KPK

The Karlsruhe Physics Course

for the secondary school A-level

The Teacher's Manual

Oscillations, Waves, Data

The Karlsruhe Physics Cours – *The Teacher's Manual*

A textbook for the secondary school A-level

- Electrodynamics
- Thermodynamics
- **Oscillations, Waves, Data**
- Mechanics
- Atomic Physics, Nuclear Physics, Particle Physics

Herrmann

The Karlsruhe Physics Course

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I

Physical Foundations

1. KPK-specific features

The chapters about „Oscillations and waves“ of the present volume is again characterized by some KPK-specific features. This is particularly apparent in the chapters on oscillations. When mechanical vibrations are treated not the movement is in the focus, but the balance of energy and momentum. In this way, the correspondence between mechanical and electrical vibrations becomes particularly clear.

The description of forced mechanical oscillations becomes more coherent if the average energy flow from the exciter to the oscillator is plotted over the frequency instead of the amplitude of the position of the oscillating body.

2. Harmonic analysis

For the description of vibrations and waves, we use what in the context of university physics is called Fourier analysis. The corresponding procedure used to be considered difficult and unsuitable for the school. Since there are fast computers, the situation has changed. Composing a periodic function out of sine functions can easily be shown with a spreadsheet program. For the decomposition with an FFT (Fast Fourier Transform), numerous easy-to-use programs are available.

In favor of a treatment of the Fourier decomposition is the fact that it is not only omnipresent in physics but also beyond. Sunlight is said to consist of light of different wavelengths, or that a tone contains certain frequencies. These types of speech are based on the fact that one decomposes the light or sound signal into harmonic parts.

Thanks to the Fourier analysis tool a consistent treatment of the concept of coherence is also possible.

The treatment of the harmonic analysis is a good opportunity to present an important mathematical method and physical tool by means of a simple example without beginning with the mathematical details which would only obscure the simple basic idea of the method. Actually, we are faced with decompositions into other basic functions in the teaching of physics and mathematics:

The electric fields of a point charge and of an electrical dipole are the first two terms in the decomposition of an arbitrary field into multipole contributions. The s-, p-, d-orbitals, etc., are the first terms of the decomposition of the electron wave-function into spherical harmonics. Mean and variance are the first terms of the moment analysis of a distribution function.

II

Remarks

1. Oscillations

Definition of the term „oscillation"

When dealing with oscillations, one would like to write a definition on the board: „An oscillations is …".

However, a clear definition of the term is hardly possible. There are numerous phenomena on a scale ranging from processes that everybody would call an oscillation up to those that nobody would consider an oscillation. Where is the border between them?

We would like that, according to the definition, a „damped oscillation" is an oscillation, although it is not periodic and although its energy is not constant.

We wish that the process with the two rotating axes (Fig. 1.4 in the student's text) is no longer a oscillation, although it is periodic and even sinusoidal.

A glider on the air track, which travels back and forth between two spring buffers, should not be an oscillation according to the definition.

We did not find a definition that meets these requirements. Finally, one realizes that one can live well without such a definition.

The same problem occurs later on, if one should be tempted to define what is a wave.

Relaxation and resonance oscillations

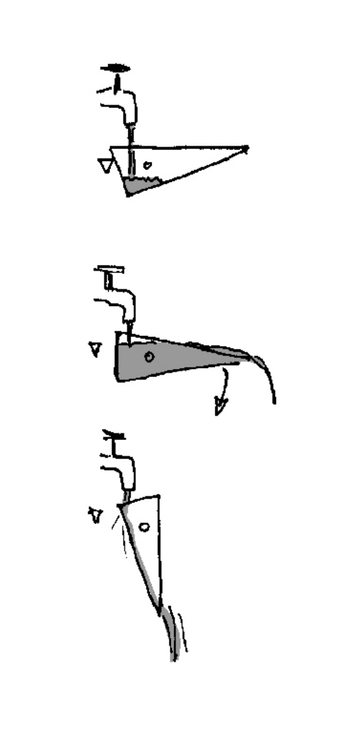

Fig. 1
Relaxation oscillator

In the lower secondary version of the KPK, we had used a broader definition of oscillation. There, the device of Fig. 1 was presented as an example of an oscillator. The system has only one energy storage. The corresponding vibrations are sometimes referred to as „relaxation oscillations" to distinguish them from the normal „resonance oscillations". Among them are the oscillation of the string of a violin, which is excited with the bow, as well as the known chemical oscillations. There is a continuous transition between resonance and relaxation oscillations.

Why only mechanical and electrical oscillations?

Anyone who is accustomed to the KPK may wonder why the analogy, which is usually so much in the foreground, is limited to the quantities of momentum, angular momentum, and electric charge. The reason is that, as regards the oscillations, it does not apply to other quantities.

In order to realize an oscillating system (a resonant oscillator), two energy storage devices of different nature are required. In the electrical case these are the capacitor and coil, characterized by a capacitance and an inductance. In a mechanical system, it can be a (inertial) body and a spring, characterized by mass (= „momentum capacity") and spring constant.

Now there exists the analogue of the capacitor with the corresponding „entropy capacitor" in thermodynamics, but there is no „inductance" for entropy currents. The same applies to the amount of substance.

Mechanical and electrical quantities, which correspond to each other, are summarized in Table 1.

The fact that the oscillating systems can be realized only with those quantities which satisfy a conservation law is probably to be regarded as a coincidence.

Mechanics	Electricity
momentum $\boldsymbol{p}$	electric charge Q
momentum current $\boldsymbol{F}$	electric current I
velocity $\boldsymbol{v}$	electric potential ϕ
displacement $\boldsymbol{s}$	magnetic flux $N\Phi$
mass m	capacitance C
1/spring constant $1/D$	inductance L
mechanical resistance R_p	electric resistance R
viscosity (momentum conductivity) η	electric conductivity σ

Table 1

Choice of systems

In the student's text, certain systems are treated or addressed. We want to justify our choice:

1. Two bodies of equal weight are connected by a spring, Fig. 1.3 in the student's text. Conceptually, it is the simplest mechanical oscillator. One clearly sees the two subsystems between which the momentum flows back and forth. It is also the perfect analog to the simple electric resonant circuit. We accept that it is a somewhat unrealistic system. A disadvantage is that in the realization on the air track it is difficult to prevent a translational movement of the whole oscillator.

2. A body is attached to the wall by a spring, Fig. 1.9 in the student's text. This oscillator is even easier to set up, and one does not have the problem of the translational movement of the whole system. Here, the momentum oscillates between body and „Earth". The heavier body, the Earth, does not participate in the energy exchange. A disadvantage of the system is that the second momentum reservoir is not easy to recognize, because one cannot see that the momentum of the Earth is changing. However, students who have learned the mechanics according to the KPK are accustomed to this property of the Earth. The system is more realistic than the system with bodies of equal weight, as in most practical applications one of the bodies has a much larger mass than the other. In each of these cases, the heavier does not contribute to the energy budget. Examples are the braking car, the ball flying against a wall, the system Earth-Moon, the system Sun-Earth, the system atomic nucleus.

3. The pendulum. From a formal point of view, it is a rather unaesthetic system: the force is not proportional to the deflection, and the system is not one-dimensional, that is, momentum, momentum current and velocity must be treated as vectors. Because of the omnipresence of such oscillations, however, it has to be treated.

4. Rotational vibrations. They are not treated because they can be so beautifully realized with Pohl's wheel, but firstly because the mechanical clocks that have been used for centuries have taken advantage of these oscillation, and secondly because the analogy to the translational oscillations can be shown so easily.

5. The electric resonant circuit. It is an important component of the radio and TV set. However, as with the pendulum clocks, it might be a species endangered with extinction.

The restoring force

The fact that the „restoring force" is proportional to the elongation for a mechanical harmonic oscillation is often formulated in the form of a rule. Apparently the statement is considered important. We do not formulate such a rule. First of all, the linear force law does not guarantee a harmonic oscillation (because to get one other ingredients are required), and secondly, one should also formulate the corresponding rules for electrical oscillations, which no one would surely do.

The restoring force

What is interesting about damped oscillations? It is hardly worth mentioning that an oscillations slowly dies away. This is also true for others, e.g. translational movements.

There is, however, an interesting phenomenon associated with the damping which occurs with oscillations and not with other movements: For a certain value of the damping constant, which is neither zero nor infinite, the system returns to the equilibrium position in a minimum time.

The most obvious and important application of such an optimum damping is the shock absorber of vehicles. Therefore, we believe that it is justified to treat damped oscillations at school.

The terms amplitude, angular frequency and phase

In the case of a harmonic oscillation, the time behavior of the most important variables is described by a „general sine function" $y(t) = \hat{y} \cdot \sin(\omega t + \phi)$

One calls $\hat{y}$ the amplitude, ω the angular frequency, and ϕ the initial phase. Obviously, different variables corresponding to one and the same oscillation have different amplitudes, and they generally have different initial phases. On the other hand, the angular frequency is the same for all quantities, which can be described by an equation of this type.

Therefore, whenever one speaks of the amplitude or the phase, one must mention the physical quantity to which these concepts refer: position, momentum, pressure, electric charge, magnetic field strength ... So there is no amplitude par excellence. Actually, in the case of mechanical oscillations one often refers to „the" amplitude while meaning the position amplitude. (This also corresponds to the etymology of the word.) We avoid this form of speaking because it gives too much importance to the position variable. Thus, we never speak of the „amplitude of the oscillation" but only of the amplitude of the position, the amplitude of the momentum, and so on. But there is no objection to speak of the „frequency of oscillation".

Shock absorbers as mechanical resistors

The shock absorber is the mechanical analogue of the ohmic electrical resistor. The linear *U-I* characteristic of the ohmic resistor corresponds to the linear *v-F* characteristic for the shock absorber.

It is customary to treat the ohmic resistor in detail in electricity. The characteristic is recorded, the linear *U-I* relationship is introduced as an important law. Rules on the parallel and series circuits of ohmic resistors are formulated. However, the mechanical analogue is hardly addressed. The characteristic is not recorded and no applications are discussed. The mechanical resistor (the analogue to the electrical resistor) does not get a graphical symbol. Presumably, this „deprivation" of mechanical friction is due to the fact that friction often occurs as a disturbing, undesired phenomenon. The fact that it is just as useful and indispensable as electrical resistance is hardly apparent.

For this reason, we give the shock absorber in the classroom more space than usual.

2. Resonance

Which variables should be plotted as a function of frequency?

An oscillating system is excited to „forced oscillations“. The response of the oscillator to the excitation depends on the frequency of the exciter. If this frequency is equal to the natural frequency of the oscillator, the reaction is maximal, that is, the oscillator oscillates most violently. At this frequency, its energy dissipation or entropy production is greatest, and therefore the energy flow from the exciter to the oscillator is maximal.

The energy dissipation occurs through friction, in Fig. 2 represented by the damper. The energy, which flows mechanically on the average into the damper (and thermally out again) can be calculated by

$$\bar{P} = \frac{k}{2} \cdot \hat{v}^2$$

Here $\hat{v}$ is the velocity amplitude of the right „link-up“ of the damper (that of the left link-up is zero). k characterizes the damper: The grater k, the harder it is.

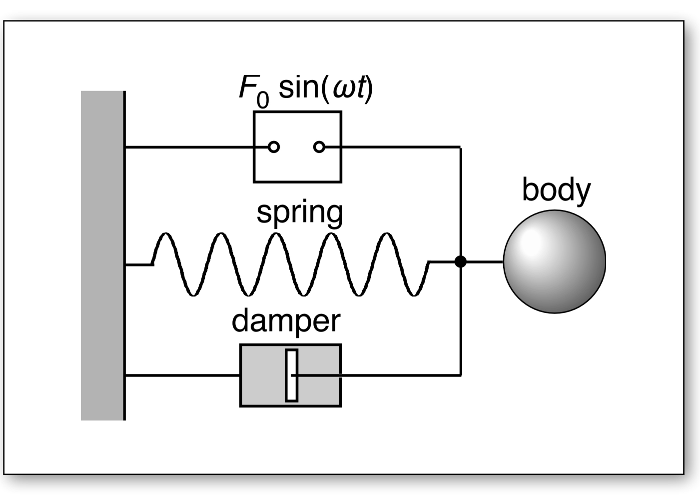

Fig. 2
Symbolic representation of an arrangement for the creation of forced oscillations

The energy dissipated in the damper is supplied to the system by the exciter. Therefore it is possible to calculate the time-averaged energy current from the quantities that characterize the exciter:

$$\bar{P} = \overline{v \cdot F}$$

Here v is the velocity of the energy output of the exciter (its right „terminal“), and F is the momentum current through this connection. When calculating the average value of the right side of the equation, notice that v and F are generally not in phase. With the phase difference ϕ we get:

$$\bar{P} = \frac{\hat{v} \cdot \hat{F}}{2} \cdot \cos\phi$$

The factor cos ϕ is known from electricity. Just like there it tells us how great the „reactive power“ is, i.e. the part of the energy that flows from the load back to the source. For cos $\phi = 0$, the net energy flow from the energy source to the load is zero. Between source and load (in our case between exciter and oscillator), the energy only flows back and forth. For cos $\phi = 1$, the net energy flow to the oscillator is optimal. This case corresponds to the resonance.

It follows from the foregoing considerations which quantities are best represented graphically as a function of the frequency so that the phenomenon of resonance is as clear as possible:

1. The time rate of the average of the dissipated energy (which is equal to the average of the energy flow from the exciter to the oscillator). As expected, this energy current has its maximum at the natural frequency of the oscillator and becomes zero for both $\omega \to 0$ and $\omega \to \infty$. Since this energy flow is proportional to the square of the velocity of the oscillating body, we can alternatively draw the velocity amplitude over the frequency. This curve also has the maximum at the right place and the correct asymptotic behavior for $\omega \to 0$ and for $\omega \to \infty$. It is less convenient to discuss the resonance on a graph showing the position amplitude of the oscillator as a function of the frequency. In such a graph, the maximum is not „in the right place“.

2. The phase shift between momentum flow and velocity at the output of the exciter. As expected it is zero at the natural frequency of the oscillator, and different from zero for smaller and larger frequencies.

Often, the phase shift between the position amplitude of the exciter and the position amplitude of the oscillator is plotted as a function of the frequency. This curve is more difficult to interpret.

Parallel and series resonant circuit

We address a problem encountered in connection with forced mechanical oscillations. However, since it is easier to discuss the electrical resonant circuit, we first consider an electrical resonant circuit.

The resonant circuit consists of a capacitor, a coil, a resistor and an excitation energy source. There are several ways to combine these 4 components: all four in parallel; all four in series; three parallel and the fourth in series ... Which of these circuits should we choose? First, we exclude those in which parallel and series connections appear mixed, we limit ourselves to the candidates with the simplest circuit diagrams, Fig. 3.

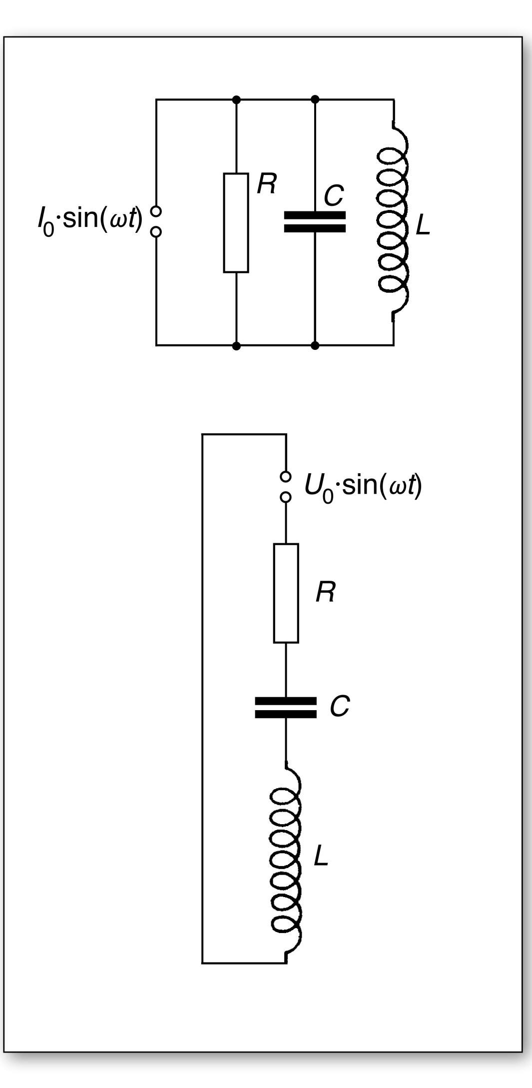

Fig. 2
(a) Parallel resonant circuit; (b) Series resonant circuit

We want to record a resonance curve, i.e we change the frequency of the exciter source and observe the reaction of the resonant circuit. But we still have to decide which variable of the source we keep constant. The most reasonable candidates for this are 1. the current amplitude or 2. the voltage amplitude. Of course one could also consider other options. So one might also keep the energy flow constant. However, in this case one can no longer recognize the resonance by looking at the energy current. Therefore, we restrict ourselves to the consideration of the electric current and the voltage.

We want to investigate how the resonant circuit reacts to a stimulus. If, in the case of the parallel resonant circuit, we would apply a given (alternating) voltage, this voltage would be present at every moment on all three other components. The „resonant circuit“ would no longer be a resonant circuit. We could just as well have connected the three components independently to three AC sources. That is why we make sure that the current amplitude has a given value, and we will examine how the resonant circuit distributes the current between the three components.

Similar arguments are valid for keeping the voltage amplitude constant in the case of the series resonant circuit.

Now the question is clearer. We have to decide between a parallel resonant circuit with a constant current amplitude exciter and a series resonant circuit with a constant voltage amplitude exciter. Both options are equivalent in many ways. The corresponding resonance curves look absolutely the same.

As with the mechanical oscillator of Fig. 2, instead of the energy current, one can also plot the velocity over the frequency, so in the case of the electrical parallel resonant circuit, the voltage instead of the energy current can be plotted over the frequency, or the current intensity in the case of the series resonant circuit. In both cases, one obtains the behavior expected from a resonance curve: the maximum is at the natural frequency, and an amplitude (voltage or current) that approaches zero for $\omega \to 0$ and $\omega \to \infty$.

So what should we decide for?

In an advanced course, one might treat both versions and emphasize the analogy or equivalence as an interesting learning goal. But even if we have to limit ourselves, the decision is not difficult. We have considered the exciter as the cause and the behavior of the resonant circuit as an effect. Since we are accustomed to the idea that the voltage plays the role of the cause of the current, we choose the series resonant circuit with an exciter of a constant voltage amplitude.

The decision was easy in the case of the electric oscillating circuit, it becomes difficult in the case of the mechanical oscillator.

One might think that one only needs to translate the electrical series resonant circuit into a mechanical arrangement. After all, we know the translation rules. Electrical charge transforms into momentum, electric current into momentum current, a coil into a spring, a massive body into a capacitor.

If we do so, we get the oscillator of Fig. 4. (The translation of the parallel resonant circuit gives the oscillator of Fig. 2.) This mechanical oscillator works in principle, and is described by the same mathematics as its electrical analogue. It has only one small flaw: We need a drive or exciter, which provides a constant velocity amplitude regardless of the frequency. This is awkward in two ways.

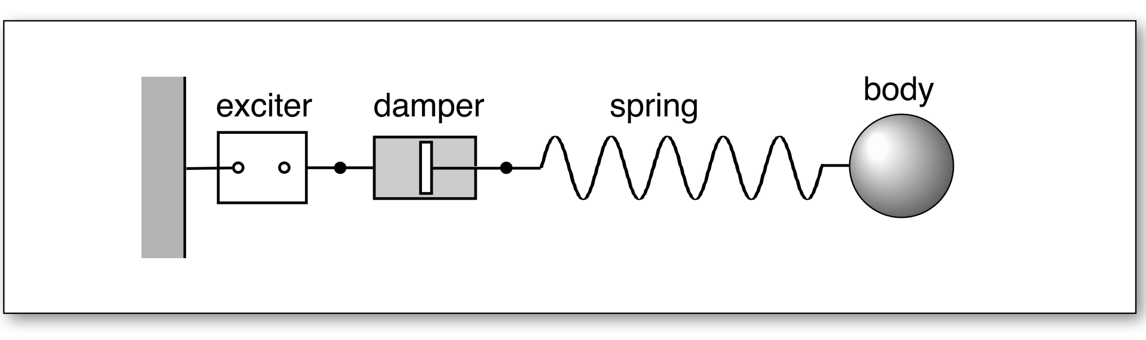

Fig. 4
Mechanical analogue of the electric series resonant circuit

1. While we are accustomed to conceiving an electrical voltage as the cause of an electric current, our feeling for mechanical processes tells us that the force, i.e, the momentum current is the cause of the velocity, and not vice versa.

2. It is not easy to find a mechanical drive that provides a periodic motion with a constant velocity amplitude. It is easy to construct a device with constant amplitude of displacement. That’s what a crank does. Actually, one could realize a drive with a constant velocity amplitude. However, this would look so unnatural as to make the whole experiment suspect.

So we are forced to make a different choice in the electrical and mechanical case.

Thus, the matter seems to be settled: One takes the mechanical oscillator of Fig. 2, which corresponds to the electrical parallel resonant circuit: spring, body, damper and exciter are mechanically connected in parallel. In university books in which the mechanical oscillator is calculated, one refers almost always to this type of oscillator.

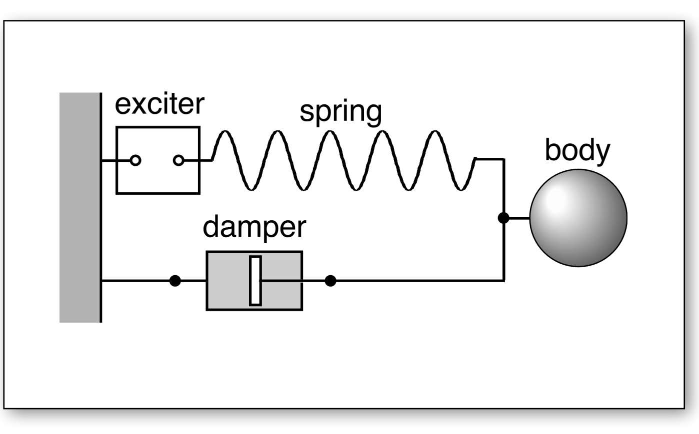

Fig. 5
Mechanical oscillator with an exciter of constant amplitude of displacement

In practice, however, another problem arises here: also this type of oscillator is difficult to realize, because one needs a drive that provides a constant force amplitude. While it is not too difficult to build such a device, it does not exist in school collections.

What one uses for school experiments is a drive with a crank that rotates at a given angular velocity. The excitation of the oscillation does not occur with a constant velocity amplitude, nor with constant force amplitude, but with constant amplitude of displacement, Fig. 5. The method works because the resulting differential equation is the same as that of a „parallel oscillator“, which is driven with a constant force amplitude.

3. Spectra

A minimum version of the Fourier decomposition

Our treatment of the decomposition of functions into a Fourier series is a compromise in which we had to balance between mathematical rigor on the one hand and brevity of the derivations and slenderness of the results on the other hand.

The shortest version would have been that we simply claim, „Any function can be decomposed into sine components“. Then, however, we would have missed some interesting phenomena of acoustics. So we decided to give the periodic functions a special meaning.

The statement that any function that is defined only in an interval of the independent variable can be periodically continued has fallen prey to our efforts of simplification.

Fourier decomposition everywhere

It may be thought that decomposing a function of time into harmonic components is too difficult for the school. We do not believe so. We even believe that it is a prerequisite for understanding many of the statements and claims that we make in class anyway, such as, „Sunlight is made up of light of many wavelengths between …“. (It would be more accurate to say, „One can decompose sunlight into …“) Or: „Our hearing is sensitive to sound with frequencies between 20 Hz and 20 kHz.“

What should we do with such statements if we do not assume that every light, every noise and every sound can be decomposed into sine parts?

Spectra

When introducing the term „spectrum“ one would have to differentiate between discrete spectra, which can be represented by a bar graph, and continuous spectra, where the ordinate is the derivative of an amplitude or energy flux density with respect to the frequency.

We have refrained from a mathematically rigorous presentation for two reasons. First, we can not assume that differential calculus has already been treated in the mathematics lessons, and secondly, the mathematical effort would be somewhat disproportionate.

Coupled oscillations

We have avoided the terms „coupled oscillations“ and „coupled oscillators“. In its original meaning, the expressions should certainly be reminiscent of the two pendulums that have been „coupled“ together by a soft spring or a thread with a small weight. One starts out from two initially independent oscillators, which are then weakly coupled to each other. Now, with the systems we are aiming at later, the single oscillators are no longer recognizable. We can not take away any springs in a way that the result is two uncoupled oscillators.

A clearer alternative would have been to characterize the various oscillators by the number of degrees of freedom. But that would have required a lengthy formal discussion that would be inappropriate for our needs. We have therefore opted for a mode of expression, which is based more on the colloquial language: a oscillator with two, three, etc. degrees of freedom is called a double oscillator, triple oscillator, etc.

Resonance curves of oscillators with several degrees of freedom

We were reluctant to give up this topic. It is not difficult to record such resonance curves. It is rather a marginal problem that prevents us from treating the subject in the classroom. While we had assumed both dampers and exciters as parallel to the spring in the treatment of resonance in the simple oscillator, there are a large number of alternatives for multiple oscillators. This would have required a cumbersome and not very productive discussion of the relationship between the resonance curve and the spectral function.

4. Waves

Their creation is not the main subject

There is a tendency in physics books to explain a phenomenon by describing how it is created or produced. The electric field is explained by the electric charge that is the cause of the field. Coherence is explained by describing light sources that produce coherent light. Electromagnetic waves are explained by the Hertzian oscillator. We think that such an approach is unnecessarily cumbersome, because often the process of creation is more complicated than the phenomenon itself. Somewhat exaggerating one could say: To explain to someone what a bicycle is, it is best to describe the bike itself and not the manufacturing process in the bicycle factory.

Also with the waves, we tried to get straight to the phenomenon itself. We describe what a wave is and what different types of waves exist.

Energy transport with waves

We have formulated the rule that the time-averaged energy flow density of a wave is proportional to the square of the amplitude of one of the quantities by which we describe the wave. Even if it is not addressed in class, the teacher should know that when two waves intersect, the respective wave variables add up, but not the energy flows. Also, one should not lose sight of the fact that in the places where „the waves compensate by interference“ only one of the wave variables is constantly (and not only its time average) zero, but another variable does not become zero. You can see it clearly with standing waves. With a standing electromagnetic wave, where the electric field strength distribution has its nodes, the magnetic field strength just has an anti-node. Similarly with a standing sound wave: The speed nodes are at the same position as the pressure antinodes.

The figures in the text that show the square of the wave function – you can see it from the fact that the background is white and not gray – are in general neither images of the energy density nor of the energy flow density. They represent these quantities only where there is a pure sine wave, but not in the interference areas.

Destructive interference is usually not complete

In the discussion of the phenomenon of interference we committed (with a somewhat uneasy feeling) an inaccuracy that is consistently committed in the textbook literature, and which seems to be hardly considered worth mentioning.

It is said that two plane waves of the same frequency and amplitude, which intersect at a certain angle, cancel each other at certain positions. These are the places where the quantity used to describe the waves adds up to zero, not only in the time average, but in every moment. In fact, that does not mean that the two waves extinguish each other. Only one of the quantities that describe the wave is zero at these points. The complementary quantity is generally not.

Fig. 6 shows the intersection of two plane electromagnetic waves. Suppose the electric field strength vectors are perpendicular to the plane of the drawing. The gray scale stands for the absolute value the of electric field strength. In the interference area horizontal lines can be seen, where the electric field strength adds up to zero. If the electric field strength vectors are perpendicular to the drawing plane o, the magnetic field strength vectors lie in the plane of the drawing, for the two individual waves perpendicular to the direction of travel of the respective wave. But this means that the magnetic field strength vectors of the two single waves cannot add up to zero, because they are never antiparallel. Where the electric field strength contributions add up to zero, the magnetic field strength do not.

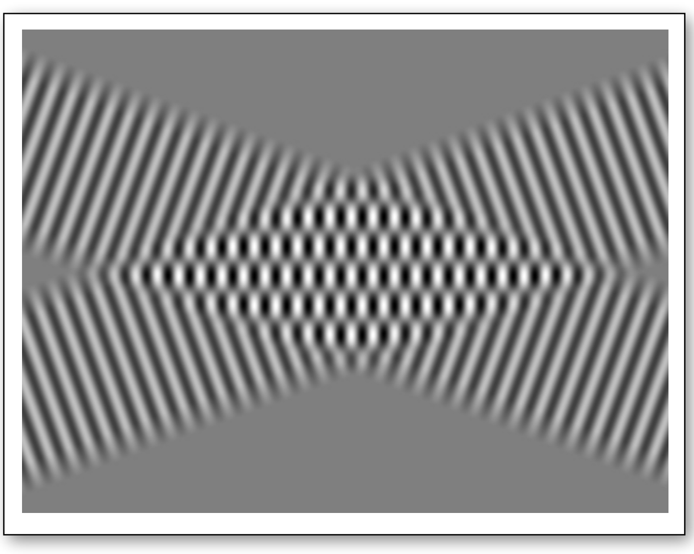

Fig. 6
If the electric field strength vectors are perpendicular to the plane of the drawing, the magnetic field strength lies in the plane of the drawing.

Therefore, the energy density of the wave is not zero there either. In contrast, the energy flux density given by the vector product $\mathbf{E} \times \mathbf{H}$ is zero because one factor, $\mathbf{E}$, is zero.

Diffraction

Diffraction is treated independently from the phenomenon of interference. In our experience, students easily confuse the two concepts. Therefore, we first discuss interference in a situation where no diffraction is involved: two sinusoidal waves passing through each other at an acute angle. One can think of these waves generated by two independent sources. The fact that this type of interference experiment causes problems in light is only due to a technical problem: there are no light sources that are sufficiently phase-stable.

More sophisticated interference experiments

An unbiased person who wants to study the interference of light would probably not begin with the diffraction phenomenon at a double slit or at a grating. The more obvious idea is that one generates two sine waves with two independent light sources. (One would in a corresponding experiment proceed to show the interference of sound waves.)

Therefore, before we come to the well-working, sophisticated interference experiments with a double slit or a grating, we try to understand why the manifest method fails.

Coherence

We discuss it in the context of the theorem of the decomposability of a wave in sine waves. This theorem is one of the most important learning objectives of this volume.

Huygens-Fresnel principle

In our opinion the Huygens-Fresnel principle is somewhat abused in the textbook literature.

To explain that from a small source emanates a circular or spherical wave, one does not need such a principle.

In the form in which Fresnel had formulated it, it says nothing else than that you can decompose a given wave into circular or spherical waves. Decompositions tell us nothing about a deeper truth. They are just a mathematical tool. They are used when they simplify the work, and they are not used when there is no benefit.

Splitting a wave field into spherical waves is useful if the problem is spherically symmetric or nearly spherically symmetric. To decompose a plane wave into spherical waves, as one does to „explain“ refraction and reflection with the Huygens-Fresnel principle, however, means to explain a simple matter in a complicated way.

5. Interference of light and X-rays

Diffractions grating and X-ray diffraction on crystals

The decision to include the diffraction grating as a subject of teaching was difficult for us. After all, it is just one of countless tools used in atomic and solid-state physics research.

The diffraction grating for visible light is especially important in the grating spectrometer, the diffraction of X-ray light for the X-ray structure analysis. Are these two methods of analysis so much more important than the many other techniques, that they should be treated in the classroom, and not the other methods? In one sense, they actually are. We owe the largest part of our knowledge of the atom and thus the experimental foundations of quantum physics to the optical spectroscopy. Historically, it was certainly more important than mass spectroscopy, electron loss spectroscopy, electron microscopy, Fourier spectroscopy, and many more. X-ray diffraction played a similar key role, namely in the elucidation of the structure of solid matter. It has been and continues to be more important than X-ray fluorescence analysis, neutron diffraction, transmission electron microscopy, scanning electron microscopy, scanning tunneling microscopy, scanning near field optical microscopy (SNOM), low energy electron diffraction (LEED), electron spin resonance, nuclear magnetic resonance (NMR), Secondary Ion Mass Spectroscopy (SIMS), and many other techniques.

6. Data transfer and storage

The amount of data in optical and acoustical perception

We start the topic by supposing that images and sounds have no redundancy. Thus, at first we do not consider the possibility of data compression. In fact, there are always sections of the path of the data flow over which this uncompressed data stream flows.

The sampling theorem

It appears in our approach in the context of acoustic data transports, but without being called by name.

Redundancy

We have not introduced a quantitative measure of redundancy. It would be easy to define such a measure:

$$R = 1 - H_0/H.$$

(H_0 = actual amount of data, H = apparent amount of data.) However, practically it would be rather useless. The problem is to determine the actual amount of data.

Games and probabilities

Dice and urn experiments play an important role in textbooks on information theory. The reason is that in this case probabilities can be easily specified. In this way we can easily show the working of Shannon's formula. Hence the section „Games“.

III

Solutions to Problems

1. Oscillations

1.1 Provisional description

1. No characteristic period

2. Momentum flows over the two shafts: over the left into the bar, over the right out of the bar. If the bar is in the symmetrical middle position, it gets just as much momentum over the left shaft as it loses through the right one. If it lies to the left of the middle position, it gets more than it looses, if it lies further to the right, it gives off more than it receives.

If at the beginning the bar is placed at the left of the middle position, it gets more momentum than it loses, until it reaches the middle position. Thus, momentum accumulates first, it moves with increasing speed to the right. As soon as it is beyond the middle position, its momentum decreases again. It comes to a standstill but continues to loose momentum; so it moves faster and faster to the left, etc.

The process is not an oscillation according to our definition because it only runs when there is a steady supply of energy.

3. No oscillation, since there is no characteristic period.

1.2 Momentum and energy

The momentum still flows back and forth between the two bodies. The total amount of the momentum of the two bodies is still the same at every moment. The absolute value of the velocity of the light one is greater at any time than that of the heavy one. The energy is no longer evenly distributed between the two bodies.

1.3 The Earth as a partner

Momentum: from the body simultaneously over both springs into the Earth and back again. (If the springs are tended from the beginning, a momentum current to the left is superimposed.)

Energy: from the body simultaneously in the two springs and back again.

1.4 Harmonic oscillations

1. See Fig. 9.

$s(t) = \hat{s} \cdot \sin(\omega t)$
$F(t) = -D \cdot s(t) = -D \cdot \hat{s} \cdot \sin(\omega t)$

$v(t) = \omega\, \hat{s} \cdot \cos(\omega t)$
$p(t) = m \cdot v(t) = m \cdot \omega\, \hat{s} \cdot \cos(\omega t)$

At the beginning ($t = 0$) the body has momentum. Then a current of negative momentum flows into the body, i.e. (positive) momentum flows out of it. As a result, the momentum of the body decreases and becomes negative. The momentum current decreases again. If it is zero, p does not change anymore. The momentum current reverses its sign, i.e. momentum flows into the body. The momentum increases and becomes positive again.

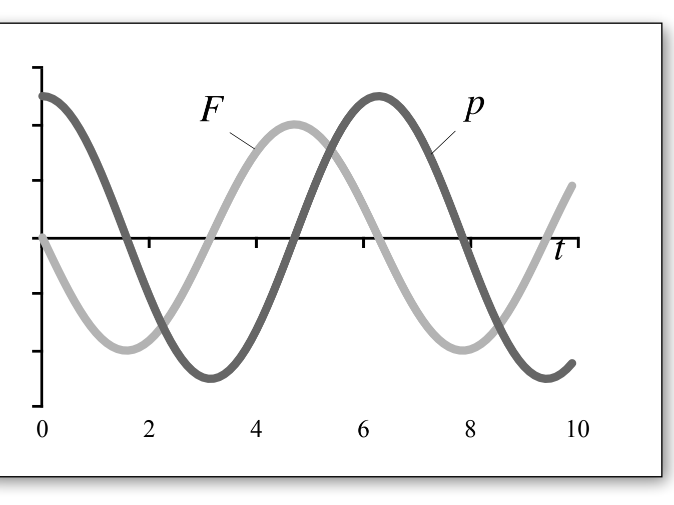

Fig. 7
Section 1.4, exercise **1**

2.

$P(t) = v(t) \cdot F(t)$

$F(t) = -D \cdot s(t) = -D \cdot \hat{s} \cdot \sin(\omega t)$
$v(t) = \omega\hat{s} \cos(\omega t)$

$P(t) = -\omega\hat{s} \cdot \cos(\omega t) \cdot D \cdot \hat{s} \cdot \sin(\omega t)$

With $2\sin(\alpha) \cdot \cos(\alpha) = \sin(2\alpha)$ we obtain:

$P(t) = -(\omega/2) \cdot D \cdot \hat{s}^2 \cdot \sin(2\omega t)$

The energy flow also follows a sine function. Like the energy in the two energy storage devices, it changes at twice the oscillation frequency.

3.

$s_A(t) = \hat{s} \cdot \sin(\omega t)$
$p_A(t) = m \cdot \omega\hat{s} \cdot \cos(\omega t)$

$s_B(t) = s_0 - \hat{s} \cdot \sin(\omega t)$
$p_B(t) = -m \cdot \omega\hat{s} \cdot \cos(\omega t)$

$F_{AB} = -D[s_B(t) - s_A(t) - s_0] = 2D \cdot \hat{s} \cdot \sin(\omega t)$

$E_F = 2D \cdot \hat{s}^2 \cdot \sin^2(\omega t)$

$E_A = E_B = (m/2) \cdot (\hat{s} \cdot \omega)^2 \cdot \cos^2(\omega t)$

$E = E_F + E_A + E_B$
$= 2D \cdot \hat{s}^2 \cdot \sin^2(\omega t) + m \cdot (\hat{s} \cdot \omega)^2 \cdot \cos^2(\omega t)$

For E to be independent of time, the factor preceding $\sin^2$ must be equal to that of $\cos^2$:

$2D \cdot \hat{s}^2 = m \cdot (\hat{s} \cdot \omega)^2$.

The equation can be simplified:

$D = (m/2) \cdot \omega^2$

and we obtain the angular frequency

$$\omega = \sqrt{\frac{2D}{m}}$$

$P_{AF} = P_{BF} = v_A \cdot F_{AB}$
$= \omega\hat{s} \cdot \cos(\omega t) \cdot 2D \cdot \hat{s} \cdot \sin(\omega t)$
$= \omega D\, \hat{s}^2 \cdot \sin(2\omega t)$

1.5 What the period length depends on

1.

$$T = 2\pi\sqrt{\frac{m}{D}}$$

$$\frac{m}{T^2} = \frac{D}{4\pi^2} = \text{const} = \frac{m'}{T'^2}$$

$$m' = m \cdot \left(\frac{T'}{T}\right)^2$$

$m = 0.25$ kg
$T = 2$ s

(a)

$T' = 3$ s

$$m' = 0.25 \cdot \left(\frac{3}{2}\right)^2 \text{ kg} = 0.56 \text{ kg}$$

(b)

$T' = 10$ s

$$m' = 0.25 \cdot \left(\frac{10}{2}\right)^2 \text{ kg} = 6.25 \text{ kg}$$

2. Two springs: D doubles, T decreases by a factor of $1/\sqrt{2}$.

Four springs: D quadruples, T decreases to one half.

3. The center of the spring does not move. Therefore, one body + half of the spring can be considered as an independent oscillator. For half of the spring $D' = 2D$. Thus, we obtain

$$T' = 2\pi\sqrt{\frac{m}{D'}} = 2\pi\sqrt{\frac{m}{2D}}$$

4. The springs are equivalent to two springs that are connected in parallel. The total spring constant is $D' = 2D$, thus we get

$$T' = 2\pi\sqrt{\frac{m}{D'}} = 2\pi\sqrt{\frac{m}{2D}}$$

1.6 The pendulum

1.

Earth:

$$T = 2\pi\sqrt{\frac{l}{g}} = 2\pi\sqrt{\frac{1.2 \text{ m}}{10 \text{ N/kg}}} = 2.2 \text{ s}$$

Moon:

$$T = 2\pi\sqrt{\frac{l}{g}} = 2\pi\sqrt{\frac{1.2 \text{ m}}{1.62 \text{ N/kg}}} = 5.4 \text{ s}$$

Neutron star:

$$T = 2\pi\sqrt{\frac{l}{g}} = 2\pi\sqrt{\frac{1.2 \text{ m}}{1\,000\,000\,000\,000 \text{ N/kg}}} = 6.9 \cdot 10^{-6} \text{ s}$$

2.

$$l = g \cdot \left(\frac{T}{2\pi}\right)^2 = 9.81 \frac{\text{N}}{\text{kg}} \cdot \left(\frac{2s}{2\pi}\right)^2 = 0.994 \text{ m}$$

3. It has to be vertical.

4. Probably the weights are not sufficient for the drive, so you have to increase their mass by about a factor of 6 (the ratio of the field strengths on the earth and on the moon).

The clock is now running, but much slower (compare with exercise 1). This error can be corrected by shortening the pendulum.

5.

$$\frac{m}{2}v^2 = m \cdot g \cdot h$$

$$\Rightarrow h = \frac{v^2}{2g} = \frac{(0.2 \text{ m/s})^2}{2 \cdot 10 \text{ N/kg}} = 0.002 \text{ m}$$

The height depends neither on the pendulum length, nor on the mass of the body.

1.7 Angular oscillations: angular momentum flowing back and forth

1. Two flywheels are arranged side by side so that the axis of one lies in the prolongation of the axis of the other. They are connected by a spiral spring: The inner end of the spiral spring is attached to one wheel, the outer to the other.

2. You can orient it as you want. It always oscillates with the same period.

1.8 Electric oscillations: electric charge flowing back and forth

1. The capacitance of a capacitor is greater, the larger the plate area and the smaller the plate spacing. The inductance of a coil is greater, the greater the cross-sectional area of the coil, the greater the number of turns per length and the greater the total number of turns. Accordingly, one can influence the period of a resonant circuit.

2. With

$$Q(t) = \hat{Q} \cdot \sin(2\pi f t)$$

we obtain

$$I(t) = \frac{dQ(t)}{dt} = \hat{I} \cdot \cos(\omega t)$$

and get

$$E_{cap} = \frac{C}{2} \cdot U^2 = \frac{Q^2}{2C} = \frac{\hat{Q}^2}{2C} \cdot \sin^2(\omega t)$$

and

$$E_{coil} = \frac{L}{2} \cdot I^2 = \frac{L\hat{I}^2}{2} \cdot \cos^2(\omega t)$$

1.9 The damping of oscillations

2. See figures 8 and 9.

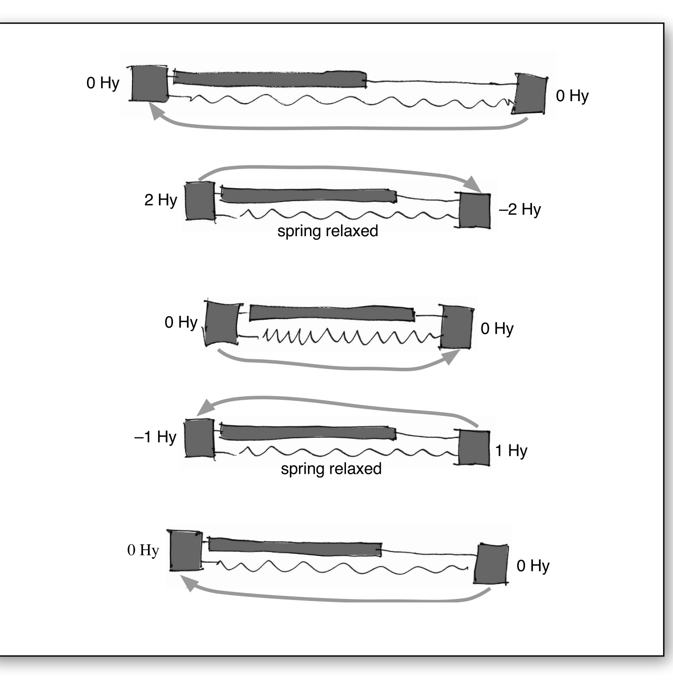

Fig. 8
Section 1.9, exercise **2**: momentum current

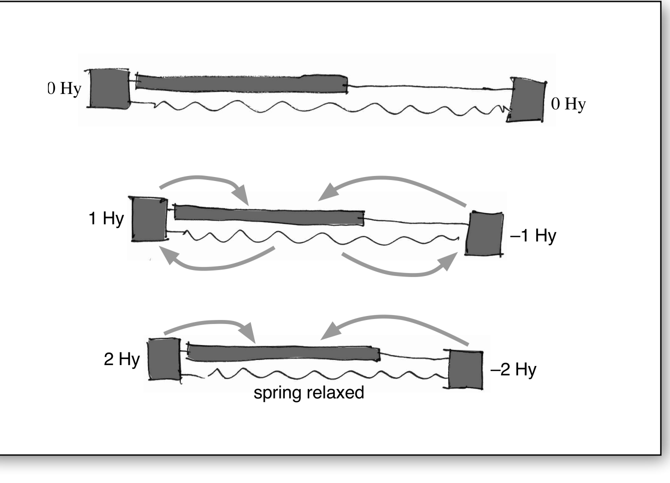

Fig. 9
Section 1.9, exercise **2**: energy current

2. Resonance

2.2 Resonance of a mechanical oscillator

1.5 Hz: The energy current is greatest for strong damping and smallest for weak damping.

1.7 Hz: The energy current is greatest for medium damping and smallest for weak damping.

1.5 Hz: The energy current is greatest for weak damping and smallest for strong damping.

2.3 How to draw a resonance curve

To each of the curve sections above the 0.5 straight line there is another one that is obtained by mirroring this section on this straight line. Thus, the deviations in the upwards direction compensate those in the downwards direction.

3. Spectra

3.1 Some mathematical results

2. The rule for obtaining the terms is

$$(-1)^{n+1} \cdot \frac{1}{(2n-1)^2} \cdot \sin\left[(2n-1)x\right]$$

(It must not be given in analytical form.) Conjecture: the more terms the sum contains, the more the graph resembles a zig-zag line (Fig. 12).

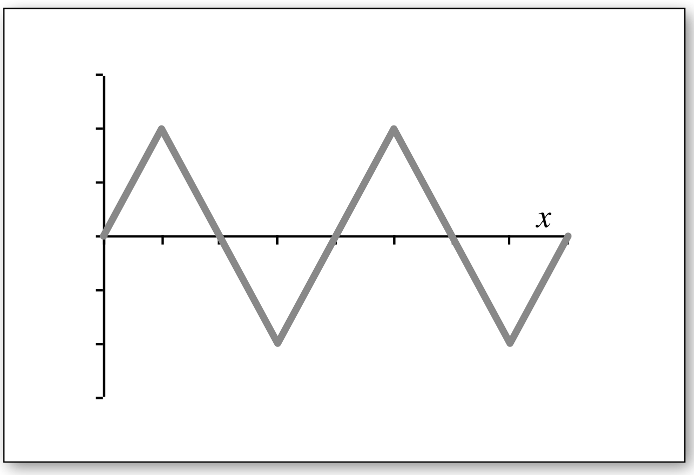

Fig. 10
Section 3.1, exercise **2**

3. The rule for obtaining the terms is

$$d_2 = \frac{a_1}{a_2} \cdot d_1 = \frac{32\ \text{cm}}{48\ \text{cm}} \cdot 0.0033\ \text{mm} = 0.0022\ \text{mm}$$

Conjecture: the more terms the sum contains, the more the graph resembles that of Fig. 13.

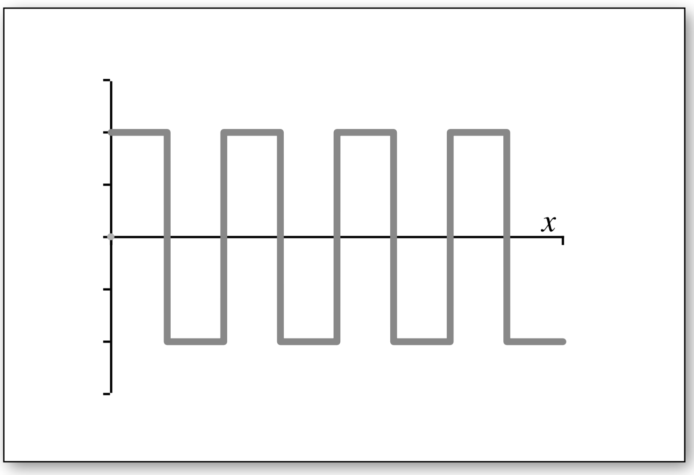

Fig. 11
Section 3.1, exercise **3**

3.2 Spectra

See Fig. 12

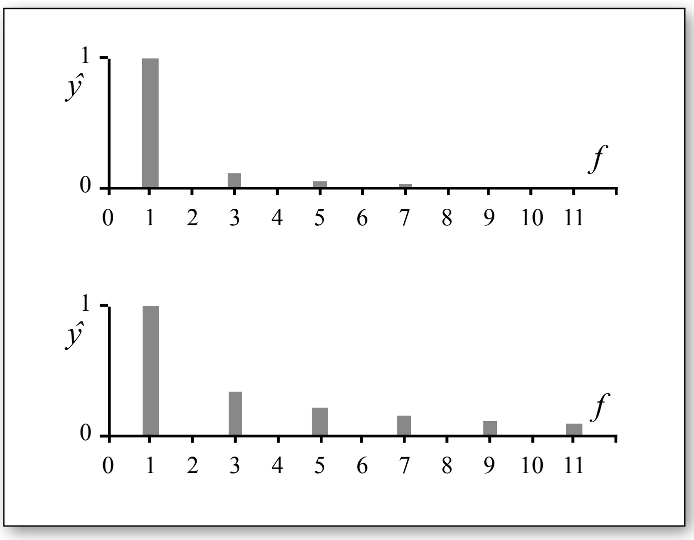

Fig. 12
Section 3.2, exercise

3.3 Double oscillators

1. For the first natural oscillation, the length of the middle spring does not change. Each of the two bodies oscillates as if it were only hanging on the adjacent outer spring. For the second oscillation, every body feels the middle one in addition to the outer spring. It is as if another spring was „connected in parallel" to the outer spring. The resulting spring constant is thereby increased, and thus the frequency becomes larger.

2. (a) The relative difference between the two frequencies is small.

(b) The first natural frequency is small compared to the second one.

3. One can consider this oscillator as a double oscillator, in which the outer springs are „infinitely soft". Therefore the frequency of the second natural oscillation is zero.

4. One connects the oscillating bodies by a soft spring.

4. Waves

4.2 The velocity of waves

Here, too, a change of a state runs through or over a „carrier“. However, the change of state is permanent. You can not send a second wave through the carrier.

Like a real wave, the domino wave also has a carrier and its own velocity.

4.4 Sine waves

1. If the new direction of the wave is the *z*-direction, *x* must be replaced by *z* in equation (4.1).

2. From a few centimeters to a few tens of meters.

3. All four snapshots look the same. Between two shots, the wave advances just one wavelength.

4. The wave runs in the negative *x*-direction.

4.5 The relationship between velocity, frequency and wavelength

1. $\lambda = v/f$ = (300 m/s)/440 Hz = 0.7 m

2. $\lambda = v/f$ = (300 000 km/s)/98.4 MHz = 3 m

4.6 Sound waves

1. Speaker, voice, musical instruments, thunderstorm, explosion

2. 150 Hz

3. 15 m and 15 mm

4. The frequency remains the same, the wavelength increases.

5. About 3000 m

4.7 Electromagnetic waves

1. In the lightning, a very strong current flows for a very short time. The magnetic field of this current changes very fast. It detaches itself from the lightning, runs away as a wave and induces an electric current in the TV aerials.

2. Transmitting antennas of radio and television transmitters, parabolic antennas of telecommunications towers, hot stove, light sources, X-ray tubes, radioactive materials.

3. For equal values of *x* and *t*, the sine term of the electric field strength has the same value as that of the magnetic field strength.

4. See Fig. 13.

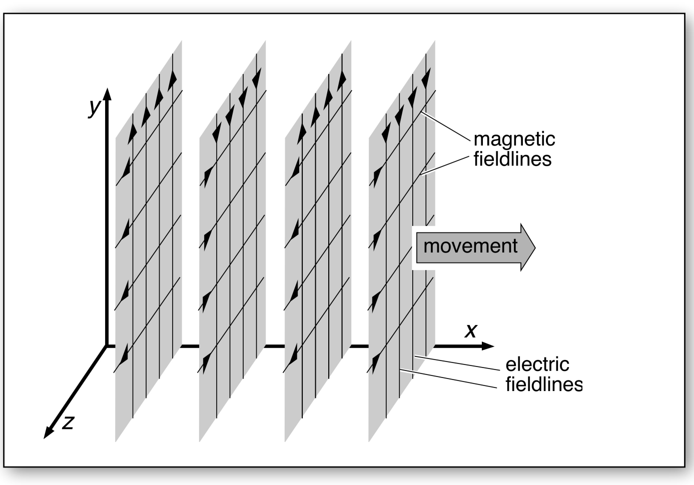

Fig. 13
Section 4.7, exercise **4**

4.8 Energy transport with waves

1. Energy of a moving body:

$$E_{kin} = \frac{p^2}{2m}$$

Energy of a tensioned spring:

$$E_F = \frac{D}{2}x^2$$

Energy of a charged capacitor:

$$E_{cap} = \frac{Q^2}{2C}$$

Energy of a coil, in which an electric current is flowing:

$$E_{coil} = \frac{L}{2}I^2$$

2. See the exercise of section 2.3

4.9 Two waves at the same place

1. Wind is not a wave. Two „winds“ can not flow through each other.

2. If there are two „sources“ of an electric or magnetic field, the total field strength is obtained by vectorial addition of the field strengths of the fields of the individual sources.

4.10 Two sine waves – interference

1. An „intermediate“ between standing and normal wave: On the one hand one recognizes a progressive movement, as with a normal wave; on the other hand, a periodic becoming small and large of the whole wave, as in the case of a standing wave.

2. When the waves oscillate „in time“: a wave whose amplitude is twice as large as that of a single wave. When they oscillate in „push-pull“: complete, permanent extinction.

4.12 Natural oscillations of wave carriers

2. Given: l = 1 m
v = 6 m/s

$\lambda_{max} = 2l$ = 2 m

In order to obtain two nodes, we must have $l = 3/2\lambda$, or
$\lambda = 2/3 l$ = 2/3 m.

With $v = \lambda f$ it follows

$f = v/\lambda$ = 9 Hz

4.13 The interference of waves

1. Here, the angle α is 180°. Using

sin (90°) = 1

we obtain with the equation

$$\sin\frac{\alpha}{2} = \frac{\lambda}{2a}$$

as expected

$a = \lambda/2$.

2.

$$\alpha = 2 \cdot \arcsin\left(\frac{\lambda}{2a}\right) = 2 \cdot \arcsin\left(\frac{550\text{ nm}}{2 \cdot 2\text{ mm}}\right) = 0.016°$$

4.14 The diffraction of waves

The wavelengths of the waves for normal television reception from a transmitting antenna on the Earth are about 1 m. They are still bent pretty well. The wavelength of the satellite programs are in the range of centimeters. The diffraction is very small here.

Clouds are „transparent“ to the waves of satellite television.

Mobile phones operate at 900 MHz and 1800 MHz. The wavelength is therefore 20 cm, and the diffraction is still quite effective. Also the cordless phone works in this frequency range.

5. Interference of light and X-rays

5.1 Coherence

1.

$$\frac{l_{coh}}{\lambda} = \frac{\lambda}{\Delta\lambda} = \frac{645 \text{ nm}}{10 \text{ nm}} \approx 64$$

2.

$$\frac{l_{coh}}{\lambda} = \frac{\lambda}{\Delta\lambda} = \frac{590 \text{ nm}}{0.6 \text{ nm}} \approx 1000$$

$$l_{coh} = 1000 \cdot 590 \text{ nm} \approx 600\,000 \text{ nm} = 0.6 \text{ mm}$$

3. To the frequencies

f_1 = 105.65 MHz
f_2 = 105.75 MHz

correspond the wavelengths:

λ_1 = 2.83957 m
λ_2 = 2.83688 m

We thus obtain

$$\frac{l_{coh}}{\lambda} = \frac{\lambda}{\Delta\lambda} = \frac{2.838 \text{ m}}{0{,}00269 \text{ m}} \approx 1000$$

$$l_{coh} = 1000 \cdot 2.8 \text{ m} \approx 2800 \text{ m}$$

5.3 Even laser light is not sufficient

1.(a) The wavelength is not needed. The interference pattern jumps after the time it takes the light to cover the 15 cm:

$$t = \frac{s}{v} = \frac{0{,}15 \text{ m}}{3 \cdot 10^8 \text{ m/s}} = 0.5 \text{ ns}$$

(b) $s = v \cdot t = 3 \cdot 10^8$ m/s · 1 s = 300 000 km

2. It may be that you have the same problems, namely, when the two speakers are connected to independent sine generators. To avoid them, connect them to the same generator.

5.4 Diffraction by pinholes and slits

1. (a)

$$d(\alpha) = \frac{\tan\alpha - \sin\alpha}{\tan\alpha} \cdot 100\%$$

$d(1°)$ = 0.015 %
$d(5°)$ = 0.38 %
$d(10°)$ = 1.5 %

(b) For small angles the adjacent has almost the same length as the hypotenuse.

2.

$$a = \frac{l}{d} \cdot \lambda = \frac{1.2 \text{ m}}{0.2 \text{ mm}} \cdot 520 \text{ nm} = 3.12 \text{ mm}$$

3.

$$\lambda = \frac{d}{l} \cdot a = \frac{0.2 \text{ mm}}{8 \text{ m}} \cdot 20 \text{ mm} = 0.5\ \mu\text{m}$$

5.5 Diffraction grating – the grating spectrometer

1. You need a light source that appears at a small angle: a lamp or also the sun. You orient the CD so that you can see the mirror image of the light source. (Attention: the sun is dazzling!) Then you tilt the CD slowly out of this position. You can see the spectrum of the light source pass by twice in succession.

2. (a)

d = 1/300 mm = 0.0033 mm
l = 2 m
a = 32 cm

$$\lambda = \frac{d}{l} \cdot a = \frac{0.0033 \text{ mm}}{2 \text{ m}} \cdot 32 \text{ cm} = 0.533\ \mu\text{m}$$

(b)

$$d_2 = \frac{a_1}{a_2} \cdot d_1 = \frac{32 \text{ cm}}{48 \text{ cm}} \cdot 0.0033 \text{ mm} = 0.0022 \text{ mm}$$

5.6 Two- and three-dimensional gratings

1. One obtains an interference image, namely a dot pattern, because for any wavelengths the condition is

$2d \cdot \sin\phi = k \cdot \lambda$ with k = 1, 2, 3, ...

fulfilled, and that for each of the sets of layers.

2. One obtains the largest occurring distance between the sets of layers. At these layers, the density of the diffraction centers is greatest compared to all other layers.

6. Data transfer and storage

6.2 Examples for amounts of data and data currents

1. 100 000 ≈65 536 = 2^{16}. A post code carries about 16 bit.

2. A little more than 13 bits

3. Almost 11 bits

4. $2^5 = 32$ signs

5. The tree has 27 ends at its lower side.
Since $2^4 = 16 < 27 < 32 = 2^5$, it follows that with three signs one receives between 4 and 5 bits.

6. A sign from source B carries 1 bit more than a sign from source A.

7. The magician needs 4 bits to identify the card. Each time the participant points to one of four piles, the magician gets 2 bits. The magician places the stack pointed to by the participant in the second position from above. After the first packing together of the piles, the card you are looking for is the 5th, 6th, 7th, or 8th card from the top. After the second round, it is the 6th from the top.

8. The beam balance can be charged with 5 kg. The smallest weight of the weight set is a 1 g weight. When asked „How heavy is the item?“, the scale can give 5000 different answers. So we have z = 5000. It follows H = 12.3 bits.

9. Image files generally have larger amounts of data than text files.

11. (a) Amount of data of one disc: 4 bit; Amount of data of a picture: 60 · 80 · 4 Bit = 19 200 bit

(b) There are $z = 16^{60 \cdot 80} = 16^{4800}$ different pictures. From this follows the amount o data:
$H = \mathrm{ld}\ 16^{4800}$ bit = 4800 · ld 16 bit = 19 200 bit.

12. A typical key has 5 notches, seen from the side. These have a different depths depending on the individual key. There are 16 different depths. Each notch contains 4 bits, all 5 notches together 20 bits. In addition, different keys still have different longitudinal profiles. From the locksmith we learn that there are 500 different longitudinal profiles. The longitudinal profile therefore contains another 9 bit. Thus, the key carries a total of 29 bit.

6.6 A few frequently used encodings

1. For example, the following sequence of numbers was generated:
1011100011111100101100011001011101100101

It corresponds to the sequence of letters:
abaadaaaaaacbadaacbaabacb

2. An uncompressed binary encoding is shown in the third column of the table. The amount of data per character is 3 bits. In column 4 there is an encoding which is compressed compared to that in column 3. The amount of data per character is:

H = 0.6 · 1 bits + 0.2 · 2 bits + 0.1 · 3 bits
+ 0.06 · 4 bits + 0.02 · 5 bits + 0.014 · 6 bits
+ 0.005 · 7 bits + 0.005 · 7 bits = 1.77 bits

		binary encoding	
sign	probability	without compression	with compression
a	0.6	000	1
b	0.2	001	01
c	0.1	010	001
d	0.06	100	0001
e	0.02	011	00001
f	0.01	101	000001
g	0.005	110	0000001
h	0.005	111	0000000

3. By encoding the text word by word: for each word in the dictionary there is a sign (which in turn can consist of different binary characters).

4. On a CD, music is stored for about 80 minutes. With

I_H = 1 Mbits/s

we obtain

$H = I_H \cdot t$ = 1 Mbits/s · 4800 s ≈ 5 Gbit

8. It just doubles the apparent amount of data, the actual amount of data remains the same. The redundancy is thus increased. Someone who gets the copy in addition to the CD will not learn anything new from the copy, no further uncertainty will be eliminated.

6.7 Games

1. For the answer to carry one bit, the possible answers must be equally probable. So the questions might be „Is it an even number?“ Or „Is it one of the numbers 1, 2, and 3?“

The answer to the question „is it the six?“ carries less than 1 bit, since the possible answers occur with different probabilities.

2. We assume that Lilly can select one word out of 30 000 (about the number of nouns in a dictionary). With 30,000 ≈ 2^{15}, Willy needs 15 yes-no questions when using the optimal strategy. For the answers to be equally probable, Willy will not begin with a question like: „Is the word ‚pencil’?“ but for instance with: „Is it alive?“, or „Can it be seen from here?“.

3. The apparent amount of data is 1 bit every morning. No insecurity is eliminated at all. Thus, the true amount of data is 0 bit. The probabilities of the two responses are 1 and 0, i.e. the most different they could be.

4. What he tells is redundant, the apparent amount of data is much larger than the actual one.

6.8 Data reduction

1. The addition reduces the amount of data. One can not conclude from the sum on the summands. The same applies to subtraction, multiplication and division. Even squaring reduces the amount of data, because you can not deduce the sign of the two equal factors from the square. When calculating the square root (from a square number), the amount of data is not reduced. However, if the result of the operation is rounded, the amount of data is reduced again.

3. No, because the information about the sign of x is not lost.

4. The grandfather often repeats, the grandmother talks about unimportant details.

6.10 Data transmission with electromagnetic waves – modulation

1. In 1 s: 77 500 oscillations
in 0.1s: 7750 oscillations
in 0.9 s: 69 750 oscillations

If you want to see the individual oscillations in the representation, you can not see the end of a 1-second interval, and vice versa.