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Teaching relativity: A paradigm change

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The teaching of relativity usually starts with kinematics: The invariance of the speed of light, clock synchronization, time dilatation and length contraction, the relativity of simultaneity, Lorentz transformation and the Minkowski diagram. The change of the reference frame is a central topic. Only afterwards problems of relativistic dynamics are discussed. Such an approach closely follows the historical development of the Special Theory of Relativity.

We believe that this access to relativity is unnecessarily complicated, and unsuitable for beginners. We present the basics of a teaching approach in which the initial postulate of relativity is the identity of energy and relativistic mass. Reference frame changes are largely avoided.

Keywords: Special relativity, additional postulate, reference frame change, relativistic dynamics.

1. Introduction

Relativity, and we mean for the moment only special relativity, is more than 100 years old, but still has not found its place in school. Just compare: Faraday-Maxwell electromagnetism, which is certainly not simpler than special relativity, would still not be included in the curricula 120 years after its creation, i.e. in 1980. There are several reasons for this deficiency. Here we want to discuss only one of them: Relativity is still taught today as it has originated historically. One starts with a very special relativistic effect, and works through with great effort to the more important and useful general statements. It is as if one enters a splendid palace not by the beautiful main portal, but by some shabby servants' entrance.

We describe the basics of a course on special relativity. The concept has been tested and is used at numerous secondary schools: parts of it already in the lower secondary school, the complete program in the upper secondary school. The course is part of the *Karlsruhe Physics Course* [1]. It can be downloaded from the Internet in various languages. A bilingual English-Chinese version was published recently.[2]

We do not describe the details of this course. We are merely presenting some ideas that underlie its development. Some of the topics we address are also discussed in articles of the column *Historical burdens on physics* [3].

We justify our paradigm change in Section 2. We choose a different "entrance" to relativity. We substantiate our choice in section 2.1. In sections 2.2 and 2.3 we begin with a critical discussion of two popular topics: the reference frame change, and the role of the observer. Section 2.4 is about naming. How is the word mass used and how do we want to use it. Section 2.5 deals with the way to write Einstein's famous equation $E = mc^2$. We are thus prepared for our main topic, which will be treated in section 3: relativistic dynamics.

2. Paradigm change

2.1. *Additional postulate of the special theory of relativity*

The laws of the special theory of relativity, or special relativity for short, are obtained from those of classical mechanics by adding one extra postulate. Traditionally and for historical reasons, the choice was made for the invariance of the speed of light upon a change of the reference frame.

Once one has become aware that this choice as a starting point is only one of several possibilities, one discovers that completely new perspectives arise for the development of a teaching concept. We have decided to introduce the mass-energy equivalence as an additional postulate instead of the invariance of the speed of light.

With this choice, we arrive more quickly at that part of special relativity that we consider being the most important one, namely relativistic dynamics.

The traditional choice of the invariance of the speed of light has a rather incidental cause: when special relativity came into being, light was the only known system that behaved relativistically. Einstein's work – both his famous publication of 1905 *Zur Elektrodynamik bewegter Körper*[4] and his textbook *Grundzüge der Relativitätstheorie*[5] – begins with a detailed, and one can say somewhat tiring part on relativistic kinematics.

One might imagine what the course of history would have been if the first relativistic observation had been that a cup of hot coffee is heavier than a cup of cold coffee (or that the corresponding observation had been made with particles in an accelerator). The presentation of the theory of relativity in our textbooks would certainly be very different from what it is actually.

Of course, the mass-energy equivalence is not supported by our everyday experience (neither is the invariance of the speed of light). But one can make it easily plausible and discuss its consequences even in beginners' classes. It leads to surprising and at first unbelievable statements; but it does not lead to the cognitive conflicts one has to deal with in the traditional approach to relativistic kinematics, which questions our basic convictions about space and time.

2.2. *Reference frames and reference frame changes*

In the traditional approach to special relativity, the following topics are dealt with before relativistic dynamics is addressed:

The invariance of the speed of light
Clock synchronization

The relativity of simultaneity
Time dilatation and length contraction
Velocity addition
Lorentz transformation
Minkowski diagram

The problem with such an approach is that one begins with the most confusing part of the theory: the relationship between space and time.

Certainly, students can learn a lot of physics by analyzing the same process in different reference frames. But we should not forget that we are dealing with beginners, and it is better to stick to the old rule: Choose a suitable reference frame right at the beginning, i.e. a reference frame in which the description of your problem becomes as simple as possible, and don't change it anymore.

And above all, don't change the reference frame in the middle of dealing with your problem (as is usually done when discussing the twin paradox). By the way, in classical mechanics and electromagnetism, too, one can create the greatest confusion if one chooses the reference frame improperly or if one changes it in the middle of the discussion.

This is why our decision was not to make reference frame changes the main topic of our lessons and to avoid them as far as possible. Above all, the impression should not be created that special relativity is essentially a theory of reference frame changes – an impression that some presentations certainly arouse. Even the name relativity gives that impression.

2.3. *The observer*

Closely related to the question of the choice of the reference frame is the problem of the so-called observer. The observer seems to be particularly important in two areas of physics: in quantum physics (where the observer always appears as the one making a "measurement") and in the theory of relativity.

An observation is always made from a certain perspective. It thereby emphasizes something that does not play a particular role in the phenomenon to be described.

We believe the observation should not be in the foreground as long as the understanding of a process is the objective. This is especially true when teaching at school, i.e. beginners.

It is true that we get all the information about the world by observing and measuring. But the idea we form of the world is quite different from what we observe. So, if we wanted to explain the shape of the earth to someone, we would certainly not start with the shadow of the obelisk in Alexandria, but simply say: The earth is a sphere.

In our opinion when teaching physics we should primarily give a picture of what nature is like – not how it is perceived by an observer.

2.4. *The use of the term mass*

Mass is a physical quantity that until not so long ago did not cause any problems. It was known for which properties it is a measure.

With Einstein's special theory of relativity, this only changed insofar as the mass of a body became dependent on its velocity, temperature and other variables. It was no longer a quantity that had a characteristic value for a body or a particle.

Thus, a body, a particle, a field, or any other structure, has a mass that depends, among other things, on its velocity. The value that the mass assumes when the centre of mass of the particle or body is at rest is called its rest mass (symbol m_0). Even more appropriate would actually be the less common term *proper mass*, because when the centre of mass is at rest, this does not mean that the parts or particles of the system are at rest.

It is that simple, or, unfortunately, one must say: it could be that simple.

For there is an area of physics in which another use of the term mass has established itself: Particle physics. A particle has a well-defined rest mass. The rest mass is characteristic of the particle species. Among the various other parameters, such as electric charge, spin, lepton number, etc., it is considered the main characteristic. It seems to constitute the identity of the particle. For this property, a compact, plausible name was needed, and particle physicists simply called it mass. Thus, in particle physics, the term mass refers to only part of the quantity that describes the inertia of a particle.

However, this custom also spread beyond particle physics, and this results in several misunderstandings and ambiguities. What is to be understood by the mass of a macroscopic body that is at rest? Is it the mass that would be measured with a (very accurate) scale, or is it the sum of the (rest) masses of the particles that constitute the body? This is a question that particle physicists probably don't ask, but we teachers do.

We have therefore decided to use the term mass (symbol m) exclusively for the quantity that measures gravity and inertia, no matter what kind of object is considered and in what state it is. Thus, a hot cup of coffee has a larger mass than the same coffee when it is cold. A photon has a mass and a magnetic field has a mass (a liter of magnetic field near a neutron star has a mass of some hundred grams).

By the way, if one follows this use of the term mass, it makes no sense to say that mass is a "form of energy" or that mass can be converted into energy.

2.5. *The identity of mass and energy*

First, let us look at the term *mass-energy equivalence*. It is a pity that a simple fact is expressed so unclearly. The word equivalence is certainly not wrong, but why not say directly: Mass and energy are the same physical quantity.

If one were to ask someone who has never seen the equation $E = mc^2$ to express this fact in a formula, he would probably write something like this:

$$E = k \cdot m \ . \tag{1}$$

The factor k tells us how the units joule and kilogram are converted into each other. As the definitions of the units kilogram and joule are independent of the choice of the reference frame, k is a universal constant.

Its value is obtained by a measurement. One finds

$$k = 9 \cdot 10^{16} \mathrm{J/kg} \ . \tag{2}$$

But what is wrong with writing

$$E = mc^2 \ ? \tag{3}$$

Every student learns in mathematics that a linear relationship between the variables x and y is written as

$$y = a \cdot x \ . \tag{4}$$

On the right side first the factor of proportionality a, and second the independent variable. The unbiased student might interpret the famous equation (3) this way: The energy is proportional to the square of the speed of light – and not: energy and mass are the same physical quantity. One might object: This can easily be explained to the students. Of course it can. But doesn't the statement become clearer if one writes $E = k \cdot m$? Would the iconic character of equation (3) survive if it were formulated in this way?

3. The laws of dynamics

In the *Karlsruhe Physics Course*[1], the extensive quantities energy, momentum, electric charge and entropy are introduced as basic quantities. Especially momentum and entropy have a very direct and vivid interpretation. Momentum is a measure of the "amount of motion", that is, what one would colloquially call "impetus" or "drive". Entropy measures almost perfectly what would colloquially be called the amount of heat (not to be confused with the rather difficult concept of heat that has established itself in physics).

Therefore, in the context of relativity, it is natural to ask in the first place for the dependence of different quantities on momentum. Momentum is our independent variable. We give momentum to a body or particle and ask how it reacts to it: How does its mass (= energy) behave? What happens to its velocity? In other words: We ask for the functions $E(p)$ and $v(p)$.

3.1. *The energy momentum relationship*

To derive $E(p)$, we take over as much as possible from non-relativistic physics. In addition, we only require the identity of mass and energy, i.e. we assume the validity of equation (1). We start with the change dE of the energy, that results from a change of the momentum dp.

$$\mathrm{d}E = v\mathrm{d}p \ . \tag{5}$$

With $p = m \cdot v$ we obtain

$$\mathrm{d}E = \frac{p}{m}\mathrm{d}p \ . \tag{6}$$

Replacing m with E/k, and reordering returns

$$E\mathrm{d}E = kp\mathrm{d}p \ . \tag{7}$$

We thus obtain

$$\mathrm{d}E^2 = k\mathrm{d}p^2 \tag{8}$$

and

$$E^2 = kp^2 + C \ , \tag{9}$$

where C is the constant of integration.

The value of C can easily be determined, because for $p = 0$ the energy E assumes the value of the rest energy E_0. Thus, C must be equal to ${E_0}^2$. We therefore get

$$E^2 = {E_0}^2 + kp^2 \tag{10}$$

and for the sought-after relationship between energy and momentum we get:

$$E(p) = \sqrt{{E_0}^2 + kp^2} \ . \tag{11}$$

The red line in Figure 1 shows the graphic representation of relation (11). Two limiting cases are of particular interest.

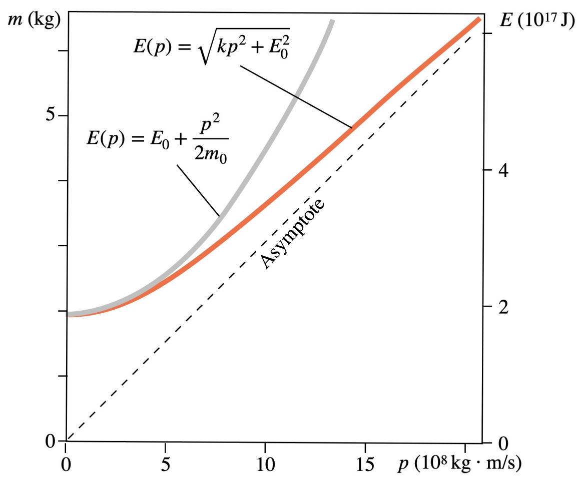

Fig. 1. Relationship between mass/energy and momentum (red line). For large values of the momentum the curve approaches the asymptote (dashed line), for small values the classical quadratic relation (grey line).

For small momentum values, equation (11) changes to

$$E(p) = E_0 + \frac{kp^2}{2E_0} = E_0 + \frac{p^2}{2m_0} \ . \qquad (12)$$

We obtain the classical kinetic energy, increased by the rest energy (grey line in Figure 1).

If the momentum is very large, so that ${E_0}^2$ can be neglected in comparison with kp^2, equation (11) turns into

$$E(p) = \sqrt{k}p \ , \qquad (13)$$

see the dashed line in Figure 1. For bodies whose rest mass is 0 kg, equation (13) applies for all values of the momentum, not only for large values (Figure 2). Thus, in the highly relativistic limiting case, energy and momentum are proportional to each other. This shows that there is a similarity between these quantities, which becomes even clearer when we solve equation (11) according to ${E_0}^2$

$${E_0}^2 = E^2 - kp^2 \ . \qquad (14)$$

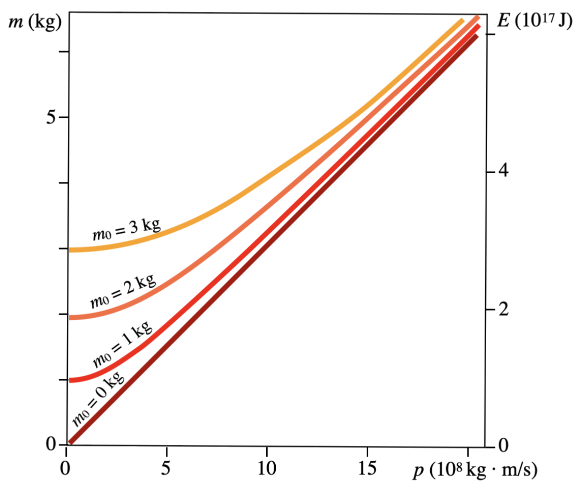

Fig. 2. Energy momentum relationship for four different rest masses. For photons (rest mass zero) the relation is linear.

We thus have the rule: If the momentum of a body changes, its energy changes in such a way that the difference $E^2 - kp^2$ retains its value. This value is the square of the rest energy E_0.

3.2. *The velocity momentum relationship*

Now our second question: How does the velocity of a body depend on its momentum? We solve $p = mv$ for v, then apply equations (1) and (11) and obtain

$$v(p) = \frac{p}{m} = \frac{kp}{E} = \frac{kp}{\sqrt{{E_0}^2 + kp^2}} \ . \tag{15}$$

If we replace the rest energy with the rest mass we get

$$v(p) = \frac{kp}{\sqrt{k^2{m_0}^2 + kp^2}} \ . \tag{16}$$

Figure 3 shows the dependence of the velocity on the momentum for different rest-masses. From equation (16) follows that the velocity of a body approaches a terminal value as the momentum increases. It is

$$\lim_{p\to\infty} v(p) = \lim_{p\to\infty} \frac{kp}{\sqrt{k^2{m_0}^2 + kp^2}} = \sqrt{k} \ . \tag{17}$$

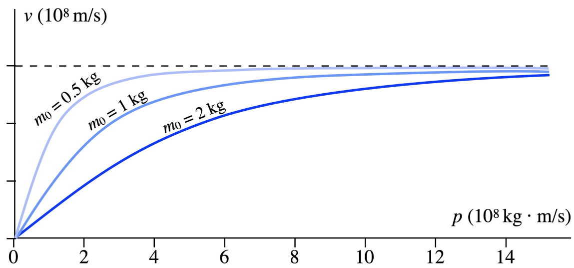

Fig. 3. Dependence of the velocity on the momentum for different rest masses.

Up to now, k only played the role of a conversion factor, but now it gets a physical meaning. Its value is the square of the terminal speed. Since k is a universal constant, its square root, i.e. the terminal speed, is also a universal constant. The terminal speed is the same for all bodies and particles and is independent of the reference frame.

This can be seen in Figure 3. The diagram also shows: the smaller the rest mass of a body is, the "faster" it approaches the terminal speed.

Let us come back to equation (16). We see: If one supplies momentum to a body, its velocity initially increases linearly with the momentum, while its mass almost does not change. This is the Newtonian limiting case. When its momentum has become very large, its velocity no longer changes, but its mass increases.

But what is the value of k and thus the value of the terminal speed? So far, nothing has been said about it. The answer to this question can only be obtained by a measurement. There are several ways to do that: Either one increases the momentum of a particle until its velocity no longer changes (in a particle accelerator) and then measures its velocity, or one measures the velocity of photons, i.e. particles of rest mass zero. Photons always move with the terminal speed.

Because of the great importance of the terminal speed, one gives it its own symbol

$$c := \sqrt{k}\ . \tag{18}$$

The measurement results in

$$c = 3 \cdot 10^8 \mathrm{m/s} \tag{19}$$

and therefore

$$k = 9 \cdot 10^{16} \mathrm{J/kg}\ . \tag{20}$$

The constant c is also called speed of light. But our derivation shows that light does not play a particular role in special relativity. That is why we prefer to call c terminal speed.

3.3. *Mass and inertia*

From classical physics we are used to consider mass as a measure of inertia. Let us first clarify what is meaningfully understood by inertia.

To determine the inertia of an object, we supply a certain amount of momentum to the object and we look at the resulting change in velocity. The more momentum $\mathrm{d}p$ is needed to achieve a desired change in velocity $\mathrm{d}v$, the greater the inertia.

Therefore we can define the inertia as

$$T := \frac{\mathrm{d}p}{\mathrm{d}v}\ . \tag{21}$$

We first consider a classical motion, i.e. a motion with $v \ll c$. We know the $p - v$ relationship to be

$$p = m \cdot v\ . \tag{22}$$

This results in

$$T = m\ , \tag{23}$$

which is no surprise.

If however the movement is relativistic, i.e. if no longer $v \ll c$, things become more complicated. From equation (15) we obtain

$$p(v) = \frac{m_0 v}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{24}$$

and

$$T(v) = \frac{\mathrm{d}p}{\mathrm{d}v} = \frac{m_0}{\left(1 - \frac{v^2}{c^2}\right)^{\frac{3}{2}}}\ . \tag{25}$$

The inertia now depends on the velocity. It can no longer be described by a single number. By the way, it is also not identical with the so-called relativistic mass.

We know a similar behavior from other contexts. The current-voltage relationship of an ohmic resistor can be characterized by a single number, its resistance. In general, however, the resistive behavior of an electrical component cannot be characterized by a single number. What we need is the $U - I$ characteristic. The situation is like that of inertia. In general, one cannot say that the mass is a measure for the inertia of a body. Rather, the inertial behavior of a body is characterized by a characteristic curve, equation (25). Sometimes the quantity defined by equation (25) is called the longitudinal mass. We think this is rather clumsy. The simple facts are thereby somewhat obscured.

4. Conclusion

The development of a teaching concept for the school, in our case for the secondary school, is a balancing act.

On one hand, teaching at school differs fundamentally from popular science presentations. The latter can limit themselves to showcasing the spectacular, the impressive and the surprising of the scientific results – one can almost say: to exhibit them like objects in a museum.

School teaching has to meet other requirements. The statements must be logically coherent. They have to fit into the previous teaching and form a foundation for the future teaching, for example, at the university.

On the other hand, we must make sure that we do not treat high-school students like university students, that we do not overburden them. Let us not forget: One can calculate and prove without generating understanding.

It should also be borne in mind that most high school students a priori have no particular interest in physics.

We have tried to develop a course under these constraints. We would like to emphasize once again that the above remarks do not represent the content of our course. We have presented only what we believe is different in our approach from that of other textbooks.

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