The transformation of a main sequence star into a red-giant star in the core-and-shell model

H. Hauptmann, F. Herrmann and K. Schmidt

Abteilung für Didaktik der Physik, Universität, 76128 Karlsruhe, Germany

Abstract

As a star is evolving from the hydrogen to the helium burning stage, i. e. from the sun-like state to the state of a red-giant star, the temperature of its core increases whereas the volume decreases. At the same time the temperature of its shell decreases while the volume of the shell increases. This behaviour of the core and of the shell is explained qualitatively by a simple model. The star is considered to be composed of two homogeneous subsystems: the active core where the heat production takes place and the shell which plays the part of a thermal insulator. Each of these subsystems is stabilized by a feed-back mechanism which is working thanks to the negative heat capacity of both systems.

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I. INTRODUCTION

In the evolution of a sun-like main sequence star to the state of a red giant, the hot core of the star is shrinking and its temperature rises from typically 15 Million to 100 Million Kelvin. At the same time the shell's radius increases by typically a factor of 25, whereas the shell's temperature decreases. Astrophysicists are able to explain these phenomena exhaustively. Their method is to simulate a star by applying field differential equations – every local variable depends on the distance from the center of the star – and to solve them numerically[1,2]. L. M. Celnikier proposed a way of obtaining the essential dependencies by applying an anlytical trick [3].

Our objective is more modest and more pretentious at the same time: We want to give purely qualitative answers to the following questions: Why does the shell blow up as the core is shrinking? Why does the core's temperature increase as that of the shell is decreasing? What are the feed-back mechanisms, which keep both the core and the shell in a stationary state? Thus, we don't ask for numbers. In order to find answers to these questions, we make use of a simple model. Indeed, the model we propose is the simplest model which is able to explain the above-mentioned effects.

A star is a very stable and long-lived system. With a lifetime of the order of billions of years the steady states of stars outrun every stationary process we know on earth. In order to maintain a steady state some feedback mechanism is needed. Since the core and the shell of a star are two subsystems with very distinct properties, a star's stability is due to two independent feedback processes.

In a recent article[4] we had considered a star as a single homogeneous system. In this case, every local variable had only one value at a time. This model was able to explain the stability of the heat production rate of a star. It was thus a model of the core alone. Accordingly, there was only one feedback loop. In the present article, the model will be completed by a second loop.

II. THE NEGATIVE HEAT CAPACITY

The fusion reaction which is responsible for the heat production, takes place only in the inner regions of a star, since only there the temperature is high enough to maintain the reaction. The material which surrounds the reaction zone can be considered to play the role of a thermal insulator. We thus can decompose the star, albeit somewhat crudely, into two sub-systems: the core or heat source of the star, and the shell or thermal insulator, Fig. 1. The radius of the core is roughly 10% of the radius of the whole star. The heat transport through the shell is mainly due to the diffusion of photons. Thus, it is driven by a temperature gradient.

Both, the core and the shell owe their stability to a property of stars which usually is referred to as negative heat capacity. A negative heat capacity of a system means that its temperature decreases when a heat flow enters it . However, the temperature decrease has to be paid for. It can be realized only if the gas expands simultaneously by a sufficient amount.

Thermodynamically speaking, a negative heat capacity can only be observed on a system which, in addition to the temperature, has one more degree of freedom. In our case this is the volume. If the volume is held constant, the system's temperature will increase when heat is supplied. On the other hand, if the temperature is held constant, heat supply results in an increase of the system's volume. There is a kind of trade-off between both variables. Actually, one can even obtain a temperature decrease upon heat supply if the volume increase is sufficient. Which of the various possiblities actually happens depends on the state equations of the system. In the case of a gravitationally confined gas, the system decides to react as described above: Upon heat supply its temperature goes down, while its volume increases strongly. When giving heat away, the opposite happens.

The reason why the system reacts this way, is that in the $1/r^2$ potential of the gravitational field an expansion is very easy. The gas behaves like a spring, whose spring constant gets weaker when expanding. The greater the diameter of the star, the easier is a further expansion. For those who are familiar with the virial theorem, the explanation is even simpler: When supplying heat energy to the star, its volume is increasing. Thereby energy is transferred into the gravitational field. According to the virial theorem, the amount of energy stored in the field is twice the amount which had been supplied to the star in the form of heat. Therefore, the internal energy does not increase but decrease.

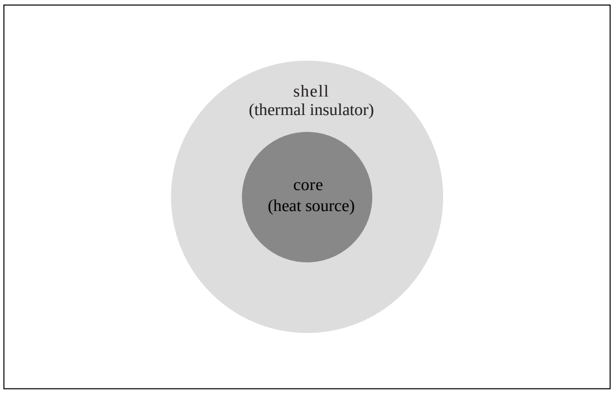

Fig. 1. A star can crudely be divided into two subsystems. The core is that part where the heat production takes place whereas the shell plays the part of a thermal insulator. Two distinct feedback mechanisms are responsible for the stability of the two subsystems.

III. THE FEEDBACK MECHANISM OF THE CORE

How does a negative heat capacity result in the stability of the fusion reaction? [4] In the normal steady state of the core the heat production rate by the fusion reaction is equal to the heat loss towards the shell. Assume now, that a perturbation is causing a decrease of the heat production rate below its “control value”. The outward heat flow will then be greater than the heat production rate and the core experiences a net energy loss. Due to the negative heat capacity, this energy loss causes the temperature of the core to increase. As a consequence the nuclear reaction rate, and thereby the heat production rate, increase. Thus, a negative feedback stabilizes the system.

IV. THE FEEDBACK MECHANISM OF THE SHELL

The stabilizing feedback mechanism of the shell must be different from that of the core, since the heat input from the core is a given quantity and cannot be controlled via the temperature of the shell. The variable to be controlled in this case is not the heat input, but the heat flow through the shell. This flow is a diffusive transport of energy by photons and depends on the transparency of the shell.

Imagine now a perturbation of the stationary state which may be caused by a change of the heat input above its former stationary value. Due to the negative heat capacity, the temperature of the shell will decrease and its radius increase. How will the transparency be influenced by such changes? To get a quantitative answer several things have to be taken into account: the influence of the temperature on the absorption and emission process and the change of the shell's geometry. However, among these dependencies the change of the geometry is dominant, and in our model star we shall refrain from considering other dependencies, Fig. 2.

If the inflation of the shell's material would take place in only one direction of space, the particles which are responsible for the absorption and emission of the photons, would spread only in the direction of the drift movement of the photons. The mean free path would increase but the thickness of the layer which has to be traversed would increase by the same factor. Thus, the transparency would not change. However, since the star is a sphere, when inflating, the average cross section of the “heat conductor” increases. As a consequence, the particles are also spreading in the traverse direction. Thereby, the gas becomes more transparent. It is the same effect as when inflating a party balloon: the balloon becomes more transparent.

As a result, an additional heat input causes the shell's transparency to increase, and a new stationary state with a greater radius of the star will establish.

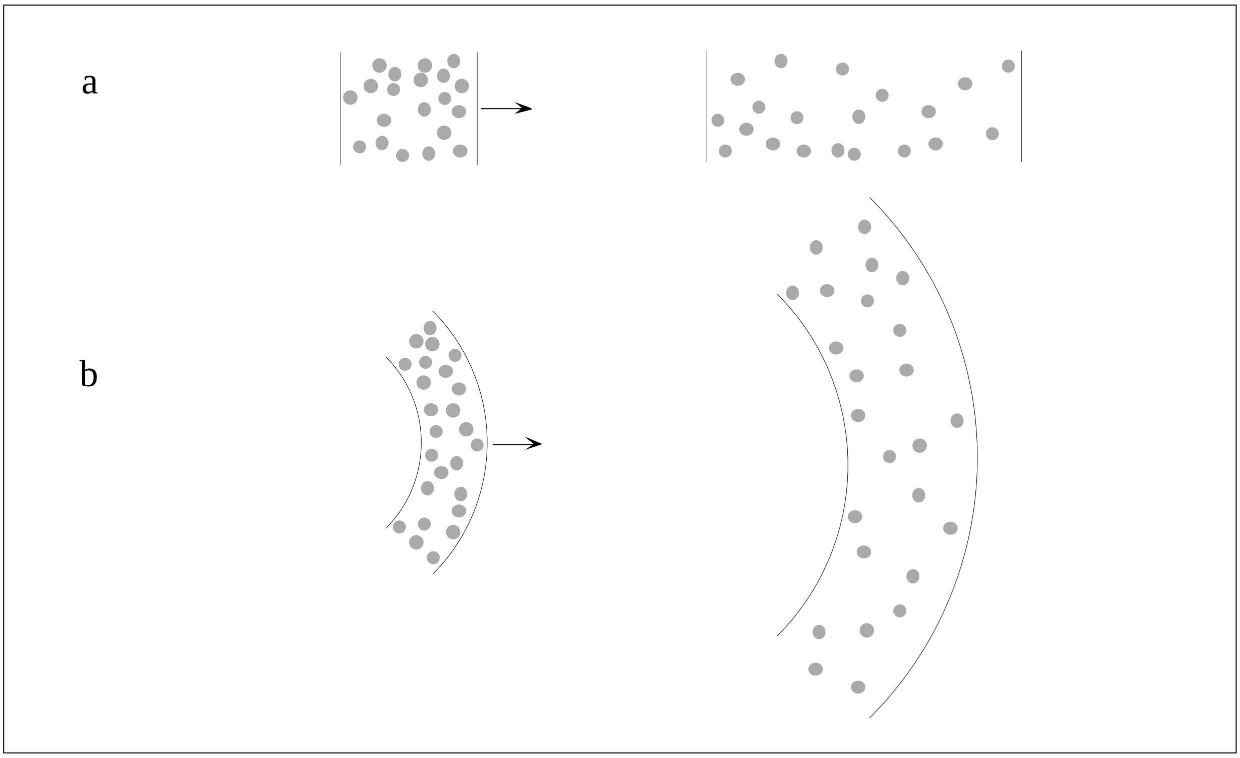

Fig. 2. (a) An expansion in one direction of space leads to an increase of the mean free path of the photons, but the thickness of the layer to be traversed increases by the same factor. Thus, the transparency of the layer remains unchanged. (b) An expansion in two dimensions makes the material more transparent. The effect is even more pronounced in a star's shell which is expanding into three dimensions.

V. THE TRANSITION FROM THE HYDROGEN TO THE HELIUM STAGE

We begin by considering the core. When the hydrogen stock is going to be exhausted, the heat production will decrease. This means a perturbation of the balance between heat production and heat loss towards the shell. There is a net loss of heat. As a result, the temperature increases and the volume decreases. When the temperature is sufficiently high, helium burning will set in. The corresponding heat production causes the contraction process to stop. The temperature and the volume of the core will again admit stationary values. The heat production rate is now higher than it was in the hydrogen stage.

The shell's behaviour is opposite: As the core is evolving from the hydrogen to the helium stage, the shell's heat supply from the core increases. Thus, the total energy of the shell (internal plus field energy) will increase. This means an increase of the radius and thereby of the transparency. The shell will expand until its heat output again equals the heat supply from the core. Thus, when the helium stage is reached, the heat flow is greater than before, the shell has a greater radius and it is more transparent.

One might expect that the same mechanism is operating again when the star is evolving to the further stages of stellar life. However, the feed-back mechanism described in the preceding chapter only works under the conditions of a radiative heat transport. In the following stages of stellar evolution, convection takes over the heat transport. Since convection is much more efficient than radiative diffusion, it will dominate the heat transport and the mechanism discussed before does not work anymore.

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[4] F. Herrmann and H. Hauptmann, “Understanding the stability of stars by means of thought experiments with a model star,” Am. J. Phys. **65**, 292-295 (1997).