 25 \text{ K} \cdot 30 \text{ Ct/s} = 750 \text{ W}$$

(b)

Given: $T = (273 + 25)\text{K} = 298 \text{ K}$

$I_S = 30 \text{ Ct/s}$

Wanted: P

$$P = T \cdot I_S = 298 \text{ K} \cdot 30 \text{ Ct/s} = 8940 \text{ W}$$

1.11 Entropy production by entropy currents

1. Given: $P = 20 \text{ kW}$

$T_1 = (273 - 5)\text{K} = 368 \text{ K}$

$T_2 = (273 + 20)\text{K} = 293 \text{ K}$

Wanted: $I_{S2}, I_{S1}, I_{S \text{ produced}}$

$$P = T \cdot I_S \Rightarrow I_S = P/T$$

(a) $I_{S2} = P/T_2 = 20 \text{ kW}/293 \text{ K} = 68.3 \text{ Ct/s}$

(b) $I_{S1} = P/T_1 = 20 \text{ kW}/368 \text{ K} = 74.6 \text{ Ct/s}$

(c) $I_{S \text{ produced}} = I_{S1} - I_{S2} = (74.6 - 68.3) \text{ Ct/s} = 6.3 \text{ Ct/s}$

2. Given: $P = 1000 \text{ W}$

$T_1 = 373 \text{ K}$

$T_2 = 1000 \text{ K}$

Wanted: $I_{S2}, I_{S1}, I_{S1} - I_{S2}$

$$P = T \cdot I_S \Rightarrow I_S = P/T$$

(a) $I_{S2} = P/T_2 = 1000 \text{ W}/1000 \text{ K} = 1 \text{ Ct/s}$

(b) $I_{S1} = P/T_1 = 1000 \text{ W}/373 \text{ K} = 2.7 \text{ Ct/s}$

(c) $I_{S1} - I_{S2} = 1.7 \text{ Ct/s}$

1.13 Entropy sources for heat engines

1. Given: $\vartheta_A = 150 \text{ }^\circ\text{C}$ $\vartheta_B = 50 \text{ }^\circ\text{C}$

$I_S = 100 \text{ Ct/s}$

Wanted: P

$$T_A - T_B = 100 \text{ K}$$

$$P = (T_A - T_B) \cdot I_S = 100 \text{ K} \cdot 100 \text{ Ct/s} = 10 \text{ kW}$$

2. Given: $P = 1000 \text{ MW}$

$T_A = 750 \text{ K}$

$T_B = 310 \text{ K}$

Wanted: I_S, P_B

$$T_A - T_B = 750 \text{ K} - 310 \text{ K} = 440 \text{ K}$$

$$P = (T_A - T_B) \cdot I_S \Rightarrow I_S = P/(T_A - T_B)$$

$$I_S = 1000 \text{ MW}/440 \text{ K} = 2.27 \text{ MCt/s}$$

$$P_B = T_B \cdot I_S = 310 \text{ K} \cdot 2.27 \text{ MCt/s} = 704 \text{ MW}$$

3. One could run a heat engine

- between the water of a cold lake in the mountains and the warmer water of a lake in the valley;

- between the water of the ocean at the equator and the water of the polar ocean;

- between an iceberg, that has been hauled with a tugboat to the equator and the warm water of the ocean;

- between the Earth and outer space (that has a temperature of 2.7 K);

- between a volcano and the water of the sea;

- between the water at the surface of the ocean and the water in deeper layers.

1.14 Energy loss and efficiency

1. Given: $P_{\text{in}} = 20 \text{ kW}$ $P_{\text{out}} = 18 \text{ kW}$

Wanted: V

$$P_V = (20 - 18) \text{ kW} = 2 \text{ kW}$$

$$V = (P_V/P_{\text{in}}) \cdot 100 \% = (2 \text{ kW}/20 \text{ kW}) \cdot 100 \% = 10 \%$$

2. Given: $V = 40 \%$

$P_{\text{in}} = 10 \text{ W}$

$T = 300 \text{ K}$

Wanted: P_{out}, I_S

$$V = (P_V/P_{\text{in}}) \cdot 100 \%$$

$$\Rightarrow P_V = (V/100 \%) \cdot P_{\text{in}} = (40/100) \cdot 10 \text{ W} = 4 \text{ W}$$

$$P_{\text{out}} = P_{\text{in}} - P_V = 10 \text{ W} - 4 \text{ W} = 6 \text{ W}$$

$$I_{S \text{ produced}} = P_V/T = 4 \text{ W}/300 \text{ K} = 0.013 \text{ Ct/s}$$

3. Given: $V = 8 \%$

$P_{\text{out}} = 46 \text{ kW}$

$T = 300 \text{ K}$

Wanted: $P_{\text{in}}, P_V, I_{S \text{ produced}}$

$$46 \text{ kW corresponds to } 92 \% \text{ of } P_{\text{in}}$$

$$P_{\text{in}}/P_{\text{out}} = P_{\text{in}}/46 \text{ kW} = 100 \%/92 \%$$

$$P_{\text{in}} = 46 \text{ kW} \cdot (100/92) = 50 \text{ kW}$$

$$P_V = P_{\text{in}} - P_{\text{out}} = (50 - 46) \text{ kW} = 4 \text{ kW}$$

$$I_{S \text{ produced}} = P_V/T = 4000 \text{ W}/300 \text{ K} = 13.3 \text{ Ct/s}$$

1.15 the efficiency of combustion processes

$$\eta_{\text{nuclear}} = (550 \text{ K} - 300 \text{ K})/550 \text{ K} = 0.45$$

$$\eta_{\text{coal}} = (800 \text{ K} - 300 \text{ K})/800 \text{ K} = 0.63$$

1.16 Measuring entropy

2. From the body whose entropy is to be measured an entropy current flows to the measuring instrument. In this process additional entropy is created. This entropy falsifies the result of the measurement.

1.17 The heatability

$$1. \alpha(100\text{K}) = 7.0 \frac{\text{mol} \cdot \text{K}}{\text{Ct}} \quad \alpha(300\text{K}) = 14.6 \frac{\text{mol} \cdot \text{K}}{\text{Ct}}$$

$$2. \alpha(P) = 0 \frac{\text{mol} \cdot \text{K}}{\text{Ct}}$$

All the entropy is used in the melting process as long as this process lasts.

$$3. \Delta T = \alpha \frac{\Delta S}{n} = \alpha \frac{\Delta S}{m} \cdot M$$

Here $M = m/n$ is the mass per amount of substance.

	Cu	Al
$\alpha \left(\frac{\text{mol} \cdot \text{K}}{\text{Ct}} \right)$	12.11	11.93
$M \left(\frac{\text{g}}{\text{mol}} \right)$	63.5	27
$\Delta T(\text{K})$	61.5	25.8

The copper heats up more than aluminum by a factor of $61.5/25.8 \approx 2.4$.

$$4. \Delta S = \frac{n}{\alpha} \Delta T = \frac{m}{M \cdot \alpha} \Delta T = \frac{100 \text{ kg} \cdot 80 \text{ Ct}}{18(\text{g/mol}) \cdot 3.89 \text{ mol} \cdot \text{K}/\text{Ct}} = 114 \text{ kCt}$$

1.18 The relation between energy supply and temperature change

1. Given: $m = 0.5 \text{ kg}$

$P = 500 \text{ W} = 500 \text{ J/s}$

$\vartheta_1 = 25 \text{ }^\circ\text{C}$

$\vartheta_2 = 100 \text{ }^\circ\text{C}$

Wanted: t

$$\Delta E = cm \Delta T$$

$$P = \Delta E/t \Rightarrow t = \Delta E/P$$

$$\Rightarrow t = c m \Delta T / P = 4180 \text{ J}/(\text{kg} \cdot \text{K}) \cdot 0.5 \text{ kg} \cdot 75 \text{ K}/500 \text{ (J/s)} = 313.5 \text{ s} \approx 5 \text{ min}$$

2. Given: Water current strength = 0.1 kg/s

$t = 5 \text{ min} = 300 \text{ s}$

$\vartheta_1 = 15 \text{ }^\circ\text{C}$

$\vartheta_2 = 45 \text{ }^\circ\text{C}$

Wanted: ΔE

$$m = 0.1 \text{ kg/s} \cdot 300 \text{ s} = 30 \text{ kg}$$

$$\Delta E = cm \Delta T = 4180 \text{ J}/(\text{kg} \cdot \text{K}) \cdot 30 \text{ kg} \cdot 30 \text{ K} = 3.76 \text{ MJ}$$

$$3. c = \frac{T}{M \cdot \alpha} = \frac{300 \text{ K}}{63.5(\text{g/mol}) \cdot 12.11 \text{ mol} \cdot \text{K}/\text{Ct}} = 390 \text{ Ct/kg}$$

4. Water: $3 \text{ }^\circ\text{C}/\text{m}^3 \Rightarrow 0.3 \text{ Cent/kg}$

$$\text{Energy: } 10 \text{ Cent/kWh} = 2.8 \text{ Cent/MJ}$$

We suppose that upon heating the temperature of the water increases by 20 K.

$$\frac{\Delta E$$

2. Gases

2.1 Gases and condensed substances

1. A tire smoothes the unevenness of the road out, since it is compressed by a small bump. Because water is incompressible a tire that is filled with water could not do the job.

2. the air within the balloon expands at constant pressure. Thereby its density decreases. (Part of the air leaves the balloon.) Thus, the density of the air in the balloon is smaller than the ambient air. The air in the balloon, together with the balloon moves upwards.

2.2 Qualitative relations between S , T , V and p

1. (a) Bubbles are rising. Air is getting out of the bottle. At the beginning the cool air in the bottle had normal pressure. After the bottle has been closed, entropy is supplied to the air at $V = \text{const}$. According to line 2 of Table 2.1 the pressure must increase. When opening the bottle air is flowing out so that the pressure can equalize.

(b) Water is entering the bottle. The heated air had normal pressure. When the bottle was still closed entropy is withdrawn at $V = \text{const}$. Thereby, pressure goes down and becomes smaller than the normal ambient pressure. Upon opening the bottle, water is pushed inside, so that the pressure can equalize.

2. In both gases the temperature increases. The temperature increase is greater in the gas with $V = \text{const}$. The process with $p = \text{const}$ can be executed in two steps. First one supplies entropy at $V = \text{const}$. Thereby the temperature as well as the pressure increase. This is the process that has been realized with the other gas. Next, one lets expand the gas until the pressure is back to its initial value. Thereby the temperature decreases (inverse of the process of line 4 in Table 2.1).

3. According to line 2 of Table 2.1 the temperature increases if entropy is supplied at $v = \text{const}$. Line 4 tells us, that the temperature decreases if the volume increases at $S = \text{const}$. Thus entropy supply and volume increase have opposite effects on the temperature. If the volume increase is sufficiently great, it will „win“ over the the entropy supply: the temperature will decrease.

2.3 Quantitative relations between S , T , V and p

1. From the ideal gas equation we get

$$\rho = \frac{Mp}{RT}$$

where ρ is the mass density and M the mass per amount of substance. Since the pressure is the same inside and outside of the balloon, we get

$$\frac{\rho_{\text{balloon}}}{\rho_{\text{outside}}} = \frac{T_{\text{outside}}}{T_{\text{balloon}}} = \frac{293}{473} = 0.619$$

Thus, the density of the air within the balloon becomes

$$\rho_{\text{balloon}} = 0.619 \cdot 1.29 \frac{\text{kg}}{\text{m}^3} = 0.799 \frac{\text{kg}}{\text{m}^3}$$

The volume is

$$V = \frac{4}{3} \pi r^3 = 905 \text{ m}^3$$

The mass turns out to be

$$m = (\rho_{\text{outside}} - \rho_{\text{balloon}}) \cdot V \\ = (1.29 - 0.799) \frac{\text{kg}}{\text{m}^3} \cdot 905 \text{ m}^3 = 444 \text{ kg}$$

2.

(a) 10 Liters

(b) Since the right hand side nRT of the ideal gas equation is constant during the expansion the volume increases by the same factor as the pressure decreases. We thus get 900 liters.

(c) $m = M \cdot n = 2 \frac{\text{g}}{\text{mol}} \cdot 36.4 \text{ mol} = 72.8 \text{ g} = 0.0728 \text{ kg}$

(d) $n = \frac{pV}{RT} = \frac{9 \cdot 10^6 \text{ Pa} \cdot 10^{-2} \text{ m}^3}{8.3 \text{ Ct/mol} \cdot 298 \text{ K}} = 36.4 \text{ mol}$

3.

$T_n = 15 \text{ }^\circ\text{C} = 288 \text{ K}$

$T_h = 40 \text{ }^\circ\text{C} = 313 \text{ K}$

$p_0 = 1 \text{ bar}$

$V_0 = 1 \text{ Liter}$

(a) Heating the air in the closed bottle from T_n to T_h ($n = \text{const}$ and $V_0 = \text{const}$). The pressure increases from p_0 to p_h :

$$\frac{p_h}{p_0} = \frac{T_h}{T_n}$$

$$p_h = p_0 \cdot \frac{T_h}{T_n} = p_0 \cdot \frac{313}{288} = 1.087 \text{ bar}$$

While letting the air flow out at the constant temperature T_h and $V_0 = \text{const}$ the amount of substance decreases from n_0 to n_{end} :

$$\frac{p_h}{p_0} = \frac{n_0}{n_{\text{end}}}$$

$$\Delta n = n_0 - n_{\text{end}} = n_0 - \frac{n_0}{1.087} = 0.080 n_0$$

8 % of the air will flow out.

(b) Cooling the air in the closed bottle from T_h to T_n ($n = \text{const}$ and $V_0 = \text{const}$). The pressure decreases from $p_0 = 1 \text{ bar}$ to p_n :

$$\frac{p_n}{p_0} = \frac{T_n}{T_h}$$

$$p_n = p_0 \cdot \frac{T_n}{T_h} = p_0 \cdot \frac{288}{313} = 0.920 \text{ bar}$$

While letting the air flow in at the constant temperature T_n and $n = \text{const}$ the volume decreases from V_0 to V_{end} :

$$p_n \cdot V_0 = p_0 \cdot V_{\text{end}}$$

$$\Delta V = V_0 - V_{\text{end}} = V_0 - 0.92 V_0 = 0.08 V_0 = 80 \text{ ml}$$

4. $V = 8 \text{ liters}$, $p_0 = 3 \text{ bar}$, $T_0 = 293 \text{ K}$, $T_1 = 353 \text{ K}$

While heating we have $V = \text{const}$ and $n = \text{const}$, thus we get:

$$\frac{p_1}{p_0} = \frac{T_1}{T_0}$$

$$p_1 = p_0 \cdot \frac{T_1}{T_0} = 3 \text{ bar} \cdot \frac{353 \text{ K}}{293 \text{ K}} = 3.61 \text{ bar}$$

5. $T_0 = 288 \text{ K}$, $T_1 = 295 \text{ K}$

While heating we have $p = \text{const}$ and $V = \text{const}$, thus we get:

$$n_0 \cdot T_0 = n_1 \cdot T_1$$

$$n_1 = n_0 \cdot \frac{T_0}{T_1} = n_0 \cdot \frac{288 \text{ K}}{295 \text{ K}} = 0.976 n_0$$

$$\Delta n = n_0 - n_1 = n_0(1 - 0.976) = 0.024 n_0$$

$$\frac{\Delta n}{n_0} = 0.024$$

2.4 % of the air flow out.

6. $p_1 = 15 \text{ bar}$, $p_2 = 2 \text{ bar}$, $T_1 = 673 \text{ K}$, $\beta = 0.30$

$$T_2 = T_1 \cdot \left(\frac{p_2}{p_1} \right)^{\frac{\beta}{\beta+1}}$$

$$= 673 \text{ K} \cdot 0.133^{0.23} = 673 \text{ K} \cdot 0.629 = 423 \text{ K}$$

$\vartheta = 150 \text{ }^\circ\text{C}$

7. We consider a process with $n = \text{const}$. In one state pressure, volume and temperature are p , V and T , in the other they are p_0 , V_0 and T_0 . We have

$$p \cdot V = n \cdot R \cdot T \text{ and } p_0 \cdot V_0 = n \cdot R \cdot T_0, \text{ respectively.}$$

Dividing the first equation by the second we obtain:

$$\frac{p}{p_0} \cdot \frac{V}{V_0} = \frac{T}{T_0}$$

Thereof we get

$$\frac{V}{V_0} = \frac{p}{p_0} \cdot \frac{T_0}{T}$$

By replacing in equation (1) we get

$$\frac{T}{T_0} = \left(\frac{p}{p_0} \right)^\beta \cdot \left(\frac{T_0}{T} \right)^\beta$$

or

$$\frac{T}{T_0} = \left(\frac{p}{p_0} \right)^{\frac{\beta}{\beta+1}}$$

2.4 The operating mode of thermal engines

1. We consider Fig. 2.11. The piston would hardly move when the pressure decreases. Therefore it would not release energy. Moreover, the temperature of the liquid would not decrease. Thus, after the pressure decrease there would come out just as much entropy as previously had flown in.

2. $p_1 = 1 \text{ bar}$, $p_2 = 30 \text{ bar}$, $T_1 = 300 \text{ K}$, $\beta = 0.40$

$$T_2 = T_1 \cdot \left(\frac{p_2}{p_1} \right)^{\frac{\beta}{\beta+1}} = 300 \text{ K} \cdot 30^{0.29} = 804 \text{ K}$$

$\vartheta = 1077 \text{ }^\circ\text{C}$

3. When the piston arrives at the right end, there is still vapor under high pressure in the cylinder. When the outlet is opened it expands into the ambient atmosphere. It could have supplied energy when expanding. However, this energy is lost.

2.5 Why the air above the Earth's surface gets cooler with altitude

$p_1 = 100 \text{ kPa}$, $p_2 = 80 \text{ kPa}$, $T_1 = 293 \text{ K}$, $\beta = 0.40$

$$T_2 = T_1 \cdot \left(\frac{p_2}{p_1} \right)^{\frac{\beta}{\beta+1}} = 293 \text{ K} \cdot 0.8^{0.29} = 274 \text{ K}$$

$\vartheta = 1 \text{ }^\circ\text{C}$

2.6 Thermal convection

1. Water is heated in a pot on the stove. The entropy enters the water from below. It is released at the side walls and the upper free water surface, where it is used for the evaporation.

2. Because of the high temperature the gases of the flame (mainly the nitrogen of the air) have a lower density than the ambient air. Therefore they flow upwards and entrain the solid particles that also are in the flame.

2.7 Irreversible processes

1. It corresponds to the inverse process of that of line 3: The volume increases, the temperature remains constant, the pressure decreases and the entropy increases.

2. For the air at the left side, that is not too near to the opening, the expansion is reversible. Thus its temperature decreases, see inverse of the process of line 4 of Table 2.1. The air at the right side, that is far enough from the opening, experiences a reversible compression, its temperature increases. After waiting a while thermal equilibrium will have established. Now both temperatures are equal.

3. This is the *Gibbs paradox*. The quantum nature of matter makes that a substance cannot transform continuously into another substance. Obviously, this answer cannot be expected from the students. But it would be quite good if they understood the problem.

3. Substance-like quantities

3.1 Substance-like quantities

1. Distance, electric tension
2. Time, frequency, angle, electric resistance

3.3 Energy currents

Equation (1a): into a thermal power plant, out of a heat pump

Equation (1b): energy transport by means of a bicycle chain, drive belt

Equation (1c): energy transport by means of an electric cable

3.4 Mass and gravitational potential

1. $I_m = 900\,000 \text{ kg/s}$, $h_2 - h_1 = 11 \text{ m}$

$$P = g \cdot (h_2 - h_1) \cdot I_m$$
$$= 10 \text{ N/kg} \cdot 11 \text{ m} \cdot 900\,000 \text{ kg/s} = 99 \text{ MW}$$

2. $I_m = 10\,000 \text{ kg/s}$, $h_2 - h_1 = 300 \text{ m}$

$$P = g \cdot (h_2 - h_1) \cdot I_m$$
$$= 10 \text{ N/kg} \cdot 300 \text{ m} \cdot 10\,000 \text{ kg/s} = 30 \text{ MW}$$

4. Amount of substance and chemical potential

4.1 The amount of substance

1.

H₂O: $m/n = 18.01494 \text{ g/mol} \approx 0.018 \text{ kg/mol}$

O₂: $m/n = 31.998 \text{ g/mol} \approx 0.032 \text{ kg/mol}$

CO₂: $m/n = 44.009 \text{ g/mol} \approx 0.044 \text{ kg/mol}$

Ag₂S: $m/n = 247.804 \text{ g/mol} \approx 0.248 \text{ kg/mol}$

Pb(NO₃)₂: $m/n = 331.198 \text{ g/mol} \approx 0.331 \text{ kg/mol}$

C₁₂H₂₂O₁₁: $m/n = 342.296 \text{ g/mol} \approx 0.342 \text{ kg/mol}$

2. $m/n = 0.342 \text{ kg/mol}$

$n = 0.29 \text{ mol}$

3. The mass of 1 l of water is 1 kg.

$m/n = 0.018 \text{ kg/mol}$

$$n = \frac{1 \text{ kg}}{0.018 \text{ kg}} \cdot \text{mol} = 55.5 \text{ mol}$$

4. For propane we have $m/n = 0.044 \text{ kg/mol}$

$$n = \frac{m}{0.044 \text{ kg/mol}} = \frac{12 \text{ kg}}{0.044 \text{ kg}} \cdot \text{mol} = 273 \text{ mol}$$

4.5 The relation between chemical potential and pressure

1. $p_1 = 60 \text{ bar}$, $p_2 = 110 \text{ bar}$

$$\mu(p_2) - \mu(p_1) = RT \ln\left(\frac{p_2}{p_1}\right) = 2.5 \text{ kG} \cdot \ln\frac{110}{60} = 1.515 \text{ kG}$$

2. $p_{\text{nitrogen}} = 0.78 \text{ bar}$, $p_{\text{oxygen}} = 0.21 \text{ bar}$,

$p_0 = 1 \text{ bar}$, $\mu(p_0) = 0 \text{ kG}$

$$\mu_{\text{nitrogen}} = RT \ln\left(\frac{p_{\text{nitrogen}}}{p_0}\right) = 2.5 \text{ kG} \cdot \ln 0.78 = -0.62 \text{ kG}$$

$$\mu_{\text{oxygen}} = RT \ln\left(\frac{p_{\text{oxygen}}}{p_0}\right) = 2.5 \text{ kG} \cdot \ln 0.21 = -3.90 \text{ kG}$$

3. The pressure approaches 0 bar; the chemical potential assumes greater and greater negative values or, mathematically spoken, approaches infinity.

4. (a) Since the mass density of alcohol is lower than that of water the alcohol floats on the water.

(b) Little by little the alcohol distributes evenly within the water. It follows the drive of the chemical potential difference. At the end chemical equilibrium has established.

5.

(a) $c_0 = 1 \text{ mol/l}$, $\mu(c_0) = 8.36 \text{ kG}$, $\mu(c) = 0 \text{ kG}$

$$\mu(c_0) - \mu(c) = RT \ln\left(\frac{c_0}{c}\right) \Rightarrow 8.36 \text{ kG} = 2.5 \text{ kG} \cdot \ln\left(\frac{c_0}{c}\right)$$

$$\Rightarrow \ln\left(\frac{c_0}{c}\right) = \frac{8.36}{2.5} = 3.34 \Rightarrow \frac{c_0}{c} = e^{3.34} = 28.22$$

$$\Rightarrow c = 0.035 \text{ mol/l}$$

(b) Calculation of the chemical potential in the gaseous phase:

$p = 150 \text{ kPa}$, $p_0 = 100 \text{ kPa}$, $\mu(p_0) = 0 \text{ kG}$

$$\mu(p) = \mu(p_0) + RT \ln\left(\frac{p}{p_0}\right) = 0 \text{ kG} + 2.5 \text{ kG} \ln 1.5 = 1.014 \text{ kG}$$

The gaseous phase and the dissolved phase are in chemical equilibrium, i.e. their chemical potentials are equal: $\mu(c_1) = 1.014 \text{ kG}$

$$\mu(c_0) - \mu(c_1) = RT \ln\left(\frac{c_0}{c_1}\right)$$

$$\Rightarrow 8.36 \text{ kG} - 1.014 \text{ kG} = 2.5 \text{ kG} \cdot \ln\left(\frac{c_0}{c_1}\right)$$

$$\Rightarrow \ln\left(\frac{c_0}{c_1}\right) = \frac{8.36 - 1.014}{2.5} = 2.94 \Rightarrow \frac{c_0}{c_1} = e^{2.94} = 18.9$$

$$\Rightarrow c_1 = 0.053 \text{ mol/l}$$

6. $p_1 = 2 \text{ bar}$, $p_2 = 6 \text{ bar}$

Because of the chemical equilibrium between the gaseous and the dissolved phase we have:

$$\mu(p_1) = \mu(c_1) \text{ and } \mu(p_2) = \mu(c_2)$$

From

$$\mu(p_2) - \mu(p_1) = RT \ln\left(\frac{p_2}{p_1}\right) = \mu(c_2) - \mu(c_1) = RT \ln\left(\frac{c_2}{c_1}\right)$$

we get

$$\frac{p_2}{p_1} = \frac{c_2}{c_1} = \frac{6}{2} = 3$$

5. Phase transitions

5.3 The heat balance for phase transitions

1. From Fig. 5.7 we get:

At 100 °C

– 1 kg liquid water contains 4600 Ct;

– 1 kg gaseous water contains 10 700 Ct.

$$f = 10700/4600 \approx 2.3$$

Gaseous water contains 2.3 times as much entropy as liquid water.

2. Given: $m = 10 \text{ kg}$

$$\vartheta = 90 \text{ °C}$$

Wanted: ΔS

$$\Delta S = \Delta S_{\text{heating}} + \Delta S_{\text{vaporizing}}$$

$$\Delta E = T \Delta S_{\text{heating}} = c \cdot m \cdot \Delta T$$

$$\Delta S_{\text{heating}} = c \cdot m \cdot \frac{\Delta T}{T} = 4180 \frac{\text{Ct}}{\text{kg}} \cdot 10 \text{ kg} \cdot \frac{10 \text{ K}}{368 \text{ K}} = 1136 \text{ Ct}$$

$$\Delta S_{\text{vaporizing}} = 10 \cdot 6048 \text{ Ct} = 60 480 \text{ Ct}$$

$$\Delta S = (1136 + 60 480) \text{ Ct} = 61 616 \text{ Ct}$$

3. Given: $\Delta S_{\text{melting}} = 6000 \text{ Ct}$

Wanted: m

In order to melt 1 kg of ice 1222 Ct are needed. With 6000 Ct

$$(6000/1222) \text{ kg} = 4.91 \text{ kg ice can be melted.}$$

4. Given: Temperature change from 20 °C to 0 °C

Mass of the mineral water = 0.25 kg

Wanted: Mass of the molten ice

$$\Delta E = T \Delta S_{\text{cooling}} = c \cdot m \cdot \Delta T$$

$$\Delta S_{\text{cooling}} = c \cdot m \cdot \frac{\Delta T}{T} = 4180 \frac{\text{Ct}}{\text{kg}} \cdot 0.25 \text{ kg} \cdot \frac{20 \text{ K}}{283 \text{ K}} = 74 \text{ Ct}$$

74 Ct must be extracted from the mineral water. They are needed to melt the ice.

To melt 1 kg of ice 1222 Ct are needed.

With 74 Ct $(74/1222) \text{ kg} = 0.061 \text{ kg ice can be molten.}$

5. Given: Temperature increase from 15 °C to 60 °C

Masse of the milk = 0,2 kg

Wanted: Mass of the vapor

$$\Delta E = T \Delta S_{\text{cooling}} = c \cdot m \cdot \Delta T$$

$$\Delta S_{\text{heating}} = c \cdot m \cdot \frac{\Delta T}{T} = 4180 \frac{\text{Ct}}{\text{kg}} \cdot 0.2 \text{ kg} \cdot \frac{45 \text{ K}}{310 \text{ K}} = 121 \text{ Ct}$$

121 Ct must be supplied to the milk. They are taken from the condensing vapor.

When 1 kg of vapor condenses 6048 Ct are supplied. To get 121 Ct, $(121/6048) \text{ kg} = 0.020 \text{ kg} \approx 20 \text{ g of vapor must condense.}$

5.4 Phase transitions in a thought experiment

1. See fig. 4

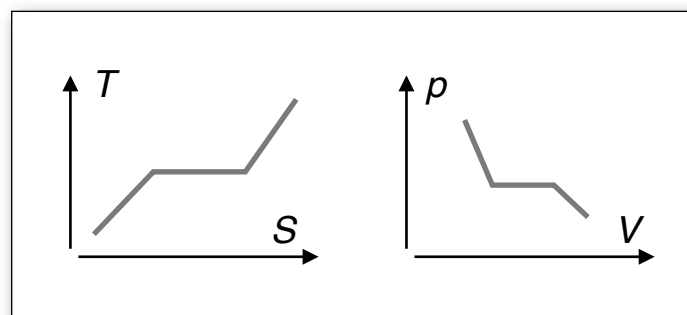


Fig. 4
Section 5.4, exercise 1

2. As long as the phase transition has not started, one feels that the pressure increases. During the phase transition the pressure no longer increases although the piston moves downwards. When finally all the gas has condensed, suddenly one feels a great resistance: The piston can no longer be moved down.

5.5 Partial pressure and atmospheric humidity

$T = 293 \text{ K}$, relative humidity = 40 %

(a) Saturation pressure = 2337 Pa

Partial pressure = 2337 Pa · 0.4 = 935 Pa

(b) We assume the total pressure to be 100 000 Pa.

$$\frac{\text{Partial pressure of the water}}{\text{total pressure}} = \frac{935}{100 000} = 0.00935 = \frac{n_{\text{water}}}{n_{\text{total}}}$$

0.935 % of the molecules are water molecules.

(c)

$$n = \frac{p \cdot V}{R \cdot T} = \frac{935 \text{ Pa} \cdot 1 \text{ m}^3}{8.31 \text{ Ct/mol} \cdot 293 \text{ K}} = 0.384 \text{ mol}$$

$$m = M \cdot n = 18 \text{ g/mol} \cdot 0.384 \text{ mol} = 6.9 \text{ g}$$

5.6 Boiling and evaporating

From Fig. 5.4 we read that the equilibrium pressure for the transition liquid → gaseous at 80 °C is 0.5 bar. Thus, when the atmospheric pressure is 0.5 bar the boiling temperature of the water is 80 °C.

5.8 Everything evaporates, everything dissolves

Mainly the same question as in exercise 5b of section 4.45.

(a) Calculation of the chemical potential of CO₂:

$p = 0.3 \text{ kPa}$, $p_0 = 100 \text{ kPa}$, $\mu(p_0) = 0 \text{ kG}$

$$\mu(p) = \mu(p_0) + RT \ln \left(\frac{p}{p_0} \right) = 0 \text{ kG} + 2.5 \text{ kG} \ln 0.003 = -14.5 \text{ kG}$$

(b) The gaseous and the dissolved phase are in chemical equilibrium, i.e. their chemical potentials are equal:

$$\mu(c_1) = -14.5 \text{ kG.}$$

We use the value from Table 5.1:

$$\mu(\infty) = \mu(1 \text{ mol/l}) = 8.36 \text{ k}$$

$$\mu(c_0) - \mu(c_1) = RT \ln \left(\frac{c_0}{c_1} \right)$$

$$\Rightarrow 8.36 \text{ kG} + 14.5 \text{ kG} = 2.5 \text{ kG} \cdot \ln \left(\frac{c_0}{c_1} \right)$$

$$\Rightarrow \ln \left(\frac{c_0}{c_1} \right) = \frac{8.36 + 14.5}{2.5} = 9.14 \Rightarrow \frac{c_0}{c_1} = e^{9.14} = 9321$$

$$c_1 = 0.000107 \text{ mol/l}$$

6. Coupled currents

6.1 One substance – several energy carriers

1. Mass, amount of substance, momentum, entropy

Only the gravitational potential is different at the input and the output. The energy carrier that is responsible for the energy transfer is the mass.

2. Undershot water wheel; the energy carrier is momentum. Mass and entropy don't play a role in the energy transfer because the gravitational potential and the temperature are the same at the input and at the output.

3. Because temperature and gravitational potential are the same at the inlet and the outlet.

4. With $M = m/n$ and $m = \rho \cdot V$ we get

$$n = \frac{m}{M} = \frac{\rho}{M} \cdot V$$

It follows:

$$I_n = \frac{\rho}{M} I_v$$

and

$$\mu = \frac{V}{n} \cdot p = \frac{M}{\rho} \cdot p$$

Thus, the energy consumption of the pump becomes:

$$P = (\mu - \mu_0) \cdot I_n = \frac{M}{\rho} (p - p_0) \cdot \frac{\rho}{M} I_v = (p - p_0) \cdot I_v$$

$$= 4.5 \text{ bar} \cdot 12 \text{ l/s} = 450 \text{ kPa} \cdot 0.012 \text{ m}^3/\text{s} = 5400 \text{ J/s}$$

6.3 Equilibria

1. Sugar in tea. The chemical potential of the sugar is independent of the position.

2. An electric conductor that is made of a single material is charged electrically. The charge distributes in such a way that the electric potential is the same everywhere.

3. Water is poured into a container. The surface arranges in such a way that the water has the same gravitational potential at every place at the surface.

4. Condition for an equilibrium of motion:

$$v(x) = \text{const}$$

Water in the middle of a river where the water is flowing quietly without turbulence. Neighboring water portions have the same velocity.

5. From

$$\mu = \frac{V}{n} \cdot p$$

$$p = \rho \cdot g \cdot h$$

we get

$$\mu(h) = \frac{V}{n} \cdot \frac{m}{V} \cdot g \cdot h = M \cdot g \cdot h = M \cdot \psi(h)$$

and

$$\mu(h) + M \cdot \psi(h) = 0$$

6. See Fig. 5

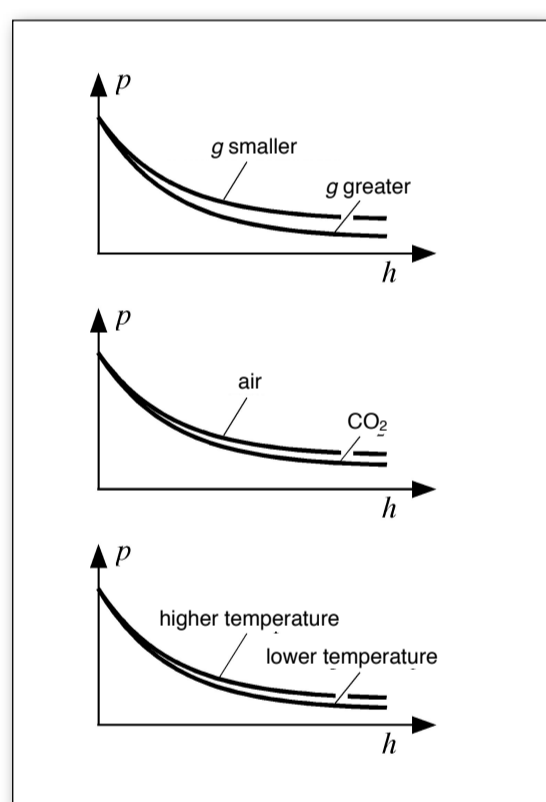


Fig. 5
Section 6.3, exercise 6

7. $p_0 = 1 \text{ bar}$, $g = 10 \text{ N/kg}$, $T = 293 \text{ K}$,

$R = 8.3 \text{ Ct/mol}$

$M = 0.8 \cdot M_{\text{nitrogen}} + 0.2 \cdot M_{\text{oxygen}} = 0.0288 \text{ kg/mol}$

$$p = p_0 \cdot e^{\frac{M \cdot g \cdot h}{R \cdot T}} = \dots = 0.62 \text{ bar}$$

8. The two metals are in electrochemical equilibrium:

$$\mu_1 - F \cdot \phi_1 = \mu_2 - F \cdot \phi_2$$

$$\Delta\phi = \frac{\Delta\mu}{F}$$

$$\text{Na-Cu: } \Delta\phi = \frac{198}{96\,500} \text{ V} = 2.1 \text{ mV}$$

$$\text{Pb-Al: } \Delta\phi = \frac{24.2}{96\,500} \text{ V} = 0.25 \text{ mV}$$

9. If the lamp is connected with copper wires a new contact aluminum-copper is generated where a second contact voltage appears. Both contact voltages compensate mutually; the net voltage is 0V.

6.4 Thermo-electric energy exchangers

One chooses the conductors in such a way that one of them has positive, the other one negative charge carriers. If both charge carriers are dragged by the entropy current from hot to cold, both contribute to the net drive of the charge in the same sense of rotation.

6.5 Electro-chemical energy exchangers

In order to obtain $\Delta\phi = 1 \text{ V}$ it must be

$$\Delta\mu = \Delta\phi \cdot z \cdot F = 1 \text{ V} \cdot 96\,500 \text{ C/mol} = 96.5 \text{ kJ}$$

From that we get:

$$96.5 \text{ kJ} = 2.5 \text{ kJ} \cdot \ln \frac{p}{p_0}$$

$$\ln \frac{p}{p_0} = 38.6$$

$$\frac{p}{p_0} = e^{38.6} = 5.8 \cdot 10^{16}$$

With $p_0 = 1 \text{ bar}$ we get $p = 5.8 \cdot 10^{16} \text{ bar}$, a value that is unrealistically high.

7. Thermal radiation

7.2 The light gas – thermal radiation

$$f_{\max} = 0,588 \cdot 10^{11} \cdot 500 \text{ Hz} = 29.4 \text{ THz}$$

$$\lambda = \frac{c}{f} = \frac{3 \cdot 10^8 \text{ m/s}}{29,4 \cdot 10^{12} \text{ Hz}} = 10,2 \text{ } \mu\text{m}$$

7.3 Emission and absorption, black bodies

$$1. f = 6.5 \cdot 10^{14} \text{ Hz}$$

$$T = \frac{f_{\max}}{0.588 \cdot 10^{11}} \cdot \text{K} = \frac{6.5 \cdot 10^{14}}{0.588 \cdot 10^{11}} \text{K} = 11\,050 \text{ K}$$

3. Energy current density at the surface of the sun:

$$j_{E, \text{sun}} = \sigma T^4 = 5.67 \cdot 10^{-8} \cdot 5800^4 \text{ W/m}^2 = 64 \text{ MW/m}^2$$

Surface area of the sun:

$$A = 4\pi r^2 = 4\pi \cdot 7^2 \cdot 10^{16} \text{ m}^2 = 6.16 \cdot 10^{18} \text{ m}^2$$

Energy current from the sun:

$$P_{\text{sun}} = j_{E, \text{sun}} \cdot A = 64 \text{ MW/m}^2 \cdot 6.16 \cdot 10^{18} \text{ m}^2 = 3.94 \cdot 10^{26} \text{ W}$$

From the surface of a sphere with radius

$$r_{\text{sun-Earth}} = \text{distance sun - Earth} = 1.5 \cdot 10^{11} \text{ m}$$

the Earth cuts an area of πr_{Earth}^2 . Thus, the Earth is hit by an energy current of:

$$P_{\text{Earth}} = P_{\text{sun}} \cdot \frac{\pi r_{\text{Earth}}^2}{4\pi r_{\text{sun-Earth}}^2} = 3.94 \cdot 10^{26} \text{ W} \cdot \frac{6371^2}{4 \cdot 1.5^2 \cdot 10^{22}} = 1.78 \cdot 10^{11} \text{ W}$$

7.4 Reversible and irreversible processes with light

1. See Fig. 6. the process at the right is never observed.

2. See Fig. 7. Both processes can be observed.

3. See fig. 8. Each light beam coming from the filament is scattered at the surface of the light bulb. From here the light continues in all directions. We consider one more a thin light beam of this scattered light, which points somewhere to the outside. It will be scattered once more by the shade of the lamp.

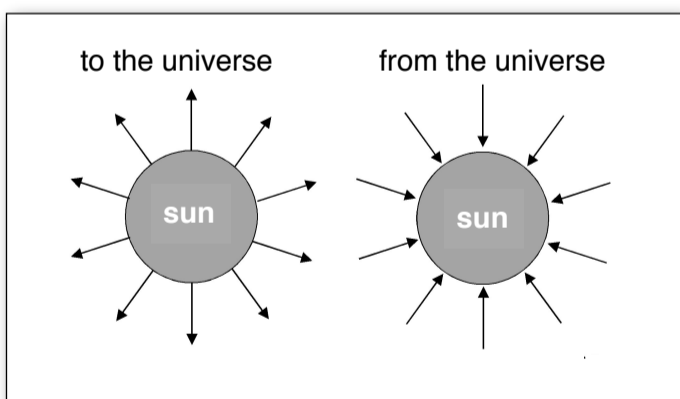


Fig. 6
Section 7.4, exercise 1

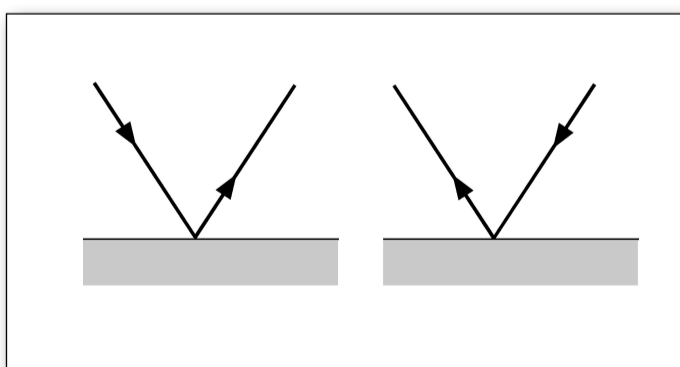


Fig. 7
Section 7.4, exercise 2

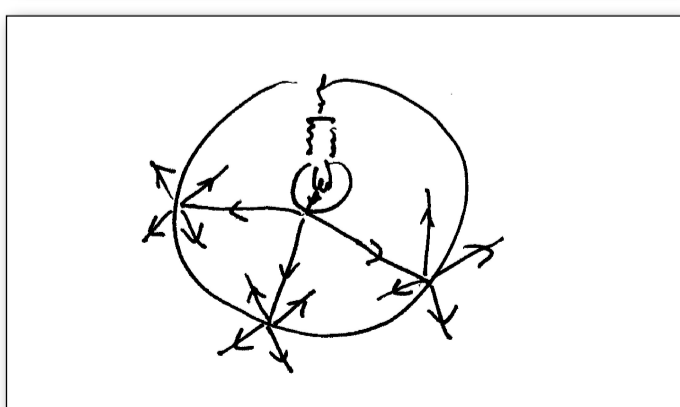


Fig. 8
Section 7.4, exercise 3

8. Entropy currents in the gas layers of planets and stars

8.1 The troposphere

1. For the current density of the energy current at the surface of the Earth coming from the sun we have:

$$j_E = 1368 \text{ W/m}^2$$

The total energy current hitting the Earth is:

$$P = \pi \cdot r_{\text{Earth}}^2 \cdot j_E$$

On the average it is distributed on the whole surface area $4\pi \cdot r_{\text{Earth}}^2$ of the Earth. Thus we also get:

$$P = 4\pi \cdot r_{\text{Earth}}^2 \cdot j_{E \text{ average}}$$

Thus we get

$$j_{E \text{ average}} = j_E/4 = 342 \text{ W/m}^2$$

From this 31 % are back-scattered directly, and we remain with:

$$0.69 \cdot 342 \text{ W/m}^2 = 236 \text{ W/m}^2$$

2. Without an atmosphere there would be no clouds, and thus no back-scattering by clouds. Perhaps there would be more ice and thus more back-scattering by the ice. We estimate $j_E = 300 \text{ W/m}^2$.

With

$$j_E = \sigma \cdot T^4 \text{ and } \sigma = 5.67 \cdot 10^{-8} \text{ W/(m}^2 \cdot \text{K}^4) \text{ we get}$$

$$T = \sqrt[4]{\frac{j_E}{\sigma}} = \sqrt[4]{\frac{300}{5.67 \cdot 10^{-8}}} \text{ K} = 270 \text{ K}$$

8.2 Stratification of liquids

1. The water with a temperature of 4 °C is at the bottom; above it there is water of higher and of lower temperature side by side; at the same height there is water of 3 °C and of 5 °C, above this there is water of 2°C and of 6 °C and so on.

2. Heating from above makes that the density becomes higher above than below. Therefore convection sets in. That means that the water is „stirred“, whereby the stratification becomes indifferent. Heating from below makes that the density becomes higher below and lower above. The stratification becomes stable, the temperature below will be higher than that above.

8.3 Stratification of gases

Way of the entropy: From the radiator to the air that is passing by. With this air upwards and to the sides. The entropy goes from the air to the cooler objects around. The temperature is higher above than below, thus the stratification is stable. The establishment of an indifferent stratification is steadily perturbed by the heating of the air at the radiator.

8.4 The convective energy transport through the troposphere

1. See also exercise 3., section 7.3.

Calculation of the energy current density j_E of the sunlight at the location of Venus:

With $j_{E, \text{sun}} = 64 \text{ MW/m}^2$ we get:

$$\frac{j_E}{j_{E, \text{sun}}} = \frac{4\pi r_{\text{sun}}^2}{4\pi r_{\text{sun-Venus}}^2}$$

$$j_E = j_{E, \text{sun}} \cdot \frac{r_{\text{sun}}^2}{r_{\text{sun-Venus}}^2} = 64 \frac{\text{MW}}{\text{m}^2} \cdot \frac{(7 \cdot 10^5 \text{ km})^2}{(1.16 \cdot 10^8 \text{ km})^2} = 2330 \text{ W/m}^2$$

$$j_{E, \text{average}} \frac{j_E}{4} = \frac{2330 \text{ W/m}^2}{4} = 583 \text{ W/m}^2$$

$$T = \sqrt[4]{\frac{j_E}{\sigma}} = \sqrt[4]{\frac{583}{5.67 \cdot 10^{-8}}} \text{ K} = 318 \text{ K}$$

We have admitted that no sunlight is directly back-scattered.

2. See also exercise 7., section 6.3

$$h = 2850 \text{ m}$$

$$p_0 = 1 \text{ bar}, M = 0.0288 \text{ kg/mol}, g = 10 \text{ N/kg},$$

$R = 8.3 \text{ Ct/mol}, T = 293 \text{ K}$. (For T we have taken 293 K as an average temperature.)

$$p = p_0 \cdot e^{\frac{M \cdot g \cdot h}{R \cdot T}} = \dots = 0.71 \text{ bar}$$

$$\beta = 0.4, T_0 = 310 \text{ K} \text{ (estimated for the equator at sea level)}$$

$$T = T_0 \cdot \left(\frac{p}{p_0} \right)^{\frac{\beta}{\beta+1}} = 310 \text{ K} \cdot 0.71^{0.286} = 281 \text{ K}$$

8.7 Heat transport in the sun and in other stars

Temperature inside the sun:

$$T_i = 15\,000\,000 \text{ K}$$

Temperature at the sun's surface:

$$T_a = 6\,000 \text{ K}$$

Energy current from the sun:

$$P = 4 \cdot 10^{26} \text{ W}$$

The energy current is the same inside and outside. Thus:

$$P = T_i \cdot I_{S,i} \Rightarrow I_{S,i} = \frac{P}{T_i}$$

$$P = T_a \cdot I_{S,a} = T_a (I_{S,i} + I_{S,erz}) \Rightarrow I_{S,erz} = \frac{P}{T_a} - I_{S,i}$$

From both equations we get:

$$I_{S,erz} = \frac{P}{T_a} - \frac{P}{T_i} = P \left(\frac{1}{T_a} - \frac{1}{T_i} \right)$$

Since $T_i \gg T_a$, $1/T_i$ is negligible against $1/T_a$. Practically all the entropy that arrives at the surface, is created on the way from the center to the surface:

$$I_{S,erz} \approx \frac{P}{T_a} = \frac{4 \cdot 10^{26} \text{ W}}{6 \cdot 10^3 \text{ K}} = 6,7 \cdot 10^{22} \text{ Ct/s}$$
