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Simple examples of the theorem of minimum entropy production

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Abstract The theorem of minimum entropy production governs the distribution of a voltage on two resistors connected in series and the distribution of an electric current on two resistors connected in parallel. It is suggested that this important theorem be used in introductory physics courses.

Zusammenfassung Die Verteilung einer elektrischen Spannung auf zwei in Serie geschaltete Widerstände und eines elektrischen Stroms auf zwei parallelgeschaltete Widerstände wird durch den Satz über die minimale Entropieproduktion bestimmt. Es wird vorgeschlagen, diesen wichtigen Satz bereits in einer Anfängervorlesung zu behandeln.

An important thermodynamic theorem claims that, in a steady state, the entropy production has its minimum value if the system is not too far from equilibrium. Usually, this theorem is formulated in a very general way (de Groot 1952, Prigogine 1962, Lavenda 1979, Landau and Lifshitz 1960). Thus, one might have the impression that it can only be understood in the context of an advanced physics course. However, there are some very elementary manifestations of this theorem, so that it is possible to teach it to students with almost no background in thermodynamics. It could be introduced, for example, in an introductory course on electricity.

Let U_0 be the potential difference of a voltage-stabilised power supply and R_1 and R_2 two resistors, figure 1(a). When the switch S is closed U_0 distributes in a particular way on R_1 and R_2, namely such that

$$U_0 + U_1 + U_2 = 0 \qquad (1)$$

and

$$\frac{U_1}{R_1} = \frac{U_2}{R_2}. \qquad (2)$$

Equations (1) and (2) together determine the values of U_1 and U_2:

$$U_1 = -\frac{R_1}{R_1 + R_2} U_0 \qquad (3a)$$

$$U_2 = -\frac{R_2}{R_1 + R_2} U_0. \qquad (3b)$$

This distribution of U_0 can be considered as a process. In a real circuit the capacitances between the various parts of the circuit are not zero, and we can draw a more accurate picture of our circuit by taking into account two of them, figure 1(b). Now, the distribution of U_0 over R_1 and R_2 depends on time:

$$U_0 + U_1(t) + U_2(t) = 0. \qquad (4)$$

Generally, $U_1(t)$ and $U_2(t)$ will not satisfy equation (2). Immediately after closing the switch ($t = 0$) the distribution is determined by the capacitances C_1 and C_2 alone:

$$C_1 U_1(t=0) = C_2 U_2(t=0).$$

With a time constant $(C_1 + C_2)(1/R_1 + 1/R_2)^{-1}$ the definite voltages, determined by (2), will establish themselves:

$$\frac{U_1(t=\infty)}{R_1} = \frac{U_2(t=\infty)}{R_2}.$$

This state of the circuit is called a steady state. The system will not leave it without an exterior influence.

The steady state is distinguished from all other states with $U_1(t)$ and $U_2(t)$ from equation (4) by the smallest value of the entropy production σ. If the concept of entropy is not known to the students, one may consider instead the dissipated power P_{diss}. If the temperature T has a fixed value, e.g. ambient temperature, the dissipated power has a minimum whenever the entropy production has one, since

$$P_{diss} = T\sigma.$$

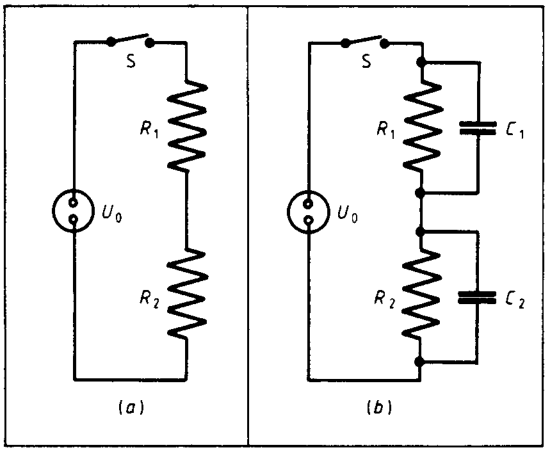

Figure 1 When the switch S is closed U_0 distributes on R_1 and R_2 according to equation (2). (*b*) is a more realistic representation than (*a*) because it shows two of the capacitances existing in a real circuit. In (*b*) it is seen that the distribution of U_0 is the result of the establishment of a steady state.

The dissipated power in our example is:

$$P_{\rm diss} = \frac{U_1^2}{R_1} + \frac{U_2^2}{R_2}.$$

With $U_1 + U_2 + U_0 = 0$ we get

$$P_{\rm diss} = \frac{U_1^2}{R_1} + \frac{(U_0 + U_1)^2}{R_2}.$$

To get the particular value $U_1^{\rm min}$ of the voltage over R_1 for which $P_{\rm diss}$ is a minimum we put $\mathrm{d}P_{\rm diss}/\mathrm{d}U_1 = 0$:

$$\frac{2U_1^{\rm min}}{R_1} + \frac{2(U_0 + U_1^{\rm min})}{R_2} = 0$$

Thus, we obtain

$$U_1^{\rm min} = -\frac{R_1}{R_1 + R_2} U_0$$

and with (1)

$$U_2^{\rm min} = -\frac{R_2}{R_1 + R_2} U_0$$

i.e. the same values as those given by equations (3a) and (3b).

It is easy to show that the theorem equally applies to the distribution of a current I_0 on two resistors R_1 and R_2 connected in parallel. States which are different from the steady state can be obtained in this case when an inductance is connected in series with each resistor. A calculation analogous to the one above shows that the power dissipation is at a minimum if $R_1 I_1 = R_2 I_2$, I_1 and I_2 being the electric currents through R_1 and R_2 respectively.

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