

Momentum flow in the gravitational field

G Heiduck, F Herrmann and G Bruno Schmid

Institut für Didaktik der Physik, Universität Karlsruhe, Kaiserstrasse 12, 7500 Karlsruhe 1, West Germany

Received 10 March 1986, in final form 3 September 1986

Abstract In gravitation an action-at-a-distance description of the interaction between two bodies is still in use. However, the momentum current picture presents a local-causes description of this interaction. The suggested approach allows for an easy way to visualise and quantitatively sketch the stress distribution in a weak static gravitational field by means of momentum current density field lines. Computer sketches of such streamlines in the common field of the earth and the moon are presented. It is shown that two massive bodies are 'pushed together' by their common gravitational field.

Zusammenfassung Es ist immer noch üblich, Gravitationswechselwirkungen wie Fernwirkungen zu beschreiben. Das Impulsstrombild liefert eine Beschreibung dieser Wechselwirkungen im Sinne einer Nahwirkungsauffassung. Der hier vorgeschlagene Weg gestattet es, Spannungsverteilungen in schwachen, statischen Gravitationsfeldern durch Impulsstromlinien anschaulich darzustellen. Es werden Computer-Zeichnungen solcher Stromlinien für das Gravitationsfeld des Systems Erde–Mond vorgestellt. Man erkennt, dass zwei Körper durch ihr gemeinsames Gravitationsfeld 'aufeinanderzgedrückt' werden.

1. Introduction

Two massive bodies, e.g. the earth and the moon, continuously exchange momentum: the momentum of one of them changes at the expense of the momentum of the other. In the language of Newtonian physics, these momentum changes are caused by the forces the two bodies exert on one another. However, the situation can also be described from a local-causes point of view: the momentum of the bodies changes because a momentum current is flowing from one body to the other. This way of speaking presupposes that a force is a (negative) momentum current and that mechanical stress is a (negative) momentum current density (Planck 1908, Weyl 1924, Herrmann and Schmid 1984, 1985a).

Since mechanical stress is a tensor, it might seem impossible to give an easy-to-read graphical representation of a mechanical stress distribution. However, interpreting mechanical stress as momentum current density suggests a simple and straightforward representation of such distributions in terms of three streamline pictures: one for the current of the x component of momentum (or x -momentum for short), one for the current of the y component and one for the current of the z component. Such pictures demonstrate the distributions of the momentum currents within a

field. In a previous article this representation has been applied to electrostatic and magnetostatic fields (Herrmann and Schmid 1985b). In the present paper the same representation will be used to demonstrate the momentum flow in weak static gravitational fields.

In § 2 the stress tensor of weak static gravitational fields is introduced. Section 3 presents momentum streamline pictures of the field of two bodies with the same mass ratio as that of the earth and the moon.

2. The stress tensor of weak static gravitational fields

Just as electromagnetic forces can be described by Maxwell's stress tensor, gravitational forces can also be deduced from a stress tensor. The stress tensor for weak static gravitational fields can be written in a cartesian coordinate system as (Einstein 1918, Misner *et al* 1970)

$$\sigma_{ik} = -\frac{1}{8\pi G} \left[\sum_{j=1}^3 \left(\frac{\partial \phi}{\partial x_j} \right)^2 \delta_{ik} - 2 \frac{\partial \phi}{\partial x_i} \frac{\partial \phi}{\partial x_k} \right] \quad i, k = 1, 2, 3. \quad (1)$$

Here, ϕ is the gravitational potential and G the gravitational constant.

This expression has the same form as the negative of Maxwell's stress tensor for electrostatic fields:

$$\sigma_{ik} = \frac{\epsilon_0}{2} \left[\sum_{j=1}^3 \left(\frac{\partial \varphi}{\partial x_j} \right)^2 \delta_{ik} - 2 \frac{\partial \varphi}{\partial x_i} \frac{\partial \varphi}{\partial x_k} \right] \quad i, k = 1, 2, 3. \quad (2)$$

(ϵ_0 = dielectric constant of the vacuum, φ = electric potential). In the momentum current picture, the negative stress tensor is interpreted as the momentum current density tensor. In order to visualise momentum current density fields, we choose the matrix representation of this tensor in a cartesian coordinate system. The rows (or columns) of this symmetric matrix represent the 'vector' current densities of the x -, y - and z -momentum current, respectively. Thus, the tensor current density field can be visualised by three streamline pictures: one for the flow of x -momentum, one for the flow of y -momentum and one for the flow of z -momentum. The program from which our figures have been created is based upon simple extension of the program available in Merrill (1976).

3. Example of a momentum current distribution

Figure 1 shows the x -momentum current distribution within the gravitational field of two spherical bodies

Figure 1 Sketch of the streamlines of the instantaneous x -momentum current density in a plane through the centres of mass of both the earth (E) and the moon (M) (x - y plane). Because they would be too dense in the sketch, all streamlines have been suppressed within the two circular regions nearest the earth and the moon are too small to be seen on this figure.

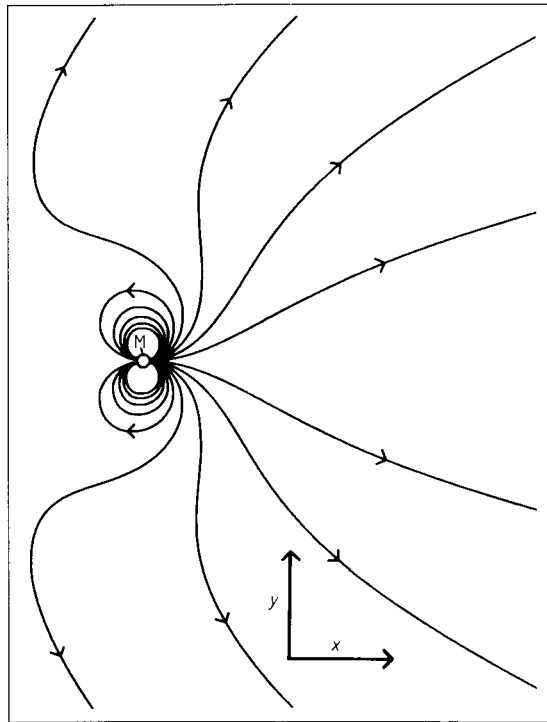
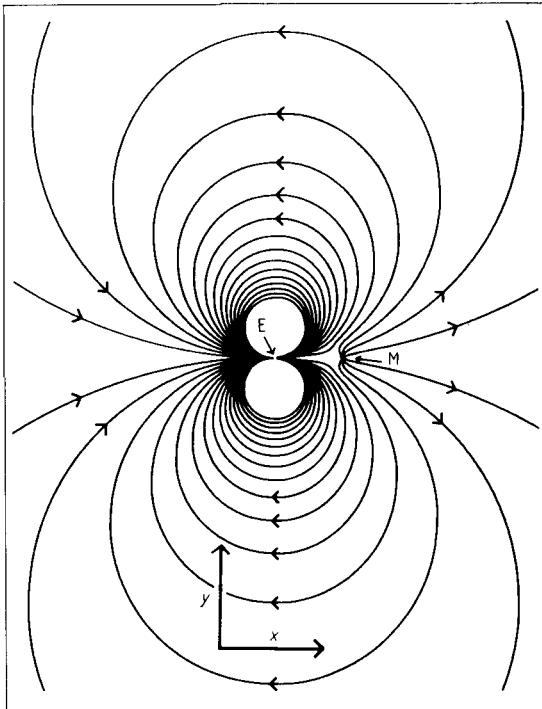


Figure 2 Details of the x -momentum current density field lines of figure 1 in the neighbourhood of the moon (M). Again the streamlines close to the moon are too dense to be represented in the sketch.

with a mass ratio equal to that of the earth and the moon. The centres of both bodies are located in the drawing plane. In figure 2 the same field is shown in the vicinity of the moon on an enlarged scale. Figure 3 shows the distribution of the y -momentum current in the x - y plane which is identical with the z -momentum current distribution in the x - z plane.

Two kinds of streamlines can be distinguished:

(1) Those lines which go back to the body from which they originate. These are responsible for the static pressure of the field on the body. This is the pressure responsible for a gravitational collapse.

(2) Those lines which connect the earth and the moon. These start at the moon and end on the earth and describe the momentum flow *between* the two bodies. Figure 1 shows that the x -momentum current in the moon is decreasing because an x -momentum current is flowing through the field to the earth and causes the momentum of the earth to increase. Figures 1 and 2 show that the momentum current does not take the shortest way from right to left: it originates at the right-hand side of the moon, follows a wide curve from right to left and enters the earth from the left-hand side. Figure 3 shows that no y -momentum (and no z -momentum) is flowing from the moon to the earth or from the earth to the moon.

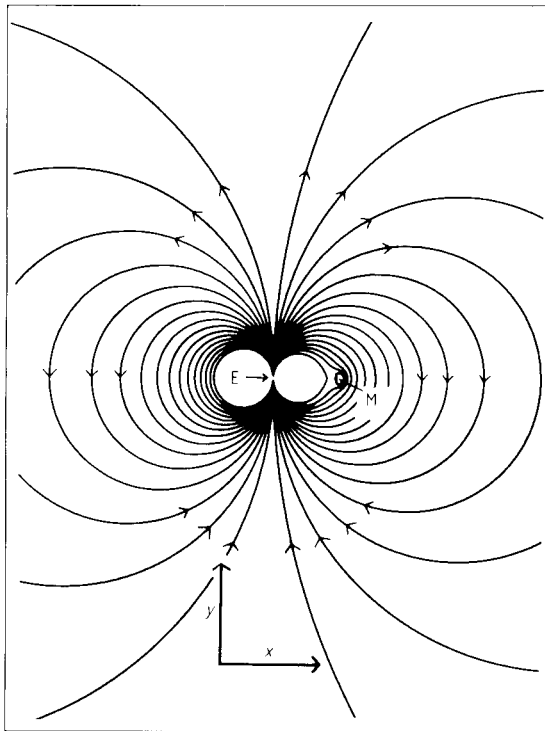


Figure 3 Sketch of the streamlines of the instantaneous y -momentum current density in the x - y plane. The streamlines close to the earth are not represented. The distribution is identical with that of the z -momentum flow in the x - z plane.

Expressed in terms of mechanical stress, this means that the bodies are not being pulled together by the gravitational field but, rather, they are being pushed together from the outside under the pressure of their common field. Schematically, such a momentum flow is represented in the yoke-and-spring assembly of figure 4. Here, bodies A and B do not interact via a gravitational field but via four springs and two bars. Springs 1 and 2, in which the x -momentum current is flowing from left to right, are under pressure, springs 3 and 4, in which x -momentum is flowing from right to left, are under tension (Herrmann and Schmid 1984). Similarly, in the gravitational field, there is local pressure in the x direction everywhere where the momentum current streamlines point in the positive x direction and local tension everywhere where they point in the negative x direction.

In elementary mechanics one usually speaks about a gravitational force acting 'along the line of centres'.

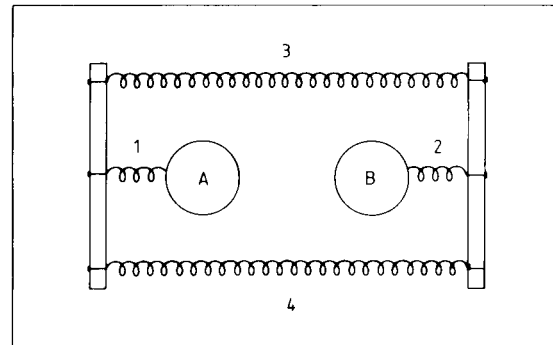


Figure 4 A simple mechanical example of how two objects A and B can be pushed together via the illustrated spring-and-yoke arrangement. The gravitational field of two massive objects serves the purpose of such an arrangement.

This way of speaking is misleading. It suggests that this line plays a particular role in the transmission of the force. Figures 1 and 3 show that this cannot be true: on the line connecting the centres of the spheres there is even a point where the current density vanishes altogether.

4. Conclusions

The language of Newtonian mechanics is a language of action-at-a-distance. The language of momentum currents is a local-causes language. From a local-causes point of view, it is natural to describe 'forces' as 'momentum currents'. Since the flowing quantity, momentum, is a vector, its current density is a tensor. A pictorial representation of this tensor field is obtained when the current of the three cartesian components of momentum are represented by three streamline pictures. From these pictures, it is straightforward to read certain properties of the field, e.g. that two massive bodies are pushed together by their common gravitational field.

References

- Einstein A 1918 *Sitzungsber. k. Preuss. Akad. Wiss.* **VIII** 154
- Herrmann F and Schmid G B 1984 *Am. J. Phys.* **52** 146
- 1985 *Eur. J. Phys.* **6** 16
- 1985 *Am. J. Phys.* **53** 415
- Merrill J R 1976 *Using Computers in Physics* (Boston: Houghton Mifflin) pp 47–54
- Misner C W, Thorne K S and Wheeler J A 1970 *Gravitation* (San Francisco: Freeman) § 20.5
- Planck M 1908 *Phys. Z.* **9** 828
- Weyl H 1924 *Die Naturwissenschaften* **12** chap. III