Two ways to slower aging - or just one?

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In the context of teaching relativity, one encounters two situations in which two observers age at different rates. Usually, these effects are treated as if they were independent of each other. Often one is referred to as special-relativistic, the other as general-relativistic. We show that both are special cases of an effect which follows naturally from the fact that space and time form a unity. They exist in a flat space-time and therefore neither of them deserves the label general relativistic.

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1. Introduction

When teaching relativistic physics, two situations or experiments are usually addressed in which two observers, comparing their clocks, discover that one has aged more than the other, two "clock effects". Let us briefly recall them. Our observers are Alice and Bob.

The first experiment is as follows: Alice and Bob adjust their watches. Alice makes a long journey, moving at constant speed. A comparison of the watches after Alice's return shows that more time has passed for Bob than for Alice. Bob has aged more than Alice.

In the second experiment Alice and Bob start in a skyscraper on a floor half way up. They adjust their watches. Then Bob goes up and Alice goes down, and they stay there for a while. Finally, they return to their starting point. Comparing their watches again, they find that more time has passed for Bob than for Alice; he has aged more. Since in the following we repeatedly refer to the two clock effects, we will give them their own names: we call the first one the travel effect, the second one the skyscraper effect.

One often finds the claim that the travel effect is an effect of special relativity (SR), whereas the skyscraper effect is an effect of general relativity (GR), see for example Wikipedia under the keyword *Global Positioning System* [1] or *Satellite navigation* [2]. Also in schoolbooks one can find this assignment.

In fact, the two clock effects are only two special cases of a phenomenon which should better be formulated in general terms. In particular, the skyscraper effect has nothing to do with the curvature of space. Therefore, it is not appropriate to call it a GR effect. Actually, this conclusion could be drawn from the known literature, see for instance [3].

In Sec. 2, we suggest how to address the interlinking of space and time in the classroom. In Sec. 3 we briefly recall how to calculate the difference in aging in both cases. In Sec. 4 we show that the skyscraper effect can be transformed into a kind of travel effect by simply changing the frame of reference.

2. Spacetime in elementary education

Special relativity has consequences and makes predictions that do not correspond to our everyday experience. The students need to take note of them, and to accept them as a fact.

We suggest telling something like the following story.

Bob wants to go shopping, Alice wants to exercise on the sports field. They separate and arrange to meet again later. Before they separate, they start their pedometers and they adjust their watches. When they meet again, they compare their pedometers to see who has walked the longer distance, and they compare their watches to see who has aged more. While it is obvious that the distances traveled are most probably different, anyone would expect that the clocks indicate the same duration for their absence. But Alice and Bob look at their watches very carefully and discover that they also display different times. They know how to arrange it so that the pedometers show as big a difference as possible: One of them doesn't move at all, while the other runs at as high a speed as possible all the time. But they also want to know how to make the watches show the longest possible time and the shortest possible time. They discover the following: For the one who does not move at all (later in the lesson we specify: the one who is "free-floating") the most time passes. And someone who is moving with (almost) the terminal speed c (almost) no time passes at all.

3. Calculation of the in aging in both cases

Both the travel effect and the skyscraper effect are special cases of a phenomenon that we have described in general terms with the help of our the story in Sec. 2. Other special scenarios can also be found in the literature, such as the one where the twins experience the same acceleration for the same amount of time, yet age differently [4, 5].

What all scenarios have in common is that two clocks are adjusted, they are then made to move around somehow in different ways, thereby changing the velocities arbitrarily (and thus also the accelerations) and going up and down arbitrarily. When they are brought together again, they will display different durations. The reason for this behavior is that space and time form a unity: spacetime. It is obvious that, apart from a few exceptions, accelerations occur during the movements. These can be transformed away, but thereby a gravitational field shows up. We can also express it the other way round: If a gravitational field appears in a somehow chosen reference frame, it can be transformed away by a change of the reference frame. In any case, the existence of a gravitational field in itself is not a manifestation of GR [6].

We remind how the term "GR effect" is commonly used. It is an effect for whose description one needs the Einstein field equation, or in other words: for which the Riemann tensor is not equal to zero.

Regarding the skyscraper effect it occurs in a homogeneous gravitational field, *i.e.* in the case where the Riemann tensor is equal to zero everywhere. Therefore, it is not appropriate to call it the GR effect.

But why are these two special cases, *i.e.* the travel effect and the skyscraper effect so popular? Because in each of the two cases a reference frame can be found, in which the description and the corresponding calculation becomes particularly transparent.

For the travel effect, this is the reference frame in which Bob is at rest. In the case of the skyscraper effect, it is the frame in which the skyscraper is at rest. Let us briefly recall the calculation of the age difference ΔT in the two cases.

We start with the travel effect. Let T_0 be the duration that Bob measures: from the time of the separation from Alice until the time of her return. From Bob's point of view, Alice's clock - the moving clock - is running slower. He gets the duration T that her clock measures by means of the well-known formula for the "time dilation":

$$T = \frac{T_0}{\sqrt{1-\frac{v^2}{c^2}}}. \tag{1}$$

Here v is the constant velocity of Alice and c is the terminal velocity (speed of light). In order to facilitate the comparison of the result with that of the Skyscraper situation, we slightly transform the equation, assuming that the durations measured by Bob and Alice differ only by a small amount, *i.e.* that $\Delta T = T - T_0 \ll T$

We obtain

$$\frac{\Delta T}{T} = \frac{1}{c^2}\frac{v^2}{2}. \tag{2}$$

Now to the Skyscraper effect: It is usually presented as a relative frequency change $\Delta f/f$ of a light signal that Alice is sending to Bob [7]:

$$\frac{\Delta f}{f} = \frac{1}{c^2} g\Delta h. \tag{3}$$

Here g is the gravitational acceleration, Δh the height of our skyscraper. From this follows the relative time difference

$$\frac{\Delta T}{T} = \frac{1}{c^2} g\Delta h. \tag{4}$$

Let us compare the two results, Eqs. (2) and (4).

In the case of the travel effect, the relative time difference $\Delta T/T$ depends only on the velocity. The reason is that we had chosen the world lines of Alice and Bob in such a way that the velocity (more precisely: the velocity difference between Alice and Bob) is constant during their separation, except for the short reversal phase.

In the case of the Skyscraper effect, the relative time difference $\Delta T/T$ depends only on the height of the skyscraper. The reason is that the world lines of Alice and Bob have been chosen in such a way that the height difference (more precisely: the gravitational potential difference $g \cdot \Delta h$ between Alice and Bob) is constant during their separation, except for the short time intervals in which they ascend or descend in the building.

The calculation of $\Delta T/T$ would be more complicated in both cases, if one had chosen another, less suitable reference frame. However, the result would not change.

In the following section we will describe the skyscraper effect, despite the expected mathematical difficulties, in a reference frame in which gravitational forces no longer exist.

4. The skyscraper effect in unfamiliar reference frame

In the reference frame of the earth we have a (nearly) homogeneous gravitational field. It is responsible for many everyday phenomena. However, this gravitational field can be easily transformed away by describing the world in a free-falling reference frame. In such a reference frame, much of what we observed before now requires a new interpretation.

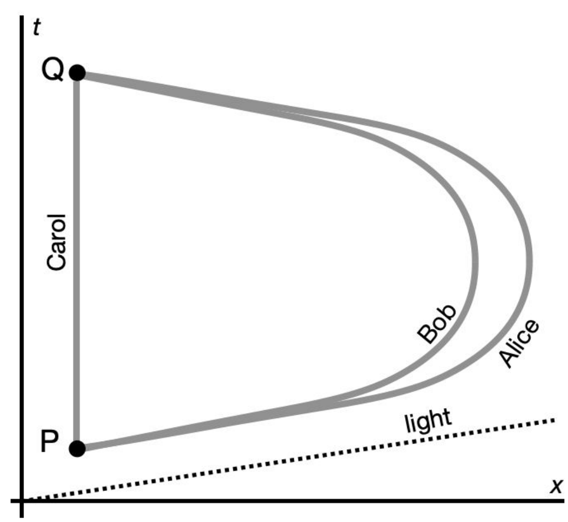

FIGURE 1. Carol's world line between the events P and Q is that of a free-falling body. In her frame of reference there are no gravitational forces. Alice's and Bob's clocks show that Bob has aged more than Alice. (The vertical axis corresponds to Carol's time, so neither to Bob's nor to Alice's).

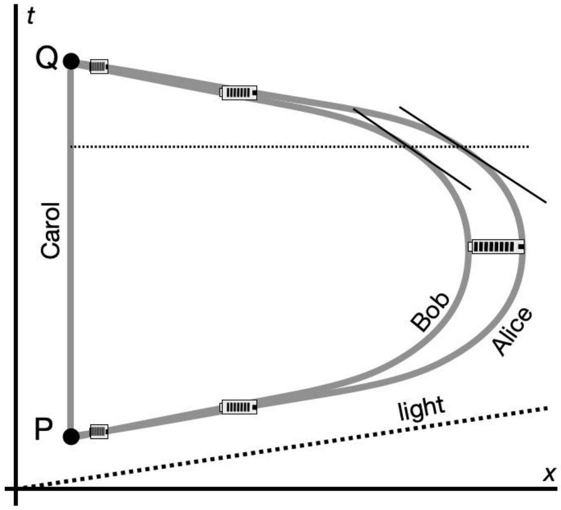

FIGURE 2. For Carol, the skyscraper is small at first. Then it gets taller and assumes its normal height at the reversal point. Thereafter it appears smaller again. For Carol, Alice's velocity is greater than Bob's at every instant, see the two tangents to the world lines.

Here is an example for such a situation that is familiar to everyone: A body hanging on a spring extends the spring due to the force of gravity acting on the body - at least this is how we earthlings describe the phenomenon. A free-falling observer sees it differently: the spring is extended, but now because one is accelerating the body by means of the spring.

Let us now describe the Skyscraper effect in such a gravity-free reference frame.

As already mentioned, Alice and Bob, after having adjusted their clocks, start at a middle floor. Then Bob goes up and Alice goes down, Fig. 1.

Besides Alice and Bob, however, we need a third participant, Carol. She also takes part in the two clock adjustments. At the very same moment when Alice and Bob separate in the middle of the skyscraper (event P in Fig. 1), Carol jumps up into the sky in such a way that she lands again just for the second clock adjustment with Alice and Bob (event Q). First, what the three of them notice when they do the second clock comparison: Again, Bob has aged more than Alice. In addition, it turns out that for Carol most time has passed, about which nobody is surprised, because her reference frame, (the reference frame in which she is at rest) was an inertial frame, and in an inertial frame the elapsed time is known to be the greatest.

We now describe the situation in Carol's reference frame. Carol's world line is a straight line. Since she is in free fall (also during the upward movement), there are no gravitational forces. But how does she describe the motion of Alice and Bob? Immediately after she jumps up, for her the skyscraper has a high speed. This means that the skyscraper is not as high as it used to be before she jumped off. This is the well-known length contraction, which is an SR effect. But then the speed of the skyscraper becomes smaller for Carol, and thus the skyscraper becomes higher, and when the speed is zero, it takes on its original height. Thereafter the skyscraper moves back again towards Carol, and does so faster and faster, whereby its height decreases again.

We can see from the two tangents in Fig. 2 that (in Carol's frame of reference) Alice moves faster than Bob at each instant of time. From this, Carol, who remembers the length contraction effect, concludes that Alice ages less than Bob. A scenario equivalent to this is described by Thorne [8].

We thus have transformed the skyscraper effect, *i.e.* the effect based on gravity, into a travel effect making the field strength of the gravitational field zero by describing the situation in Carol's reference frame. We see that the difference in aging has nothing to do with general relativity. It is just another version of the travel effect. The only difference with the travel effect is that the motion does not occur at a constant speed.

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