KPK

The Karlsruhe Physics Course

for the secondary school A-level

The Teacher's Manual

Mechanics

The Karlsruhe Physics Cours – *The Teacher's Manual*
A textbook for the secondary school A-level

- Electrodynamics
- Thermodynamics
- Oscillations, Waves, Data
- **Mechanics**
- Atomic Physics, Nuclear Physics, Particle Physics

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I

Physical Foundations

1. KPK-specific features

The characteristics of the present mechanics volume are mainly the same as those of the other volumes. The can be summarized as follows:

- The substance-like (extensive) quantities are our basic quantities. In the case of mechanics these are momentum, angular momentum and energy.
- We emphasize similarities in the structures of mechanics, electricity and thermodynamics.
- Friction is not treated as extraneous to mechanics.
- When treating the Theory of Relativity we do not begin with the invariance of the terminal velocity c upon the change of the reference frame, but with the identity of mass and energy.

Many comments about the special features of our approach to mechanics can be found in the Teacher's Manual of the junior high school course. They are not repeated here.

2. Momentum currents

The idea to interpret the quantity F as a momentum current intensity is old. In contrast to many other insights, whose origin is lost in the darkness of history, the author of this idea is quite clear. It was Max Planck [1], who proposed this interpretation of mechanical processes in 1908. Since that time it can be found in many books, in particular about the mechanics of continuous media. Unfortunately, it is never placed at the beginning of a mechanics course. Our habituation to a mechanics with forces makes us believe, that „force mechanics“ is easier to understand than „momentum flow mechanics“. Our teaching experience has convinced us, that the opposite is true. Momentum currents are not more difficult than forces, they are easier.

The description with forces, which are not a characteristic of the medium between the bodies, but which are assigned to the bodies themselves, is a sophisticated construction which Newton has realized under aggravating circumstances and for which we can admire him. He has succeeded in constructing a consistent mechanics, although an important ingredient of the theory was not available at the time: the concept of a field.

In his Principia he does not mention that something is missing (this was not the place for such judgements), but he did so in 1693 in a letter to the savant and theologian Richard Bentley:

> *That gravity should be innate inherent and essential to matter so that one body may act upon another at a distance through a vacuum without the mediation of any thing else by and through which their action or force may be conveyed from one to another is to me so great an absurdity that I believe no man who has in philosophical matters any competent faculty of thinking can ever fall into it.*

3. School physics and popular science

Popular science books are entertaining and suspenseful, and they have nice pictures. School books also try to be entertaining and suspenseful, and their pictures are getting better and better. Even so, they are lagging behind. The schoolbook's competitors are not only the popular science books, but also TV and many offers of the internet. Why are these competitors more successful? Do the good authors write popular science books whereas the schoolbooks are written by humorless pedants? Or is it that there is not enough money for the writing and production of schoolbooks? How can a schoolbook author rival the mass media TV and internet?

Actually, it is not the money that is decisive, and schoolbook authors are not the poorer authors. The problem to solve is simply not the same. There is physics that might be called „school physics“. (Some people will not agree about this claim. They say there is only one physics.)

We will contrast the characteristics of school physics with those of popular science.

1. School physics must describe the everyday phenomena. It avoids the spectacular, in order to avoid the impression that physics is responsible only for the extraordinary phenomena. That means: Not too much gimmicks, gadgets, paradoxes and thrill. This is a great restriction, since such are precisely the subjects which can help to motivate the students. If the spectacular effects are too dominant, there is the risk that only the effect will remembered, but not its meaning and not its explanation.

Popular science literature is like picking the cherries from the cake.

2. Physics at school is such that the students maintain the logical control about what they learn. The requirements relative to the consistency are high. That means effort, and it shows that one is not satisfied with arguments that would be accepted in pseudoscience or parascience. It is one of our most important meta teaching objectives to convey this high standard. Popular science presentations are not subject to this strict criteria. They rely on the confidence of the reader. That is why it is not easy to distinguish them from pseudoscientific texts.

Thus, school physics is somewhat ascetic. We school physicists should not run after the popular science authors. We would loose the race anyway. School physics is less attractive, but it is deeper. We must convince our students that they get something valuable, and thereby motivate them.

4. Experimenting

4.1 Spectacular experiments

What just has been said with respect to the text of a school book, also applies to experimenting. Spectacular experiments are entertaining and leave lasting impressions. However, usually only the spectacular effect is kept in mind and not what should have been learnt by the experiment. Thus, an experiment should not be made only because it is stunning – except perhaps in the last class before the holidays.

We leave those spectacular experiments, where you do not learn much, to the television which must keep the audience happy, or the science centers which are dependent on high visitor numbers.

School experiments are chosen according to the subject, that is to be taught. This does not mean that they are boring. Experiments should be big and authentic. It must be easy to perceive what is essential, what is not essential should remain in the background. Especially in mechanics we also make experiments that show something that is familiar to everyone. The outcome of them is known from the beginning. The only reason for doing them is to have the object of contemplation and reflection is in front of us.

We employ the air-track, but we also show that big realistic vehicles behave essentially in the same way as the air-track glider. We show with simple examples that the sensors and the computer that we employ for our measures give those results that we expect due to our common sense. In this way we create confidence in our sophisticated technical devices and avoid the impression that physics makes statements about another world.

4.2 The measuring experiment

Measuring is one of the activities of the practically working scientist. The students have to learn that in science one accepts a theory as useful only if it can be verified experimentally. That is why we also make measurements in the class room and why we check theories. The procedure need not be explained here, because every physics teacher knows it. But we would advise moderation. The pupils should know the scientific method and have enough confidence in scientific statements that they do not need the experimental evidence again and again for each claim or statement.

This remark is appropriate, since the teaching tradition cultivated in particular at the German universities has led many teachers, to believe that one should not say anything in physics lessons which is not proved by an experiment.

But what can be objected to confirm experimentally as many theoretical statements as possible? The objection is that the measurement is often based on technical tricks that have hardly any educational value. And above all it is objectionable that the quantitative experimentation costs a lot of time. Measuring and calculating are often at the expense of presenting the phenomena and at the expense of their conceptual clarification.

4.3 The precision of a measurement

When measuring something one tries to get accurate values. As a school teacher one has an idea about what is a precise measurement, what is an imprecise measurement and when we do rate an observation or measurement as qualitative. Probably as follows:

accuracy of measurement

1 %	precise measurement
5 %	measurement is all right
20 %	imprecise measurement
50 %	qualitative observation

The exact measurement is, according to habit, the best. Let us, however, consider measurement accuracies in the scientific research. The mass of the electron can be easily measured with an accuracy of about 10^{-3}. On the other hand, only an upper limit is known for the mass of the neutrino, and the accuracy of the measurements made so far is perhaps a few hundred percent. Nevertheless, one will not say that the neutrino mass experiment is the worse experiment. On the contrary, it is more difficult and more sophisticated than that with which the electron mass has been determined.

Whether a measurement is good or bad is therefore not to be seen in the measuring accuracy alone. What counts is rather how much more we know after the measurement than before. The quality of the measurement, therefore, depends on what we know about the value before the measurement. A suitable measure for the assessment of a measurement is the amount of data H or number of bits obtained with the measurement. It is calculated as follows:

$$H = \mathrm{ld}\,\frac{\Delta x_\mathrm{b}}{\Delta x_\mathrm{a}}\ \text{bit}$$

Here Δx_b is the uncertainty before and Δx_a after the measurement. If the uncertainty before the measurement was 10% percent and thereafter only 1%, then the gain is approximately 3 bits. If a value that is known with an accuracy of 1% is measured with 1% accuracy, the gain is 0 bit.

Now, the situation in school is generally different from that in research. For example, we want to measure the gravitational acceleration. If we take the view that we want to get as close to the literature value as possible, we are in an uncomfortable situation because the measurement in the classroom will surely be more inaccurate. Our quality criterion provides a negative value. Therefore, the result is somewhat frustrating.

But on can see the problem differently. One says to the pupils: Imagine, we are the researchers who make the measurement for the first time. We do not yet know the value, or better: We still know very little about it. One lets the students make an estimate. Obviously, they are reluctant. They say: We do not know. The teacher now suggests a few values, each of which is immediately recognized to be impossible or unrealistic, thus limiting the estimated interval. Let us assume that we end up with the interval between 1 and 100 m/s^2. Only now is the measurement made. Even if the measurement has an uncertainty of 10%, it turns out to be a very good measurement.

Remarks

1. Tools

What does the value of a physical quantity refer to

This section has several motivations. Firstly, the topic offers the possibility to establish links between different areas of physics. It shows the simplest form of an analogy. Pressure, temperature, electric field strength ... are analogous to each other because the values of all these quantities refer to a point, etc.

Secondly, the question as to which geometric structure the value of a quantity refers teaches us much about the quantity itself. One might think that the corresponding statements – temperature refers to a point, electric charge to a region of space – are as self-evident as the article in front of the quantity’s name, i.e. everyone knows it anyway, from the beginning. One would be wrong. Ask a physics student, to what geometrical entity refers a force, a mass, momentum, a voltage – you will be surprised. In the course of their studies, students are indoctrinated with mass points so that they can not even imagine a continuously distributed mass. The whole world seems to consist only of point-shaped objects. And do they not learn that a force is defined not only by its value but also its point of attack and its line of action? Moreover, at some moment volume forces are introduced. Under these conditions it is not evident when they finally learn (for instance, from the engineers) that a force refers to a surface area and a mass to a region of space.

Line-related quantities

We distinguish between quantities that refer to a point, a surface, and a region of space, that is, to a zero, a two-dimensional, and a three-dimensional structure. It is striking that the one-dimensional structure, the line, is missing. Actually, there are also quantities whose values refers to a line. These are quantities which can be written as a line integral, e.g. the electrical voltage. However, it would appear rather artificial to introduce to beginners the voltage as belonging to a line.

Since, in most cases, we encounter the voltage in a conservative field, the path of integration, and thus the shape of the line does not matter, and it suffices to specify its endpoints. In this case a voltage is simply a potential difference. That is why we assign a voltage not to a line but to two points. In this respect we treat it like a temperature difference or a pressure difference.

In principle, we are in a similar position for the area-related quantities. Thus, in the case of the flow of a quantity which satisfies a conservation law, only the specification of the edge of the surface matters to determine the current strength unambiguously. However, relating the current strength to the edge of the surface area would be awkward since we later introduce the current density, and this is defined for all the points of the surface through which the flow passes.

Functions with more than one independent variable

They are an important tool as they are encountered in all areas of physics: spatial distributions of pressure, velocity, mass and energy density, temperature, electrical potential, electric and magnetic field strength. In the mathematics classes such functions appear only indirectly, as functions with a parameter. The reason why the teaching of mathematics is so little concerned with the subject is that the treatment of functions of several independent variables is not very profitable, as long as the corresponding differential calculus – the vector analysis – can not yet be treated in school. Thus, we have included the distributions in space and time at a very low level in our physics lessons, so that they are later available for the description of currents and fields.

2. Momentum and momentum currents

Collision experiments

Collision experiments have advantages and pitfalls. The advantages: The actual impact process, that is, the part of the experiment in which the momentum passes from one body to the other, is so fast that one gets the feeling that it does not matter what goes on in detail. The impression arises that the outcome of the experiment can be predicted only by accounting momentum and energy. In the experiment, which we show as representative, namely the elastic collision between two bodies, which can only make a linear movement, this indeed applies.

The reason why this method works in this case is simple: the system has only two degrees of freedom, and we have two equations for determining the final state (the state after the collision), namely the balance equation for momentum and that for energy. However, here we have to do with an uncommon special case. The method does no longer work:

- if the collision is not elastic;
- if more than two bodies are involved, as for example in the case of the ball-chain (Newton's cradle);
- if the movement is not restricted to a straight line.

In order to predict the result of an experiment in these cases, other properties of the colliding bodies have to be taken into account. In the case of the ball-chain for instance the fact that a shock-wave is running without dispersion through the series of spheres.

Thus the advantage of a collision experiment is at the same time its peril. These experiments convey the impression: The energy is first here and then there, and the momentum is first here and then there. The question of what happens in between does not arise. So these experiments fit perfectly in the action-at-a-distance tradition. The question for the conductor of energy and momentum remains untackled. This way of treating collision problems does not favor a local-causes description.

Momentum conductivity

The statement „air does not conduct momentum" is valid only under restrictions. We encounter the total momentum flow in the Navier Stokes equation, which is an expression of the momentum balance and thus of Newton's laws. These equations show that there are various contributions to the momentum current.

One of the terms represent what we usually call friction. The second contribution is often called a convective momentum current. It describes that part of a momentum transport which is due to the movement of the flowing medium, and the third term corresponds to the momentum current that flows whenever the medium is under mechanical stress. This term is independent of the movement of the fluid and it has nothing to do with friction.

Each of these contributions has an electric analogue. The first one is dissipative and corresponds to an electric current through a conductor that has not zero resistance. The second one, i.e. the convective momentum current, corresponds to the charge transport in a beam of charged particles and the last one corresponds to a supercurrent.

When speaking of the electric conductivity we usually refer only to the first and to the last one: a copper wire has a certain resistance, and a superconductor has zero resistance. It does not make much sense to specify the conductivity of the vacuum for an electron beam.

In the same sense, the statements in the student's text about momentum conductors have to be understood. If it is said that air is a bad momentum conductor, this statement at first refers only to the dissipative part of the current. Later, in section 2.12, the „superconducting" current is also addressed.

Stress and strain

Mechanics is an intricate field of science because the basic quantities momentum and momentum current (force) are vector quantities. Therefore, all equations of mechanics are relations of vector quantities, and even in the simplest case of a linear relationship between two vector quantities the „factor of proportionality" is a tensor quantity. An important example is the relation between force and displacement. According to our teaching tradition this difficulty is bypassed by considering only such situations in which only one of the six independent components of the tensor is different from zero. So, in the traditional formulation of Hooke's law we remain with:

$$F = -D \cdot s \ .$$

The spring constant is the remnant of a tensor.

A consequence of this is that it is only possible to treat such forces in which the force vector is parallel to the rod which transmits the force (in which the momentum current flows). This is an arbitrary and restrictive condition. For most of the forces that we encounter in our daily lives, it is not fulfilled.

Look for instance at a big tree. Here huge forces are exerted, and school physics can not describe a single one of them. When such situations are described with momentum currents, the mathematical difficulties do not disappear, since mathematics remains the same as in the force model. Nevertheless, the momentum current description provides a considerable simplification: it allows to describe the stress state by means of streamline pictures, and these can be drawn without resort to the tensor calculus, since the conservation of the flowing quantity is automatically respected, just as the streamline picture of a water flow automatically expresses the conservation of the quantity of water.

Bending stress

Bending stress is actually a superfluous concept, because locally every stress is a compressive and/or tensile stress. The statement that a beam stands under bending stress only means that pressure and tension are spatially distributed in a particular way.

Momentum current circuits

A momentum current loop always results in another one. This is a requirement of angular momentum conservation. We do not address this complication in the classroom, because somewhere you have to stop. The usual representation with forces has, of course, the same problem. In this case however, one stops even earlier, as far as school physics is concerned: one only looks at ropes and rods in which the force is parallel to the direction of the rope or the rod, usually without saying that one considers only a special case.

The three principal stresses

The scalar quantity „pressure" emerges under special conditions from the tensor quantity „mechanical stress". Mechanical stress is a second-rank tensor. In frictionless fluids and gases, the diagonal elements of the tensor matrix are identical, all other tensor components are zero. The tensor can then be described by a single number, namely the value of the diagonal elements. This is the magnitude we call pressure.

It is not difficult to get a clear idea of the stress tensor. A small element of matter in the interior of a body can be under three mutually independent tensile or compressive stresses in three mutually perpendicular directions. In order to describe the stress state at the location of the element, six numerical values are needed:

- three of them characterize the principal directions (in order to define the orientation of a right-angled coordinate system, three numbers are required);
- the three stress values belonging to the principal directions.

In the special case of liquids and gases, the three stress values are the same. There is no longer any need to give directions; all directions are equivalent.

Another special case is that in which the stress is different from zero in a single direction. This case is found in most applications of mechanics, which are treated in mechanics lessons before hydromechanics: when a force is transmitted with a rope, or with a rod (and the force vector is parallel to the rod). For the description of such a state, it suffices to specify a single direction and a single stress value.

In most realistic situations, the stress tensor has its most general form: one needs three numbers to characterize the principal directions and three to specify the corresponding stresses. The stress state of the wood of a loaded table top is an example.

In liquids and gases pressure is the same in all directions

It is one of the objectives of the lessons to show that the pressure in friction-free liquids and gases is „the same in all directions". If the students should understand that this is a special feature, if they should understand the statement at all, they must first learn the normal case. They must realize that in general the pressure is not „the same in all directions". Thus, they must realize that an object may be under different pressures in different directions.

The fact that a solid body can be subjected to various pressures in different directions is easy to see. That there are exactly three independent directions, on the other hand, is quite difficult to understand. And with the tools available to us in the class also difficult to show. Therefore the fact is simply told by the teacher.

3. Angular momentum and angular momentum currents

Angular momentum is treated in more detail in the KPK than is otherwise customary. The reason: it is not often that one obtains in the classroom so many important results with so little effort. Quantitative relations almost fall into one’s lap. And as far as technical applications are concerned, rotational mechanics is certainly no less important than the mechanics of translational motions.

4. The gravitational field

The gravitational field strength

We introduce the magnitude gravitational field strength. Thereby we mean the „gravitostatic“ field strength resulting from the Earth’s mass, without the contribution to the gravitational acceleration, which originates from the rotation of the Earth and exists in the rotating reference system. This latter does not exceed 0.5% of the value of *g*. It is neglected by us, inasmuch as we say that only the gravitational field strength is responsible for the free fall. For the gravitational acceleration, we only give the value 9.8 N/kg, that is, with only one place behind the comma, and we do not address the differences in the magnitude of *g* for different locations on the Earth's surface.

5. Momentum, angular momentum and energy

Friction: when teaching it?

Friction is addressed for the first time as a phenomenon in which momentum flows from one body to another. However, we have moved its detailed treatment into the chapter about energy. For only here can we explain what we consider the essential of a frictional process: the production of entropy. This also means that we do not classify the so-called „static friction“ into the same category of processes as ordinary friction.

In the field of thermodynamics, the pair of concepts state/process plays an important role. The normal friction (with entropy production) would be referred to as a process in this sense, while the static friction is a state.

System dynamics programs

The topic of friction is well suited to be modeled with a system dynamics software (such as STELLA, Powersim, Dynasys, Berkeley Madonna or Coach) or for the treatment with a spread sheet (like Excel or Numbers). Usually, the relationship between the frictional momentum current and the velocity difference is known. Using the modeling system or the spread sheet, the velocity is calculated as a function of time. The learning effect is great, the time investment is appropriate. A differential equation is solved numerically, the solution of which is not trivial, but not too complex.

6. Reference frames

In which context should the subject be treated?

The subject is an inevitable evil. Imagine we have to do with a certain physical situation, that has a certain symmetry. As soon as we begin to describe the situation mathematically, we must break the symmetry.

The directions right and left are a priori equivalent. But as soon as we introduce a coordinate axis, we favor one direction over the other. A car A, which is running to the left at 110 km/h, is faster than a car B, which travels at 80 km/h to the right, and nevertheless:

$v_B > v_A$.

A force in a string is pointing to the right or to the left depending on how we orient the vector of the area to which the force refers. As soon as we have decided for a certain orientation of this vector –and we have to decide– the symmetry is broken.

These are just the most innocuous examples. Here are some more annoying. Electromagnetic induction requires a different explanation, depending on whether the coil is stationary and the magnet is moving or reversed.

The momentum balance for a rotational motion requires a very different description depending on whether it is described in the laboratory frame of reference or in the rotating reference system: in the first case the momentum changes constantly due to the centripetal force (a momentum current whose intensity vector is parallel to the radius of the rotational movement) and in the second there is equilibrium of forces between centripetal and centrifugal force. (The rate of the inflowing momentum is equal to that of the outflowing momentum.)

Up to now we have tried to avoid these complications in the class room. We have described the phenomena either independently of the reference system (for example, the electromagnetic induction), or we have described them from the beginning in that reference system, in which it presents itself in its simplest form.

For a description in different reference systems, significantly more teaching time would be needed, and it would also have distracted from the actual phenomenon. The phenomenon would appear more complicated than it is.

However, one can also see the good side of the problem. By describing a phenomenon in different reference frames, something new can be learned.

Instead of spreading the treatment of the reference system changes over the whole course, we have dedicated a separate chapter to them. So we do not emphasize the complication that occurs in each particular case, but we aim at a meta-result: Upon the change of the reference system the mathematical description changes, but not the phenomenon itself.

Reference frame and zero point

We have placed the subject in a somewhat wider context than is otherwise customary. Usually it is only asked how the description of a phenomenon is modified when one makes a change from a reference system S to a system S' where S' moves with respect to S with a constant velocity or a constant acceleration. However, this can also be expressed differently: One chooses a new zero point for the velocity or for the acceleration. Changing the reference system is therefore nothing else than shifting the zero point. But this is an operation which can also be discussed in other contexts: the choice of the zero point of temperature, pressure or electrical potential.

Velocity addition

It is awkward in two respects to speak of velocity addition:

1. What is added or composed are not velocities, but velocity differences. The electrical analog is calculating a voltage when two batteries or other electrical components are connected in series.

2. Mathematics tells us what addition means. The relativistic formula with which one calculates the velocity change upon a change of the reference system does not correspond to the addition operation.

The concept reference frame

The term is used in different ways: sometimes as a body, which is assumed to be at rest, sometimes as a coordinate system. In his explanation of the term, Einstein is very careful: he first introduces the reference body, thereafter the reference system.

Two reference systems, which do not move against each other, but differ only in the zero point of the position, or in the orientation of the axes, represent two different reference systems. When we speak of a reference system change in the class room, we do not consider this kind of changes. The clearest handling of the subject would be to present the change of the reference frame as a change of the zero-point of the velocity, and later also of the acceleration. The disadvantage of this approach would be a loss of vividness. I imagine that I am localized in one of the reference systems. In other words: for me, my body is the „natural reference system“. And then the first question is: „Where am I?“ and not „How do I move?“.

Free floating reference frames

In the treatment of space-time, a class of reference systems plays an special role: the reference systems in which the gravitational field strength is zero. We refer to them with Wheeler [2] as floating or free-floating reference systems. Today, they are often called inertial systems. The choice of this name is not fortunate since it meant something else when it was introduced by Lange in 1885. For Lange an inertial system was a reference system in which force-free bodies move in a rectilinear uniform manner. By „force-free“ was meant also free of gravitational forces. A free-falling body on the earth is not free of forces, and the reference system defined by it is, in the sense of Lange, not an inertial system. It is, however, a free-floating reference system in Wheeler's sense.

7. Terminal velocity

The axiom of special relativity

To get from the classical to relativistic mechanics, one has to introduce an additional „axiom“ (and one must know which rules of classical mechanics remain unchanged). Einstein chose the observation that the velocity of the light is independent of the reference system of as an additional axiom. This was logical, because it was just the result of the Michelson-Morley experiment. A new mechanics had to be constructed that took account of this discovery.

When the theory is introduced today, after all its consequences are experimentally confirmed, it is advisable to choose another statement of the theory as an additional axiom.

One of the new propositions is the claim that mass and energy are equivalent quantities, that is, that E and m are the same quantity, only measured in other units of measure.

In our course we choose the energy-mass equivalence as a new axiom. An advantage of this choice is that the first encounter with the theory of relativity does not occur in the context of kinematics. So one has not to do at the very beginning with the cognitive conflicts that are caused by the fusion of space and time into space-time. If one proceeds in this way, it is consistent, to write the relation between E and m not in the form

$$E = mc^2$$

This equation suggests that the energy is proportional to the square of the velocity of the light, or, in other words, if the velocity of the light is doubled, the energy quadruples – which, of course, is not meant. We write instead:

$$E = k \cdot m.$$

The velocity of light

We call the constant c not „speed of light“ for when one does so, the impression arises that light plays a special role in relativistic mechanics. c is the limiting speed for all bodies and particles. Just because light, at least according to our present knowledge, has no rest mass, it is also the only velocity that the light can have.

The term „particle accelerator“

We have used the term „particle accelerator“ with a little bit of discomfort as the generic term for the machines, which for a variety of reasons bear the most varied names: particle accelerators, boosters, synchrotrons, storage rings and colliders. Neither of the other names would have been more appropriate.

A relativistic measure for inertia

In the equation $p = m \cdot v$, m is the mass which is identical with the energy according to the equivalence principle. In the classical version of the formula it is also a measure of the inertia because it tells us how much momentum must be supplied to a body to change its velocity. This interpretation is no longer adequate at relativistic velocities. The inertia would better be described by the expression dp/dv, instead of p/v. The two expressions are identical only for non-relativistic velocities. However, $m = p/v$ reflects the inertia qualitatively also in the relativistic case, since both p/v and dp/dv go to infinity for $v \to c$.

The relativistic formulae

It is not difficult to derive the function $E(p)$. In

$$p = m \cdot v$$

we substitute our relativistic axiom

$$E = k \cdot m$$

and obtain:

$$p = \frac{E}{k} \cdot v \ .$$

We calculate from this

$$v = k \cdot \frac{p}{E} \ ,$$

substitute in into the generally valid relation

$$dE = vdp$$

and obtain

$$dE = k \cdot \frac{pdp}{E} \ .$$

Next we bring E to the left:

$$EdE = k \cdot pdp.$$

It follows:

$$d(E^2) = k \cdot d(p^2)$$

or

$$d(E^2 - kp^2) = 0.$$

Through integration, we obtain:

$$E^2 - kp^2 = \text{ const} = E_0^2$$

or

$$E(p) = \sqrt{kp^2 + E_0^2} \ .$$

From this one easily gets the relations $v(p)$ and $E(p)$. Probably, for the school these derivations are too difficult.

Momentum and energy current

We treat the identity of mass and energy. This causes another identity, which is just as simple: momentum and energy current are identical. (Attention: not momentum current and energy current.) The identity of the two quantities manifests itself in the energy-momentum tensor:

$$\begin{pmatrix} \rho_E & \frac{j_{Ex}}{c} & \frac{j_{Ey}}{c} & \frac{j_{Ez}}{c} \\ c\rho_{px} & \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ c\rho_{py} & \sigma_{yx} & \sigma_{yy} & \sigma_{yz} \\ c\rho_{pz} & \sigma_{zx} & \sigma_{zy} & \sigma_{zz} \end{pmatrix}$$

The first row contains the energy density and the three components of the energy current density (divided by c). In the first column below the energy density there are the three components of the momentum density (multiplied by c). The remaining 9 components represent the mechanical stress tensor, i.e. the current density components of the x, y, and z momentum. Since the tensor is symmetric, the energy current density (apart from the c-factors) is equal to the momentum density density vector. That the two quantities are identical (up to the factor c^2) is plausible: momentum is „mass in motion“. With the fact that mass = energy, it follows that momentum is energy is in motion. Energy in motion, however, is an energy current. This is not difficult, and it might seem appropriate to address it in the classroom. We do not do it, however, since we do not need this result again.

Mechanical stress within the gravitational field

Static gravitational fields are very similar to electrostatic fields. The two fields differ, however, in one essential point: while in an electric field there is tensile stress in the direction of the field strength vector, the gravitational field is in the field line direction under compressive stress. In the directions perpendicular to the field lines there is pressure in the electric field and tension in the gravitational field. It is easy to convince oneself of this difference by considering a hollow sphere. An electrically charged hollow sphere has the tendency to fly apart. It has no electrical field in its interior. Therefore the field must pull from the outside. A hollow sphere that has mass has the tendency to implode. It also is field-free in its interior. The field must therefore press from the outside.

We have argued here within the framework of Newtonian mechanics, i.e. in the Euclidean space. In the general relativistic description, the gravitational forces disappear, there is no longer a gravitational field strength, and the mechanical stresses in the gravitational field also disappear.

The usage of the terms mass, rest mass, energy, rest energy, invariant mass, relativistic mass and internal energy

There are two concepts, but seven different names are in use for them, and unfortunately the assignment of the names to the concepts is not unambiguous. In particular, the name mass is used in two different meanings. The problem arose with the theory of relativity. On the one hand, we have the discovery that the old quantities energy and mass are the same physical quantity: energy has the same properties as mass, namely, gravity and inertia.

On the other hand, the theory of relativity describes the physical world with four-vectors and their Lorentz-invariant magnitudes. Lorentz invariants are practical. They do not contain the arbitrariness due to the choice of the reference system. Since the mass had stood for centuries for something characteristic of a particle, something which constitutes the essential of its identity, which does not depend on the reference system, the word was intended to remain in this role in the future. Hence the name, especially in particle physics, was used for the Lorentz-invariant magnitude of the four-vector, that is, what was initially called rest mass.

Thus two concerns are competing:

- the name mass as a measure of the inertia (which for a given body can be large in one reference system and small in another);
- the name mass for a quantity which characterizes a particle and whose value is independent of the reference system.

Thus the chaos was preprogrammed.

Using the mass as a universal measure of inertia requires a new name for the value of the mass in the center-of-mass reference frame. This could be the term *rest mass* or *rest energy*. What is meant by „rest“ is that the center of mass of the system under consideration is at rest. Apart from this, there may be any amount of unrest.

Those who use the designation mass for the Lorentz invariant had to find a new name for the measure of inertia. They called it relativistic mass. And if one had to fear that someone does not know that the quantity m_0 is simply called mass, one added the adjective invariant for security; It became the *invariant mass*.

We have chosen the first alternative: mass as a measure of the inertia and gravity of a body or particle. The magnitudes (or better values) E_0 and m_0 are called *rest energy* and *rest mass*, even though nothing is at rest in a system whose total energy is equal to the rest energy – except its center of mass. The term *internal energy* would be more appropriate here.

8. Spacetime

The names Special theory of relativity and general theory of relativity

Both names are due to Einstein. It is not difficult to understand the motivation for calling them theories of *relativity*. As far as the school is concerned, however, these names are rather unsuitable to describe what is done in the classroom.

When we treat electricity in the classroom we neither call it Maxwell theory.

The first step towards an understanding is not theory. The word theory is somewhat intimidating; it suggests, that the subject is difficult and for some students even incomprehensible.

We therefore prefer the title *spacetime*, i.e. the object of our concern, and not the name of the theory that describes it.

The delimitation between special and general relativity

The boundary between the two is not clearly defined. Often one counts phenomena and effects to the General theory of relativity (GTR) as soon as accelerations occur. We believe that this is an awkwardly chosen criterion, because wether accelerations occur or not, is only a question of the reference frame, and not of the phenomenon itself. We are dealing with a real GTR phenomenon only when the curvature of space-time plays a role , i.e. when one needs to use Einstein's field equation in order to treat a problem mathematically. As long as the gravitational field is homogeneous, we do not need the GTR.

Dynamics before kinematics

In the KPK, we have chosen a structure of relativistic mechanics that deviates from the usual approach. It can be described in a somewhat cursory way: first dynamics, then kinematics.

However, the denomination kinematics is not quite appropriate, because what has to do with the description of the movements in space and time does not actually deserve the name kinematics. We prefer the name spacetime physics.

But why not treat the space-time at the beginning? Because it is clearly the more difficult part of the special theory of relativity. The main cause of problems in learning are, according to our experience, the changes of the reference system. A second, more conceptual difficulty is the fusion of space and time to spacetime.

If one starts with relativistic dynamics, one can avoid both problems. One gets important results without getting involved with them.

Where to stop?

More than in other physical domains, the physics of spacetime makes difficulties when it has to be elementarized; when a course has to be designed that does not cover the whole program. It seems as though one can only understand something of relativistic kinematics, if one has already understood everything, that is, all the implications and consequences of the Lorentz transformations. The Lorentz transformations are not mathematically difficult. But with all the trimmings, one would need more time than we have at our disposal. We therefore designed the course in such a way that the treatment of the subject spacetime can be discontinued after two hours of teaching, i.e. after having treated sections 8.1 to 8.4. At that point the most important goal has already been reached, namely, that in a flat spacetime the geodetic connection between two spacetime points corresponds to the greatest time difference.

Up to this point, no changes of the reference system are necessary, and one avoids the endless discussions about for whom a distance or duration is greater.

As regards the subsequent sections, the topics of spacetime distance, and reference system dependence of lengths and time intervals are dealt with. We have tried to get along without treating Minkowski diagrams and the problem of clock synchronization.

The terms length contraction and time dilation

Both the length of a body and the duration of an process change when the reference system is changed. As is known, this is referred to as length contraction and time dilatation.

We believe that these terms are unfortunate, because both suggest that we have to do with a process: when something is contracted, it is first longer and then it is shorter. In our case, however there is no process in the physical sense, but a physical quantity has different values in the same state in two reference systems.

In another context one would not use such a wording either. If, for example, the kinetic energy in one reference system has a larger value than in another, one would not say that the energy has increased.

The names event and spacetime point

Sometimes a point in spacetime is called an „event". We try to distinguish between the terms „event" and „spacetime point", in the same way as one usually distinguishes in the normal three-dimensional space between an object and its location, or between a container and its volume.

The movement on a world-line

It is often said that a body moves on a world line. We find this expression problematic. A body moves on its trajectory in the normal three-dimensional space. But what does it do in the spacetime? It can move only in space, not in time. We have tried to find a language that takes this fact into account. Just as there is no movement, there is no current in spacetime either.

Relativity of simultaneity and clock synchronization

It is customary to deal extensively with the relativity of simultaneity within the framework of the theory of relativity. We believe that the issue takes too much space in the curriculum. It is intricate, but hardly needed for anything that matters.

The question of whether two events occurring at different locations are simultaneous arises from the conviction that there is a time that is independent of the location and the velocity, i.e. a certain parameter that allows to position or arrange the states of the world as a whole. In order to answer the question, one has to define a method that allows to decide wether two distant events are simultaneous or not. This is done by explaining how to „synchronize" clocks, which are located in different places.

In order to gain some distance, let us ask a different but similar question: Is the „equality of position" relative? In a more fluent language: Do two events occur for one person in the same place, as for any other person? By „for one person" and „for any other person" we mean „in one reference system" and „in any other reference system". The answer is, of course, 'no'. It is so self-evident that nobody would have the idea to ask the question at all.

The fact that the statement about simultaneity is rather insignificant is best seen when viewed from the perspective of the theory of general relativity. For there the question melts away like snow in the sun; the concept of simultaneity loses its meaning, since one can no longer „synchronize" two identical clocks.

Why, however, does the relativity of simultaneity have no important consequences? Because the relationship between events that are simultaneous in one reference system and not in all the others is space-like. One of the events is not causally connected to the other. The reversal of the chronological sequence therefore has no consequences.

9. Space and gravitational field

The components of the gravitational field

As long as one is restricted to „gravitostatics", the description of the gravitational field is simple. The distribution of the field strength in the vicinity of a mass point is, except for a sign, the same as that of the electric field strength in the vicinity of a point charge. Gravitostatics and electrostatics have the same structure. But just as electrodynamics needs a second field (or better: further components of the electromagnetic field), as soon as moving and accelerated charges are considered, the gravitational field also gets more components as soon as the bodies are accelerated. The corresponding phenomena are also called gravitomagnetic effects in analogy to electrodynamics.

For the complete description of the electromagnetic field, the 6 components of the two field strengths *E* and *B* are needed. (If the the fact that the divergence of the magnetic flux density is zero is already included in the description, one remains with four independent components: the electric scalar potential and the three components of the magnetic vector potential.) For the description of the gravitational field, on the other hand, a four-dimensional symmetrical 2-nd order tensor is needed, the metric tensor, which has 10 independent components.

Inertia and gravity in different reference frames

Within the framework of classical mechanics, inertia and gravity appear as two different properties of bodies, which, however, are characterized by the value of the same quantity, the mass. Einstein [4] comment to this strange fact was: „The previous mechanics have registered this important observation, but not interpreted. A satisfactory interpretation can only be achieved in such a way that one would see: The same quality of the body expresses itself as ‚inertia' or ‚gravity' depending on the circumstances." By „depending on the circumstances" he means: depending on the reference system. This issue is discussed in the student text.

Newton's absolute space

Phenomena that are related to gravitation can be divided into two classes.

Some have to do with the gravitational interaction of bodies that are at rest relative to each other. Classically, they are described by the law of gravitation and are very similar to the phenomena of electrostatics. The bodies „feel" that part of the gravitational field, which is described by the well-known vector field of the gravitational field strength.

The other phenomena occur when bodies are accelerated. Today one describes them (together with the static phenomena) with the general theory of relativity, i.e. with the help of the metric tensor.

How did Newton deal with these two effects? The first, the gravitostatic, he described with his gravitational law. The fact that this gravitational effect is acting over a distance, was felt by him as a deficiency, and he said that very clearly. In spite of his discomfort, he did not go so far as to introduce a medium which could transmit the quantity of motion (quantitas motus). His „Hypotheses non fingo" is well-known. Actually, it would have been consistent to accept the stars as the cause of the second effect, the inertial forces, as the somewhat younger George Berkeley suggested. Here, Newton has preferred an interpretation that is actually the healthier one. The inertial forces are not caused by distant bodies, but by the „absolute space", i. e. by something that is in the same place as the body [6]. „Absolute space, by virtue of its own nature and without reference to any external object, always remains the same and immovable."

Newton was often blamed for this statement, because, as the argument goes, there is no absolute space. However, when criticizing Newton some caution is needed. With his absolute space, he describes the inertial forces as a local effect. One can therefore see Newton's absolute space as a precursor of modern space-time [7]. From today's point of view, this description is more appropriate than that of the static forces with actions at a distance.

The Michelson-Morley experiment and the ether

The experiments by Michelson and Morley have shown that the speed of light is independent of the reference system. This experiment had various consequences for physics. One of them was of epochal importance: the theory of relativity. The other one was only indirectly related to this theory: It was concluded that there is no ether. Both consequences are often formulated together – almost as if the non-existence of an ether is simply one of the numerous new statements of relativity. Sometimes it is mentioned only by the way, as in Einstein's publication from 1905 [8].

Let us show with a thought experiment that these are two different statements and that the one does not follow from the other. A car drives at high speed on a conveyor belt, which is initially at rest. The speed of the car relative to its support is almost equal to the terminal speed *c*. We now turn on the conveyor belt, so that it is moving in the same direction as the car. We would find that the car is still moving with *c*. And even if we run the conveyor belt in the opposite direction, the speed of the car would remain nearly equal to *c*. Let us now assume that this experiment was made instead of the Michelson-Morley experiment. What would have been concluded? It would have been concluded that there is a terminal velocity and that when changing the reference system velocities should not simply be added. This observation might then have led to the special theory of relativity, just as the real Michelson-Morley experiment actually led to the theory of relativity. However, one would not have deduced from the observation of the car that the carrier of the car, namely the conveyor belt, does not exist. However, such a conclusion was drawn in the cases of the Michelson-Morley experiment: from the fact that the speed of the light does not change as the reference system is changed it was concluded that the carrier of the light wave does not exist.

As long as the theory of relativity did not exist, the conclusion on the non-existence of the ether seemed to be the only possible way out of the dilemma that was caused by the Michelson-Morley result. However, Einstein's theory solved the problem in a completely different way. The fact that Einstein himself initially opined that the ether is a superfluous concept can be classed as a slip-up. Soon afterwards he writes: „According to the general theory of relativity, a space without ether is unthinkable." [9]

What could not be done

From the previous remarks it follows that we have not discussed the geometric version of the theory of gravitation, i.e. the authentic Einstein formulation in which there are no gravitational forces (or momentum currents), no energy density and even the concept of the gravitational field no longer exists. Hardly anything of what the students had learned in the mechanics class would have survived. In the teaching time, which one would be willing to dedicate to the subject, one would not have gone beyond a few nice sentences.

10. Cosmology

Space and time or spacetime?

To show that space and time form a unity is an important learning goal. We have kept the subject brief. The insight into the link between space and time becomes clear only when reference system changes are discussed. We have previously explained why we are restrained ourselves with the treatment of reference system changes. Another reason is that in the two solutions discussed in the classroom, space and time are coupled in such a way that one can discuss the curvature of the space without reference to the much less intuitive curvature of spacetime.

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III

Solutions to Problems

1. Tools

1.6 Streamlines

2. The streamline picture must not change in time.

3. Arrows of equal length but, according to the magnitude, of different thickness.

Arrows of equal length but, according to the magnitude, of different gray shading.

2. Momentum and momentum currents

2.2 Momentum currents

1.

$m_M = 70$ kg
$m_I = 52$ kg
$v_{I(before)} = 4.5$ km/h
$m_I \cdot v_{I(before)} = (m_I + m_M) \cdot v_{after}$

$$v_{after} = \frac{m_I \cdot v_{I(before)}}{(m_I + m_M)} = \frac{52\ \text{kg} \cdot 4.5\ \text{km/h}}{52\ \text{kg} + 70\ \text{kg}} = 1.9\ \text{km/h} = 1.9\ \text{km/h}$$

4.

Before: $p_A = 3$ Hy $p_F = 0$ Hy
$\Delta p_A = -2$ Hy $\Delta p_F = 2$ Hy
After: $p_A = 1$ Hy $p_F = 2$ Hy

5. At the beginning, the car may roll in the x direction, thereafter in the y direction.

30 km/h = 8.3 m/s

Before: $p_x = 10\,000$ Hy $p_y = 0$ Hy
After: $p_x = 0$ Hy $p_y = 10\,000$ Hy
$\Delta p_x = 10\,000$ Hy $\Delta p_y = -10\,000$ Hy

The momentum that makes the difference comes from the Earth.

2.3 Momentum currents in friction processes

1. The magnitude of the velocity of the block is greater than the magnitude of the velocity of the board. However, if the sign is taken into account, the velocity of the board is greater. Momentum is flowing from the board to the block, since the block is loosing negative momentum. It thus receives positive momentum. Momentum flows from the board, which has the higher velocity, to the block.

2. No. Here momentum goes from a body of lower to a body of higher velocity. Our rule cannot be applied because it is not a frictional process.

2.6 Flow equilibria

1. (a) The motor pumps momentum from the Earth into the car.
(b) The momentum slowly flows away into the air and into the Earth.
(c) The momentum flows quickly into the Earth.
(d) All of the momentum, that the motor is pumping into the car, is flowing away again.

2. There is no friction. Consequently, no momentum enters or leaves the body.

2.7 Compressional, tensional and bending stress

1. The tow coupling is under tensional stress, Fig. 2.1.

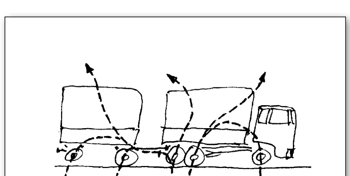

Fig. 2.1

2. Momentum flows to the right, i.e. out of the trolley, because the negative momentum of the trolley increases.

3. See fig. 2.2

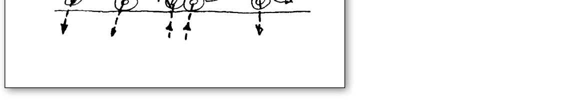

Fig. 2.2

2.9 The momentum current strength

1.

$t = 10$ s
$p = 200$ Hy

$$F = \frac{p}{t} = \frac{200\ \text{Hy}}{10\ \text{s}} = 20\ \text{N}$$

2.

$F = 6000$ N
$t = 5$ s
$p = F \cdot t = 6000\ \text{N} \cdot 5\ \text{s} = 30\,000$ Hy

2.10 Newton's law of motion

1.

$t = 5$ s
$m = 150$ kg
$F = 15$ N
$p = F \cdot t = 15\ \text{N} \cdot 5\ \text{s} = 75$ Hy

$$v = \frac{p}{m} = \frac{75\ \text{Hy}}{150\ \text{kg}} = 0.5\ \text{m/s}$$

2.

$F = 200$ kN
$t = 30$ s
$v = 54$ km/h = 15 m/s
$p = F \cdot t = 200\,000\ \text{N} \cdot 30\ \text{s} = 6 \cdot 106$ Hy

$$m = \frac{p}{v} = \frac{6 \cdot 10^6\ \text{Hy}}{15\ \text{m/s}} = 4 \cdot 10^5\ \text{kg} = 400\ \text{t}$$

3.

$m = 42$ kg
$F = 20$ N
$t = 3$ s
$v = 1.2$ m/s

Momentum that has flowed into the trolley:

$p = 20\ \text{N} \cdot 3\ \text{s} = 60$ Hy

Momentum contained in the trolley:

$p = m \cdot v = 42\ \text{kg} \cdot 1{,}2\ \text{m/s} = 50{,}4$ Hy

The missing momentum has gone into the Earth due to friction.

4.

$l = 2$ km
$d = 10$ cm
$v = 0.5$ m/s
$t = 2$s
$V = \pi (d/2)^2 l = 15.71\ \text{m}^3$
$m = \rho \cdot V = 15\,710$ kg
$p = m \cdot v = 15\,710\ \text{kg} \cdot 0{,}5\ \text{m/s} = 7855$ Hy

The momentum goes into the Earth via the valve.

$$F = \frac{p}{t} = \frac{7855\ \text{Hy}}{2\ \text{s}} = 3928\ \text{N}$$

2.11 Convective momentum transports

1.

Water current: 0.5 l/s

$v = 3$ m/s
$l = 1$ m

In one second a section of a length of 3 m of the jet traverses the cross-sectional area. This corresponds to a volume of 0.5 l. A section of 1 m length has a volume of 0.5/3 l = 0.167 l. The mass of the corresponding water is

$m = 0.167$ kg.

$p = m \cdot v = 0.167\ \text{kg} \cdot 3\ \text{m/s} = 0.5$ Hy

$$F = \frac{p}{t} = \frac{m}{t} \cdot v = 0.5\ \text{kg/s} \cdot 3\ \text{m/s} = 1.5\ \text{N}$$

2. Volume current:

$$\frac{V}{t} = 10\ \text{m}^2 \cdot 5\ \text{m/s} = 50\ \text{m}^3/\text{s}$$

Mass current:

$$\frac{m}{t} = \rho \cdot \frac{V}{t} = 1.293\ \text{kg/m}^3 \cdot 50\ \text{m}^3/\text{s} = 64.65\ \text{kg/s}$$

Momentum current:

$$F = \frac{m}{t} \cdot v = 64.65\ \text{kg/s} \cdot 5\ \text{m/s} = 323\ \text{Hy/s} = 323\ \text{N}$$

2.12 More about momentum conductors

1. The positive x direction is to the right. The connection is not conductive for x momentum, and conductive for y and z momentum.

2. The connection is not conductive for momentum whose vector is in the plane of the arrangement and conductive for momentum that is perpendicular to this plane.

3. The construction results in about 470 N.

4. Two sticks plugged into each other. Upon a tensile load they detach from each other.

5. Valve of bicycle or car tire, turnstile at the exit of the metro station, semiconductor diode

2.13 Hooke's law

1.

(a) $s = \dfrac{F}{D} = \dfrac{12\ \text{N}}{150\ \text{N/m}} = 0.08\ \text{m}$

(b) $s = \dfrac{24\ \text{N}}{150\ \text{N/m}} = 0.16\ \text{m}$

2.

(a) For $F = 15$ N we get $s = 0.32$ m;
for $F = 30$ N we get $s = 0.4$ m.

(b) For $s = 0{,}2$ m we get $F = 4$ N.

(c) With increasing prolongation it becomes ever more difficult to further extend the rope.

3. The two ends of a twine are connected with the two ends of a spring in such a way, that the twine hangs loosely as long as the string is not tended. If now the spring is stretched the momentum flows at first only though the spring and Hooke's law applies. But as soon as the twine is completely stretched no further prolongation is possible. The momentum current increases but neither the twine nor the spring becomes longer.

4. We call the springs A and B. We thus have:

$F_A = D_A s_A$ and $F_B = D_B s_B$.

Since the same momentum current flows through both springs, we have, $F_A = F_B$, thus $D_A s_A = D_B s_B$. Since $s_A = 4 s_B$ we obtain
$D_B = 4 D_A$.

5. (a) For each spring taken separately we have:

$F_1 = D \cdot s$ and $F_2 = D \cdot s$.
Thus the total momentum current is
$F' = F_1 + F_2 = 2Ds$.
For the whole arrangement we get:
$F' = D's$ with $D' = 2D$.

(b) For each spring taken separately we have:

$D \cdot s_1$ and $F = D \cdot s_2$.
Thus the total prolongation is

$$s' = s_1 + s_2 = 2\frac{F}{D} .$$

For the whole arrangement we obtain:
$F' = D's$ with $D' = D/2$.

2.14 Velocity, acceleration, angular velocity

1.

$d = 30$ cm
$\omega = 3500$ revolutions per minute

$$\omega = 3500 \cdot \frac{2\pi}{60\ \text{s}} = 366.5\ \text{s}^{-1}$$

$v = \omega \cdot r = 366.5 \cdot 0.15\ \text{m/s} = 55$ m/s.

2.

$$\omega = \frac{2\pi}{1\ \text{day}} = \frac{2\pi}{24 \cdot 60 \cdot 60\ \text{s}} = 7.3 \cdot 10^{-5}\ \text{s}^{-1}$$

$r = 6370$ km

$v = \omega \cdot r = 7.5 \cdot 10^{-5} \cdot 6.37 \cdot 106\ \text{m/s} = 478$ m/s.

3.

$$\omega = \frac{2\pi}{1\ \text{year}} = \frac{2\pi}{365 \cdot 24 \cdot 60 \cdot 60\ \text{s}} = 2.0 \cdot 10^{-7}\ \text{s}^{-1}$$

$$r = \frac{v}{\omega} = \frac{30\ \text{km/s}}{2 \cdot 10^{-7}\text{s}^{-1}} = 150 \cdot 10^6\ \text{km}$$

2.15 Momentum changes for circular movements

1. We suppose, that the straight section before the quarter circle is oriented in the positive x direction, the second straight section points in the positive y direction.

At the beginning and at the end of the circular section the steering wheel has to be turned abruptly into the new position.

The time rate of change of the x momentum jumps at the beginning of the bending of the street from zero to a negative value and then decreases steadily until the end of the curve. The rate of change of the y momentum begins with zero, takes upon increasing positive values and changes at the end of the curve abruptly to zero. For a well-designed road the curvature has no discontinuities and no abrupt movements of the steering wheel are necessary.

2.

$$\omega = \frac{2\pi}{T} = 7.85\ \text{s}^{-1}$$

$$F = \frac{\Delta p}{\Delta t} = m\omega^2 r = 0.5\ \text{kg} \cdot (7.85\ \text{s}^{-1})^2 \cdot 1\ \text{m} = 30.8\ \text{N}$$

2.16 Pulleys

1.

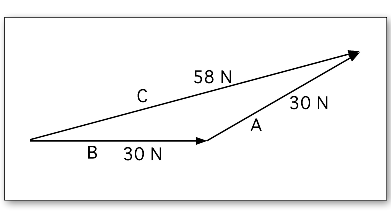

Fig. 2.3

2.

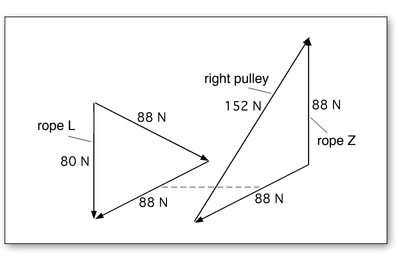

Fig. 2.4

2.17 Relationship between pressure and momentum current

1. Given: $F = 420$ N
$A_1 = 2\ \text{cm}^2$
$A_2 = 3\ \text{cm}^2$
$A_3 = 3\ \text{cm}^2$

Wanted: p_1, p_2, p_3

$$p_1 = -\frac{420\ \text{N}}{0.0002\ \text{m}^2} = -2.1\ \text{MPa}$$

$$p_2 = p_3 = -\frac{420\ \text{N}}{0.0003\ \text{m}^2} = -1.4\ \text{MPa}$$

2. Given: $m = 12$ kg
$A = 1.5\ \text{cm}^2$

Wanted: p_1, p_2, p_3

$F_3 = m \cdot g = 12\ \text{kg} \cdot 10\ \text{N/kg} = 120$ N
$F_1 = F_2 = F_3/2 = 60$ N

$$p_1 = -\frac{120\ \text{N}}{0.00015\ \text{m}^2} = -800\ \text{kPa}$$

$$p_2 = p_3 = -\frac{60\ \text{N}}{0.00015\ \text{m}^2} = -400\ \text{kPa}$$

3. The momentum current is estimated to be $F = 40$ N.

The diameter of the nail is about

$d = 1\ \text{mm}^2$

The cross-sectional area is

$$A = p\left(\frac{d}{2}\right)^2 \approx 0.8\ \text{mm}^2 = 0.000\,000\,8\ \text{m}^2$$

Thus we get

$$p = \frac{40\ \text{N}}{0.000\,000\,8\ \text{m}^2} = 50\ \text{MPa} = 500\ \text{bar}$$

If the cross-sectional area of the tip is ten times smaller, a pressure of 500 MPa = 5000 bar results.

4. From the mass of the hammer $m = 1$ kg and the estimated velocity $v = 2$ m/s we get the momentum $p = 1\ \text{kg} \cdot 2\ \text{m/s} = 2$ Hy. We estimate that the transmission of the momentum takes about 0.01 s. With $F = p/t$ we obtain

$$F = \frac{2\ \text{Hy}}{0.01\ \text{m}^2} = 200\ \text{N}$$

If the cross-sectional area of the tip of the nail is
$0.1\ \text{mm}^2 = 0.000\,000\,1\ \text{m}^2$,
the pressure will be

$$p = \frac{200\ \text{N}}{0.000\,000\,1\ \text{m}^2} = 2000\ \text{MPa} = 20\ \text{kbar}$$

2.18 Stress in three directions

1. Textiles, tissues

2. Concrete, stones, but also sand and gravel

3. Wood, some textiles, mica, graphite

3. Angular momentum and angular momentum currents

3.1 Angular momentum

Angular momentum can pass from one body to another.
Momentum can pass from one body to another.
Electric charge can pass from one body to another.

If a wheel comes to a halt due to friction of its bearing, its angular momentum flows away into the Earth.
If a vehicle comes to a halt due to friction, its momentum flows away into the Earth.

If an electrically charged body discharges due to bad insulation, its charge flows away into the Earth.

Angular momentum can assume positive and negative values.
Momentum can assume positive and negative values.
Electric charge assume positive and negative values.

Angular momentum can neither be created nor destroyed.
Momentum can neither be created nor destroyed.
Electric charge can neither be created nor destroyed.

3.2 Angular momentum pumps

In the first case nothing happens, i.e. Willy will not turn around. In the second case he begins to rotate.

3.3 What angular momentum depends on – flywheels

1. In vehicles: to avoid that momentum of the direction of movement flows away into the Earth, and for propelling the vehicle; energy transmission with drive belt or chain; gear wheel in the gear box and pulleys in the tackle.

2. When a wheel is rotating, closed momentum currents are flowing within the wheel. These are the stronger, the faster the wheel is rotating. If the angular velocity is too high, the wheel will disrupt.

3.
m = 8,5 kg
r = 20 cm
ω = 3000 revolutions per minute

$J = m \cdot r^2 = 8.5\ \text{kg} \cdot (0.2)^2\ \text{m}^2 = 0.34\ \text{kgm}^2$.

$L = J \cdot \omega = 0.34\ \text{kgm}^2 \cdot \dfrac{3000 \cdot 2\pi}{60\ \text{s}} = 106.8\ \text{E}$.

4. The angular momentum remains constant. We thus have

$L = J_1 \cdot \omega_1 = J_2 \cdot \omega_2$

We estimate the following values:

Total mass = 50 kg

Medium radius when the arms and the legs are not stretched out = 0.15 m

When the arms and one leg are stretched out, a mass of 10 kg is displaced to a radius 0.4.

We thus obtain:

$J_1 = 40\ \text{kg} \cdot (0.15\ \text{m})^2 + 10\ \text{kg} \cdot (0.4\ \text{m})^2 = 2.5\ \text{kgm}^2$
$J_2 = 50\ \text{kg} \cdot (0.15\ \text{m})^2 = 1.125\ \text{kgm}^2$

$\omega_2 = \dfrac{J_1}{J_2} \cdot \omega_1 = \dfrac{2.5}{1.125} \cdot \omega_1 = 2.22\omega_1 = 2.22$ revolutions/second

5.
r_1 = 50 000 km
r_2 = 10 km

The angular momentum before the collapse is equal to that after.

$L_1 = L_2$

$m \cdot r_1^2 \cdot \omega_1 = m \cdot r_2^2 \cdot \omega_2$

$r_1^2 \cdot \omega_1 = r_2^2 \cdot \omega_2$

$\omega_2 = \dfrac{r_1^2}{r_2^2} \cdot \omega_1 = \dfrac{50\ 000^2}{10^2} \cdot \dfrac{1\ \text{revol.}}{120\ \text{days}} = 2.4 \dfrac{\text{revol.}}{\text{second}}$

6. Rotate the upper part of your body by a certain angle, thereby holding your arms stretched out. Now bring your arms next to your body and reverse the rotation. During the forward rotation the upper part of your body had a great moment of inertia, during the return rotation the moment of inertia was small. Therefore the angle of rotation was smaller for the forward rotation than for the return rotation. (We have supposed that the moment of inertia of the lower part of your body did not change in the process.)

3.4 Angular momentum conductors

1. See Fig. 3.1a. The crank is turned. The rotatable container that is filled with water begins to rotate if the water can conduct angular momentum.

2. See Fig. 3.1b. The crank is turned. The lower axis will also rotate.

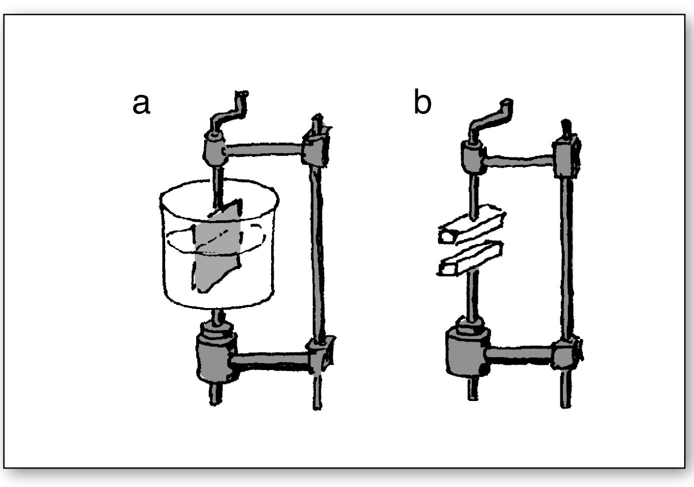

Fig. 3.1

3. Tornado

4. Crankshaft: Energy that comes from the pistons, is conducted to the shaft; camshaft: for opening and closing the valves; driveshaft: Energy transport to the wheels

3.5 Current strength and rate of change of the angular momentum

1.
m = 1200 kg
r = 1 m
ω = 3 revolutions per second
M = 120 E/s

(a) $L = m \cdot r^2 \cdot \omega = 1200\ \text{kg} \cdot 1\ \text{m}^2 \cdot 3 \cdot 2\pi \cdot \text{s}^{-1} = 22\ 619\ \text{E}$

(b) $M = \dfrac{\Delta L}{\Delta t} = \dfrac{L}{t} \quad \Rightarrow \quad t = \dfrac{L}{M} = \dfrac{22619\ \text{E}}{120\ \text{E/s}} = 188.5\ \text{s}$

2.
$\bar{M}$ = 40 E/s

ω = 8 revolutions per second
J = 2 kg m²

(a) 4 power strokes/second

(b) 10 E/power stroke

(c) $L = J \cdot \omega = 2\ \text{kgm}^2 \cdot 3 \cdot 2\pi \cdot \text{s}^{-1} = 100.5\ \text{E}$

(d) Supposition: The motor supplies angular momentum to the crankshaft only during the power stroke. During the remaining three strokes it does not supply or absorb angular momentum. Thus three quarters of the 10 E will be stored in the flywheel, i.e. 7.5 E. This represents 7.5 % of the total angular momentum of the flywheel.

3.7 More about angular momentum conductors

1. There are several solutions. Attention must be paid that the various components have the necessary degrees of freedom of movement. Thus, the driveshaft must have two cardan joints and one extendable tube.

2. If at the beginning the flywheels have the same angular momentum, Lilly will not rotate after tilting the wheels. If their angular momentum at the beginning is opposite, Lilly will begin to rotate.

4. The gravitational field

4.2 What gravity depends upon

1. We suppose $m = 70$ kg.

$F = m \cdot g$

$F_{Earth} = 70 \text{ kg} \cdot 9.8 \text{ N/kg} = 686 \text{ N}$
$F_{Moon} = 70 \text{ kg} \cdot 1.62 \text{ N/kg} = 113 \text{ N}$
$F_{neutron\ star} = 70 \text{ kg} \cdot 10^{12} \text{ N/kg} = 7 \cdot 10^{13} \text{ N}$

2.

$F = 300$ N
$g = 1.62$ N/kg

$$m = \frac{F}{g} = \frac{300 \text{ N}}{1.62 \text{ N/kg}} = 185 \text{ kg}$$

4.3 Free fall

1. We suppose $m = 70$ kg.
$t = 0{,}77$ s

$p = m \cdot g \cdot t = 70 \text{ kg} \cdot 10 \text{ N/kg} \cdot 0.77 \text{ s} = 540 \text{ Hy}$
$v = g \cdot t = 10 \text{ N/kg} \cdot 0.77 \text{ s} = 7.7 \text{ m/s}$

2.

$t = 0.5$ s

$v = g \cdot t$

$v_{Earth} = 5$ m/s; $v_{Moon} = 0.81$ m/s; $v_{Sun} = 137$ m/s

3.

$v = 15$ m/s

$$t = \frac{v}{g} = \frac{15 \text{ m/s}}{10 \text{ N/kg}} = 1.5 \text{ s} = \text{time until return}$$

Total time = $2 \cdot 1.5$ s = 3 s

4.

Total time = 5 s
Falling time = 2.5 s

$v = g \cdot t = 10 \text{ N/kg} \cdot 2.5 \text{ s} = 25 \text{ m/s}$

4.4 Falling with friction

A momentum current of

$F = m \cdot g = 0.8 \text{ kg} \cdot 10 \text{ N/kg} = 8 \text{ N}$

is flowing form the Earth into the globe.

From Fig. 4.6 in the students text one reads: $v = 20$ m/s

4.6 Circular orbits in the gravitational field

1. He gives a push to each of the two bodies, i.e. he charges them with momentum. The body with the smaller mass flies away faster.

2. Set the spaceship into rotation.

3. In $\omega = \frac{v}{r}$ we introduce $v = \sqrt{r \cdot g}$ and obtain

$$\omega = \frac{\sqrt{r \cdot g}}{r} = \sqrt{\frac{g}{r}}$$

4.

(a) $u = 2\pi r = 2\pi \cdot 384\,000 \text{ km} = 2\,412\,700 \text{ km}$

(b) $T = 27 \cdot 24 \cdot 3600\ s = 2\,332\,800 \text{ s}$

(c) $v = \frac{u}{t} = \frac{2\,412\,700 \text{ km}}{2\,332\,800 \text{ s}} = 1.03 \text{ km/s}$

(d) $v = \sqrt{r \cdot g} \Rightarrow g = \frac{v^2}{r} = \frac{1.03^2 \cdot 10^6 \text{ m}^2/s^2}{3.84 \cdot 10^8 \text{ m}} = 0.0028 \text{ N/kg}$

5. See Fig. 4.1 A hyperbolic orbit is obtained if the velocity is strongly increased.

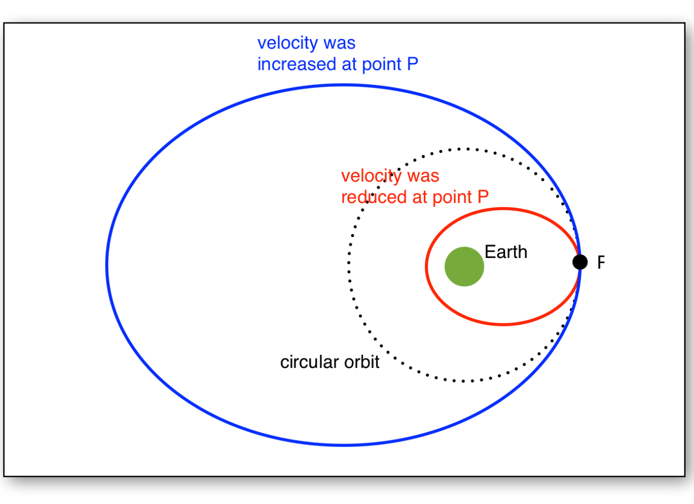

Fig. 4.1
Dashed line: initial circular orbit. Red: velocity was reduced at point P; blue: velocity was increased at point P.

4.7 The field of spherically symmetric bodies

1. With

$$g(r) = G \cdot \frac{m}{r^2}$$

we get

$$m = \frac{g}{G} r^2$$

We insert the field strength g of the Earth at the position of the Moon, and also the radius of the orbit of the Moon:

$$m = \frac{2.8 \cdot 10^{-3} \text{ N/kg}}{6.67 \cdot 10^{-11} \text{ m}^3/(\text{kgs}^2)} \cdot 3.84^2 \cdot 10^{16} \text{m}^2 = 6 \cdot 10^{24} \text{kg}$$

The mass of the Earth could also have been calculated by means of a body next to the surface of the Earth. In this case the radius of the Earth would have been needed.

2.

$r = 1.5 \cdot 10^{11}$ m

(a) $\omega = \frac{2\pi}{\text{year}} = \frac{2\pi}{365 \cdot 24 \cdot 3600 \text{ s}} = 2.0 \cdot 10^{-7} \text{s}$

$v = \omega \cdot r = 2 \cdot 10^{-7} \text{ s} \cdot 1{,}5 \cdot 10^{11} \text{ m} = 3 \cdot 10^4 \text{ m/s} = 30 \text{ km/s}$

(b) $g = \frac{v^2}{r} = \frac{9 \cdot 10^8}{1.5 \cdot 10^{11}} \frac{\text{N}}{\text{kg}} = 6 \cdot 10^{-3} \text{ N/kg}$

(c) $m = \frac{g}{G} r^2$

$$m = \frac{6 \cdot 10^{-3} \text{ N/kg}}{6.67 \cdot 10^{-11} \text{m}^3/(\text{kgs}^2)} \cdot 2.25 \cdot 10^{22} \text{ m}^2 = 2.02 \cdot 10^{30} \text{ kg}$$

3. In

$$v = \sqrt{\frac{G \cdot m}{r}}$$

m is the mass of the central body, r and v are the radius and the velocity of the satellite. Thus we get:

$$m = \frac{r \cdot v^2}{G}$$

What is needed is the orbital radius and the velocity of the planet's moon.

4.

$$\omega = \frac{2\pi}{\text{day}} = \frac{2\pi}{24 \cdot 3600 \text{ s}} = 7.3 \cdot 10^{-5} \text{ s}^{-1}$$

m = mass of the Earth = $6 \cdot 1024$ kg

With

$$\omega = \sqrt{\frac{g}{r}}$$

and

$$g = G \cdot \frac{m}{r^2}$$

we get

$$\omega = \sqrt{G \frac{m}{r^3}}$$

We solve for r:

$$r = \sqrt[3]{\frac{Gm}{\omega^2}}$$

$$r = \sqrt[3]{\frac{6.67 \cdot 10^{-11} \cdot 6 \cdot 10^{24}}{7.3^2 \cdot 10^{-10}}} \text{ m} = 4.22 \cdot 10^7 \text{ m} = 42\,000 \text{ km}$$

r is the radius of the orbit. To obtain the height above the Earth's surface the radius of the Earth must be subtracted and one finds the altitude of the satellite to be 36 000 km.

The orbital velocity turns out to be:

$v = \omega \cdot r = 7.3 \cdot 10^{-5} \text{ s}^{-1} \cdot 4.22 \cdot 10^7 \text{ m} = 3080 \text{ m/s}$.

4.8 Galilei, Kepler and Newton

1. From

$$v = \sqrt{\frac{G \cdot m}{r}}$$

one obtains

$$v^2 = \frac{G \cdot m}{r}.$$

With

$$v = \frac{2\pi r}{T}$$

we get

$$\frac{4\pi^2 r^2}{T^2} = \frac{G \cdot m}{r}$$

or

$$\frac{T^2}{r^3} = \frac{4\pi^2}{G \cdot m}.$$

The quotient on the right side of the equation has the same value for all planets and satellites of a given central celestial body.

2. .

$m_A = m_B = 1$ kg
$r = 10$ cm

For the momentum current between bodies A and B we obtain:

$$F = G \cdot \frac{m_A \cdot m_B}{r^2} = 6.67 \cdot 10^{-11} \cdot \frac{1}{0.01} \text{N} = 6.67 \cdot 10^{-9} \text{ N}$$

It is difficult to measure a momentum current of about 10^{-8} N. Several other momentum currents of the same order of magnitude disturb the experiment.

4.9 The tides

1.

$$g = G \cdot m_{Moon} \cdot \frac{1}{r^2}$$

With $m_{Moon} = 7.35 \cdot 10^{22}$ kg and $r = 3.84 \cdot 10^8$ m wird

$$g = \frac{6.67 \cdot 10^{-11} \cdot 7.35 \cdot 10^{22}}{(3.84)^2 \cdot 10^{16}} \frac{\text{N}}{\text{kg}} = 3.32 \cdot 10^{-5} \text{ N/kg}$$

To calculate the difference of g on both sides of the Earth the magnitude of g must not be known with hight precision.

$$\Delta g = G \cdot m_{Moon} \cdot \left(\frac{1}{r^2} - \frac{1}{(r + \Delta r)^2} \right)$$

$$\approx G \cdot m_{Moon} \cdot \left(\frac{1}{r^2} - \frac{1}{r^2 + 2r\Delta r} \right)$$

$$= G \cdot m_{Moon} \cdot \left(\frac{r^2 + 2r\Delta r - r^2}{r^2(r^2 + 2r\Delta r)} \right)$$

$$= G \cdot m_{Moon} \cdot \left(\frac{2r\Delta r}{r^2(r^2 + 2r\Delta r)} \right)$$

$$\approx G \cdot m_{Moon} \cdot \frac{2\Delta r}{r^3}$$

$$\Delta g = \frac{6.67 \cdot 10^{-11} \cdot 7.35 \cdot 10^{22} \cdot 2 \cdot 1.274 \cdot 10^7}{(3.84)^3 \cdot 10^{24}} \frac{\text{N}}{\text{kg}}$$

$$= 2.21 \cdot 10^{-6} \text{ N/kg}$$

2. See Fig. 4.2. The momentum current, that enters the two parts A and B of the dumbbell via the gravitational field does not have the same direction as the momentum that is accumulating in the dumbbell. The x component of the momentum that arrives in B flows through the bar of the dumbbell to A and compensates the x component of the momentum that enter in A.

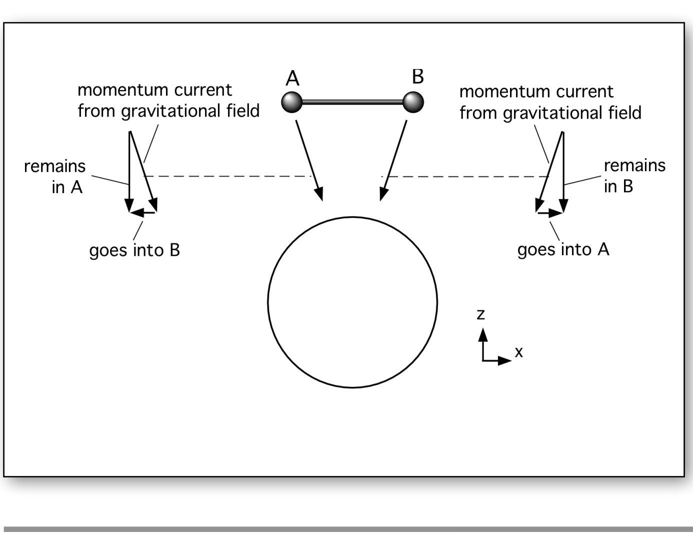

Fig. 4.2
In the dumbbell negative z momentum accumulates. Positive x momentum flows from A To B.

5. Momentum, angular momentum and energy

5.2 Die Momentum as an energy carrier

1.

$v = 20$ km/h $= 5.6$ m/s
$F = 900$ N
$P = v \cdot F = 5.6$ m/s $\cdot$ 900 N $= 5040$ W

The momentum flows away into the Earth and into the air. The energy is used for the production of entropy.

2.

$v = 10$ m/s
$P = 800$ W

$$F = \frac{P}{v} = \frac{800\ \text{W}}{10\ \text{m/s}} = 80\ \text{N}$$

3.

$m = 50$ kg
$v = 0.8$ m/s
$h = 5$ m
$F = m \cdot g = 50$ kg $\cdot$ 10 N/kg $= 500$ N
$P = v \cdot F = 0.8$ m/s $\cdot$ 500 N $= 400$ W

$$t = \frac{h}{v} = \frac{5\ \text{m}}{0.8\ \text{m/s}} = 6.25\ \text{s}$$

$E = P \cdot t = 400$ W $\cdot$ 6.25 s $= 2500$ J

4.

$s = 35$ km
$F = 900$ N
$E = F \cdot s = 900$ N $\cdot$ 35 km $= 31\,500$ kJ

5.3 Angular momentum as an energy carrier

1.

$P = 27$ MW
$\omega = 100$ revolutions/minute

$$\omega = \frac{100 \cdot 2\pi}{60}\ \text{s}^{-1} = 10.49\ \text{s}^{-1}$$

$$M = \frac{P}{\omega} = \frac{27 \cdot 10^6\ \text{W}}{10.49\ \text{s}^{-1}} = 2.57 \cdot 10^6\ \text{E/s}$$

2. Motor: Bipower SX

$M = 130$ Nm $= 130$ E/s
$\omega = 4000$ revolutions/minute

$$\omega = \frac{4000 \cdot 2\pi}{60}\ \text{s}^{-1} = 419\ \text{s}^{-1}$$

$P = \omega \cdot M = 419\ \text{s}^{-1} \cdot 130$ E/s $= 54.5$ kW

The energy current (power) of 68 kW is supplied by the motor at a higher angular velocity.

5.4 Mechanical energy storage

1.

$m = 30$ kg
$t = 6$ s
$F = 20$ N
$p = F \cdot t = 20$ N $\cdot$ 6 s $= 120$ Hy

$$E = \frac{p^2}{2m} = \frac{120^2\ \text{Hy}^2}{2 \cdot 30\ \text{kg}} = 240\ \text{J}$$

2.

$m = 200$ g
$v = 0.8$ m/s
$s = 5$ cm

$$E = \frac{m}{2}v^2 = 0.1\ \text{kg} \cdot 0.8^2\ \text{m}^2/\text{s}^2 = 0.064\ \text{J}$$

$$E = \frac{D}{2}s^2 \Rightarrow D = \frac{2E}{s^2} = \frac{2 \cdot 0.064\ \text{J}}{0.05^2\ \text{m}^2} = 51.2\ \text{N/m}$$

3. Mass of the light glider $= m$
mass of three gliders $= 3m$

The momentum p is the same before and after the collision.

$$E_{before} = \frac{p^2}{2m}$$

$$E_{after} = \frac{p^2}{3 \cdot 2m} = \frac{E_{before}}{3}$$

The total kinetic energy has decreased during the collision to 1/3 of its original value. The missing 2/3 have been spent for the production of entropy.

4.

$m = 20$ kg
$v_{before} = 0.5$ m/s
$D = 60$ N/m

(a) $p_{before} = m \cdot v_{before} = 20$ kg $\cdot$ 0.5 m/s $= 10$ Hy
$p_{after} = m \cdot v_{after} = 20$ kg $\cdot$ (−0.5 m/s) $= -10$ Hy

(b)

$$E_{kin} = \frac{m}{2}v_{before}^2 = \frac{m}{2}v^2 = 10\ \text{kg} \cdot 0.25\ \text{m}^2/\text{s}^2 = 2.5\ \text{J}$$

(c)

$$E_{spring} = \frac{D}{2}s^2 = E_{kin}$$

$$s = \sqrt{\frac{2E_{kin}}{D}} = \sqrt{\frac{2 \cdot 2.5\ \text{J}}{60\ \text{N/m}}} = 0.289\ \text{m}$$

(d) When the spring is compressed halfway its energy is:

$$E_{spring}' = \frac{D}{2}\left(\frac{s}{2}\right)^2 = \frac{E_{spring}}{4},$$

i.e. a quarter of the total energy. Thus, the kinetic energy is equal to three quarters of the total energy:

$$E_{kin}' = \frac{m}{2}v'^2 = \frac{3E_{kin}}{4} = \frac{3}{4} \cdot \frac{m}{2}v^2$$

We thus obtain

$$v'^2 = \frac{3}{4}v_{before}^2$$

$$v' = \sqrt{\frac{3}{4}}v_{before} = \sqrt{\frac{3}{4}} \cdot 0.5\ \text{m/s} = 0.43\ \text{m/s}$$

5. The energy flows back and forth between the spring and the body. The momentum flows back and forth between the body and the Earth (via the spring).

$m = 300$ g
$D = 7.5$ N/m
$v = 0.5$ m/s

In the equilibrium position the kinetic energy is equal to the energy of the spring when it in its state of maximum extension.

$$\frac{D}{2}s^2 = \frac{m}{2}v^2$$

$$D \cdot s^2 = m \cdot v^2$$

$$s = \sqrt{\frac{m}{D}} \cdot v = \sqrt{\frac{0.3\ \text{kg}}{7.5\ \text{N/m}}} \cdot 0.5\ \text{m/s} = 0.1\ \text{m}$$

6. We admit the vehicle moves in the x direction at the beginning and in the y direction at the end.

The energy has the same value in every moment. All of the x momentum flows into the Earth, and all of the y momentum that the vehicle has at the end comes from the Earth.

7.

$d = 2.2$ m
$m = 1.8$ t
$\omega = 2$ revolutions per second

$L = m \cdot r^2 \cdot \omega = 1800$ kg $\cdot$ 1.12 m² $\cdot$ 2 $\cdot$ 2π s⁻¹ $= 27\,370$ E

$$E = \frac{J}{2}\omega^2 = \frac{L \cdot \omega}{2} = \frac{27\,370\ \text{E} \cdot 2 \cdot 2\pi\ \text{s}^{-1}}{2} = 172\ \text{kJ}$$

8. Index b: before, Index a: after

$J_A = J_B = 2$ kgm²
$\omega_{A,b} = 2$ revolutions/second

(a) $L_{A,b} = J_A \cdot \omega_{A,b} = 2 \cdot 2 \cdot 2\pi\,\text{E} = 25.13$ E

(b)

$$E = \frac{L \cdot \omega}{2} = \frac{25.13\ \text{E} \cdot 2 \cdot 2\pi\ \text{s}^{-1}}{2} = 158\ \text{kJ}$$

(c)

$$L_{A,a} = L_{B,n} = \frac{L_{A,b}}{2} = 12.56\ \text{E}$$

(d)

$$E_{A,a} = \frac{L_{A,a}^{\ 2}}{2J} = \frac{(L_{A,b}/2)^2}{2J} = \frac{L_{A,b}^{\ 2}}{8J} = \frac{E_{A,b}}{4}$$

$$E_{A,a} + E_{B,a} = \frac{E}{2}$$

(e) When the wheels are coupled, half of the energy of the rotation is lost. It is spent for entropy production. The process is analogue to the inelastic collision of a vehicle against another one at rest that has the same mass.

9.

$t = 10$ s
$P = 150$ MW

$$\omega = 1600\ \text{revol./min} = \frac{1600 \cdot 2\pi}{60}\text{s}^{-1} = 167.55\ \text{s}^{-1}$$

$E = P \cdot t = 150$ MW $\cdot$ 10 s $= 1.5 \cdot 109$ J

$$E = \frac{J}{2}\omega^2 \Rightarrow J = \frac{2E}{\omega^2} = \frac{2 \cdot 1.5 \cdot 10^9\ \text{J}}{167.55^2\ \text{s}^{-2}} = 1{,}07 \cdot 10^5\ \text{kgm}^2$$

5.5 Energy storage in the gravitational field – the gravitational potential

1. See Fig. 5.1

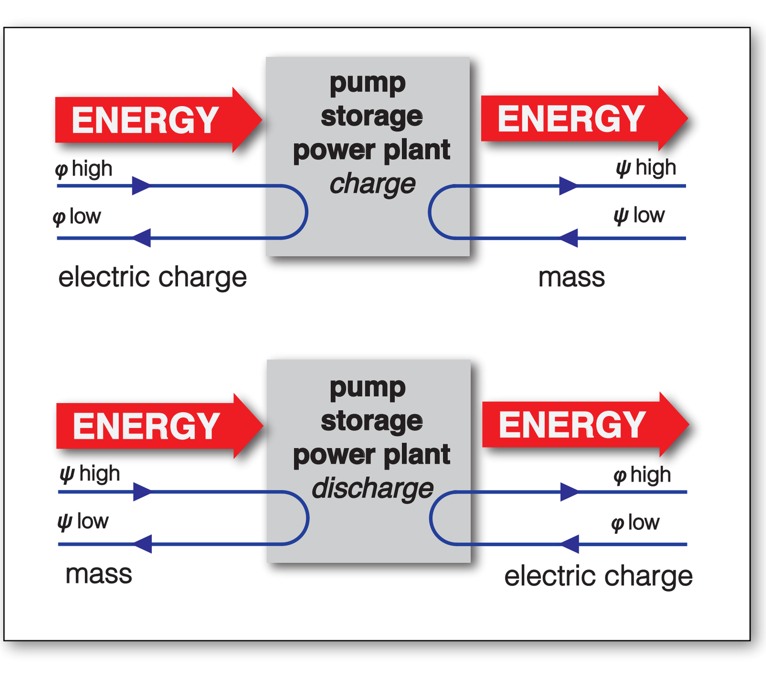

Fig. 5.1

$h_2 = 880$ m
$h_1 = 550$ m
$V = 12 \cdot 10^6$ m³

$\psi_2 = g \cdot h_2 = 9.8$ N/kg $\cdot$ 880 m $= 8624$ J/kg
$\psi_1 = g \cdot h_1 = 9.8$ N/kg $\cdot$ 550 m $= 5390$ J/kg

$\Delta\psi = \psi_2 - \psi_1 = 3234$ J/kg

$m = \rho \cdot V = 1000$ kg/m³ $\cdot$ $12 \cdot 10^6$ m³ $= 12 \cdot 10^9$ kg

$E = m \cdot \Delta\psi = 12 \cdot 10^9$ kg $\cdot$ 3234 J/kg $= 38.8 \cdot 10^{12}$ J

$$P = \frac{E}{t} \Rightarrow t = \frac{E}{P} = \frac{38.8 \cdot 10^{12}\ \text{J}}{1.06 \cdot 10^9\ \text{W}} = 36.6 \cdot 10^3\ \text{s} \approx 10\ \text{h}$$

2.

$\Delta h = 120$ m
$I_V = 12\,000$ m³/s

$I_m = 12 \cdot 10^6$ kg/s

$\Delta\psi = g \cdot \Delta h = 9.8$ N/kg $\cdot$ 120 m $= 1176$ J/kg

$$P = \frac{E}{t} = \frac{m \cdot \Delta\psi}{t} = \Delta\psi \cdot I_m$$

$$= 1176\ \text{J/kg} \cdot 12 \cdot 10^6\ \text{kg/s} = 14 \cdot 10^9\ \text{W} = 14\ \text{GW}$$

3.

$v = 5$ m/s

$$\frac{m}{2}v^2 = m \cdot g \cdot h$$

$$\frac{v^2}{2} = g \cdot h$$

$$h = \frac{v^2}{2g} = \frac{5^2 \cdot \text{m}^2/\text{s}^2}{2 \cdot 9.8\ \text{N/kg}} = 1.28\ \text{m}$$

4. (a) When thrown off the stone is charged with energy and with negative momentum (positive direction is downwards).

(b) When moving upwards the stone transfers its energy to the gravitational field. At the point of return its energy is zero. Positive momentum flows continuously into the stone, so that its negative momentum decreases until becoming zero in the turning point.

(c) When moving downwards it gets energy from the gravitational field and its (positive) momentum increases.

5.

$r = 10$ cm
$m = 2$ kg
$v = 0.8$ m/s

Total energy at the beginning (index a):

$J = m \cdot r^2 = 2$ kg $\cdot$ 0.12 m² $= 0.02$ kgm²

$$\omega = \frac{v}{r} = \frac{0.8\ \text{m/s}}{0.1\ \text{m}} = 8\ \text{s}^{-1}$$

$$E_a = \frac{m}{2}v^2 + \frac{J}{2}\omega^2$$

$$= \frac{2\text{kg}}{2} \cdot 0.8^2\ \text{m}^2 + \frac{0.02\ \text{kgm}^2}{2} \cdot 8^2\ \text{s}^{-2} = 1.28\ \text{J}$$

The total energy at the beginning is equal to the energy at the end (index e):

$E_a = E_e = m \cdot g \cdot h$

$$h = \frac{E_a}{m \cdot g} = \frac{1.28\ \text{J}}{2\ \text{kg} \cdot 9.8\ \text{N/kg}} = 0{,}065\ \text{m}$$

5.6 Tackle, gear drive chain and belt drive

1. At B three times as great as at A, Fig. 5.2a

2. At B four times as great as at A, Fig. 5.2b

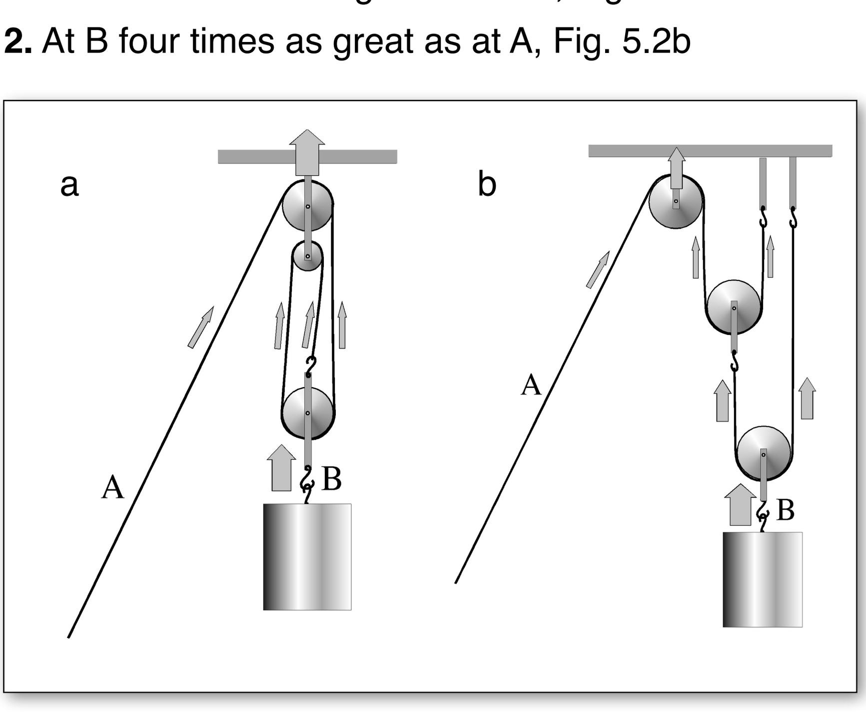

Fig. 5.2

5.7 Friction

1. See Fig. 5.3

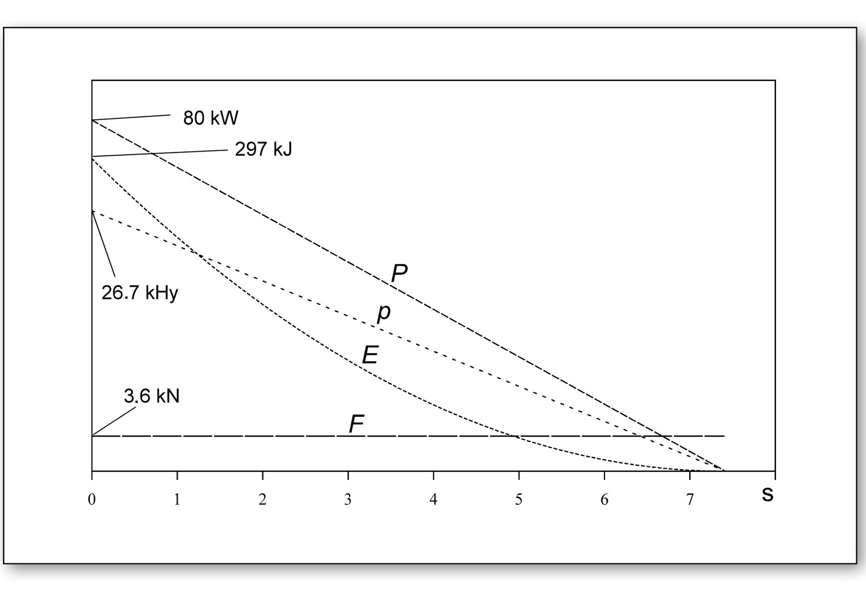

Fig. 5.3

2. The slower the vehicle, the less efficient the brake. It is easy to bring the vehicle from a high to a low velocity, but it cannot be brought to halt. To stop it completely a second brake is necessary.

6. Reference frames

6.2 Phenomena in different reference frames

1.

Reference frame Earth:

	A	B	total
before			
v	3 m/s	–3 m/s	
p	6 Hy	–6 Hy	0 Hy
E_{kin}	9 J	9 J	18 J
after			
v	–3 m/s	3 m/s	
p	0 Hy	0 Hy	0 Hy
E_{kin}	0 J	0 J	0 J

Reference frame of body A before collision:

	A	B	total
before			
v'	0 m/s	–6 m/s	
p'	0 Hy	–12 Hy	–12 Hy
E_{kin}	0 J	36 J	36 J
after			
v'	–3 m/s	–3 m/s	
p'	–6 Hy	–6 Hy	–12 Hy
E_{kin}	9 J	9 J	18 J

2. See Fig. 6.1

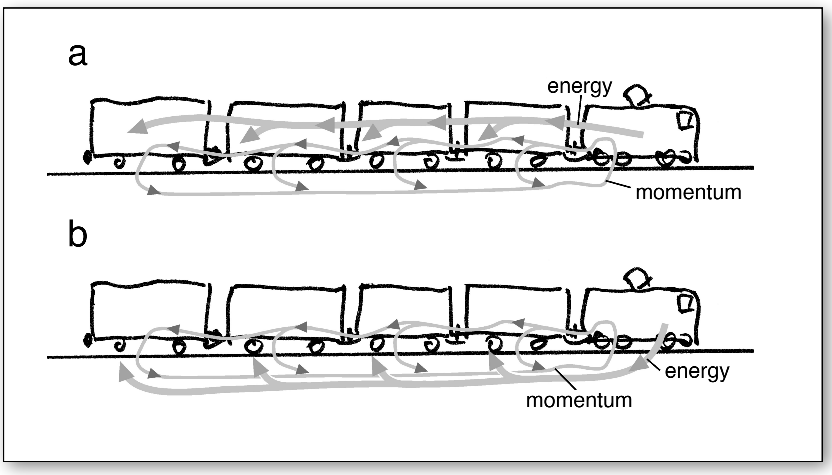

Fig. 6.1

3. The rocket engine starts, the spaceship accelerates. In Willy's reference frame the gravitational field strength is no longer zero. (We had asked the same question in exercise **2**, section 4.6. There we gave a different answer.)

6.3 Free float frames

1. $v = g \cdot t \approx 10 \frac{\text{m}}{\text{s}^2} \cdot 5\ \text{s} = 50 \frac{\text{m}}{\text{s}}$

Willy feels weightless. For him the gravitational field strength is zero.

At the floor of the drop tower there is a catapult. With this catapult Willy is thrown upwards with a velocity v = 50 m/s. Willy describes the whole process as free floating. Its duration is now 2 · t = 10 s.

2. Both of them do not feel any gravitational field; they are free floating during the duration of the experiment. Free floating reference frames move at constant velocity with respect to one another. Willy says that Lilly is approaching him with the constant velocity of 50 m/s. Lilly says the same about Willy. After both have met at halfway, they move away one from the other with the same velocity.

3. The skydiver feels the gravitational field; for him the field strength is not zero. Thus, momentum is flowing into him. Due to air friction an equal momentum current is leaving him, so that his velocity remains constant. He is in a state of flow equilibrium.

7. Terminal velocity

7.2 Energy has the properties of mass

1.

$E = 5 \cdot 10^{16}$ J

$$m = \frac{E}{k} = \frac{5 \cdot 10^{16}\ \text{J}}{9 \cdot 10^{16}\ \text{J/kg}} = 0.56\ \text{kg}$$

2. We calculate the surface of a sphere with a radius of 150 million kilometer:

$A = 4\pi r^2 = 4\pi \cdot 1.5^2 \cdot 10^{22}\ \text{m}^2 = 287 \cdot 10^{21}\ \text{m}^2$

The total energy flow through this surface is:

$P = A \cdot 1400\ \text{W} = 396 \cdot 10^{24}$ W

Thus the energy loss of the Sun per second is:

$E = 396 \cdot 10^{24}$ J

$$m = \frac{E}{k} = \frac{396 \cdot 10^{24}\ \text{J}}{9 \cdot 10^{16}\ \text{J/kg}} = 4.4 \cdot 10^{9}\ \text{kg}$$

3. 1400 J arrive on a square meter.

$$m = \frac{E}{k} = \frac{1400\ \text{J}}{9 \cdot 10^{16}\ \text{J/kg}} = 1.56 \cdot 10^{-14}\ \text{kg}$$

$$\frac{m}{t} = 1.56 \cdot 10^{-14}\ \text{kg/s}$$

$$t = \frac{m}{1.56 \cdot 10^{-14}\ \text{kg/s}} = \frac{0.001\ \text{kg}}{1.56 \cdot 10^{-14}\ \text{kg/s}} = 6.4 \cdot 10^{10}\ \text{s}$$

≈ 2000 Jahre

4.

$E = 500$ kJ

$$m = \frac{E}{k} = \frac{500\ \text{kJ}}{9 \cdot 10^{16}\ \text{J/kg}} = 5.6 \cdot 10^{-12}\ \text{kg}$$

A car needs about 10 liters per 100 km, i.e. about 10 kg/100 km. With v = 100 km/h it follows, that the car needs 10 kg/h, or $3 \cdot 10^{-3}$ kg/s. The process of acceleration lasts about 10 s. Thereby the car loses $30 \cdot 10^{-3}$ kg = 30 g. This decrease is $5 \cdot 10^{9}$ as great as the increase due to the higher kinetic energy.

7.6 What happens to the velocity when the reference frame is changed

1.

Velocity relative to the Earth:

Uranus $v_{UE} = 0{,}9\ c$
Wostok $v_{WE} = -0{,}5\ c$
Shenzhou $v_{SE} = 0{,}5\ c$

It follows:

$v_{EW} = 0{,}5\ c$
$v_{ES} = -0{,}5\ c$

$$v_{UW} = \frac{v_{UE} + v_{EW}}{1 + \frac{v_{UE} \cdot v_{EW}}{c^2}} = 0.966\ c \qquad v_{US} = \frac{v_{UE} + v_{ES}}{1 + \frac{v_{UE} \cdot v_{ES}}{c^2}} = 0.727\ c$$

2.

$v' = 140$ km/h

$v_0 = 30$ km/s

$$v - v_0 = \frac{v' + v_0}{1 + \frac{v' \cdot v_0}{c^2}} - v_0 = v' \cdot \frac{1 - \frac{v_0^2}{c^2}}{1 + \frac{v' \cdot v_0}{c^2}}$$

The second summand in the denominator is small compared to one, and much smaller than the second summand in the numerator. It can be neglected against the one.

$$v - v_0 \approx v' \cdot \left(1 - \frac{v_0^2}{c^2}\right)$$

$$= 140\ \text{km/h} \left(1 - \frac{3^2 \cdot 10^8}{3^2 \cdot 10^{16}}\right) = (1 - 10^{-8}) \cdot 140\ \text{km/h}$$

The deviation from 140 km/h is certainly smaller than the accuracy of the measurement of the velocity.

7.7 How energy depends on momentum

1. See Fig. 7.1

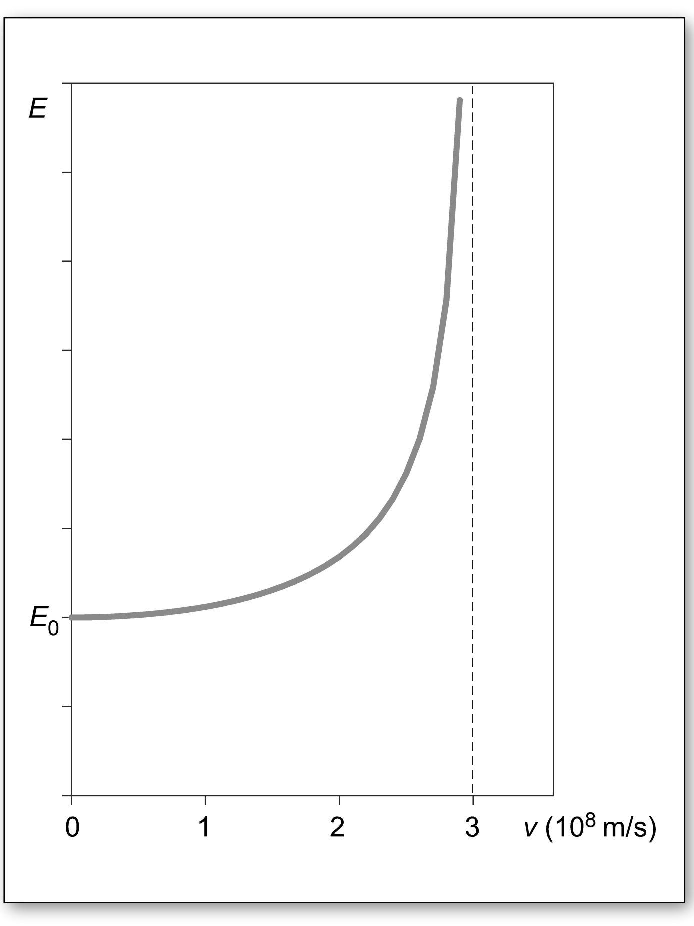

Fig. 7.1

For small velocities the energy/mass is almost independent of the velocity. For $v \rightarrow c$ it goes asymptotically to infinity.

The condition for $E(v) = 2E_0$ is

$$E(v) = \frac{E_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

It follows

$$v = \frac{\sqrt{3}}{2} c = 0.866\ c = 2.6 \cdot 10^{8}\ \text{m/s}$$

2. See Fig. 7.2

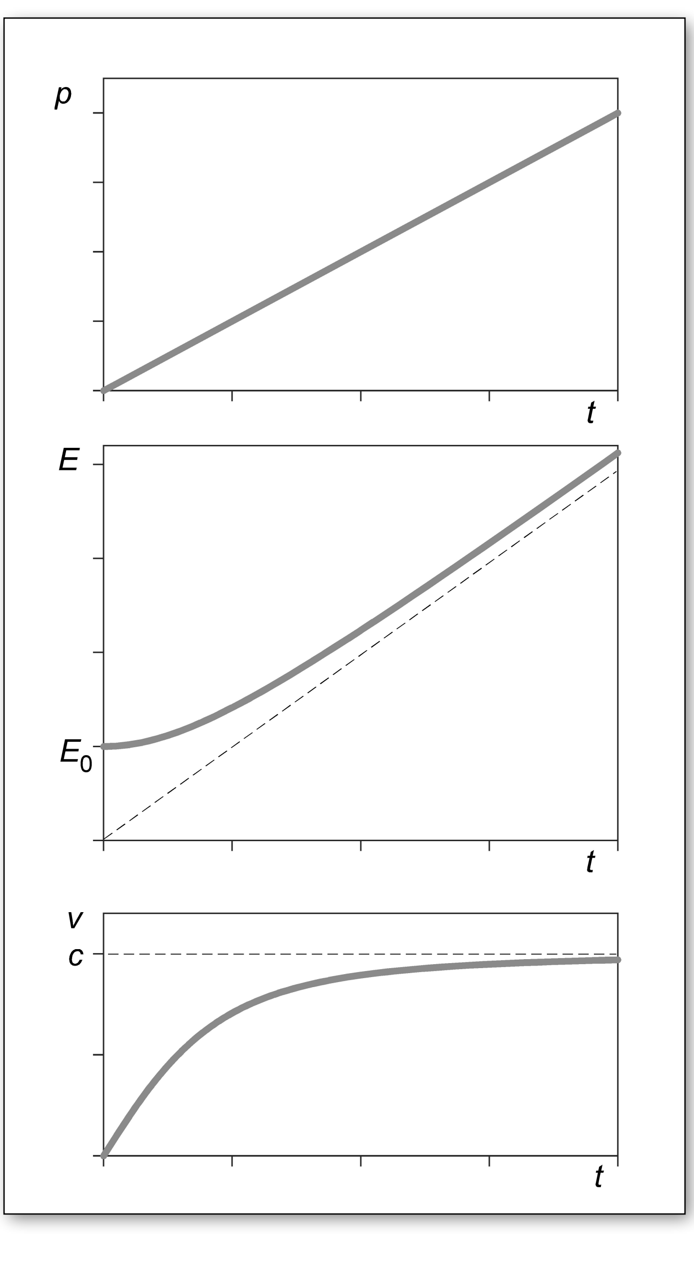

Fig. 7.2

7.8 Particle accelerators

From

$$E(v) = \frac{E_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

we get

$$v = c \cdot \sqrt{1 - \left(\frac{E_0}{E}\right)^2}$$

Proton-Synchrotron-Booster:
$E = 1.5\ E_0$
$v = 0.75\ c$

Proton-Synchrotron:
$E = 20\ E_0$
$v = 0.9987\ c$

Super-Proton-Synchrotron:
$E = 400\ E_0$
$v = 0.999996875\ c$

LHC:
$E = 7000\ E_0$
$v = 0.9999999898\ c$

7.10 Clocks in the gravitational field

1.

$t = 2$ years $= 6.3 \cdot 10^{7}$ s
$\Delta h = 400$ m

$$\Delta t = \frac{t \cdot g \cdot \Delta h}{k} = \frac{6.3 \cdot 10^{7}\text{s} \cdot 9.8\ \text{N/kg} \cdot 400\ \text{m}}{9 \cdot 10^{16}\ \text{J/kg}} = 2{,}7 \cdot 10^{6}\text{s}$$

2. There are no consequences for the everyday life. Willy learns that he ages more quickly than Lilly only through the news he gets from Lilly, and he can notice it when Lilly is visiting him: She is younger than she would be, if she had lived at the same height as Willy.

8. Spacetime

8.1 Problems of presentation and designation

1. See Fig. 8.1

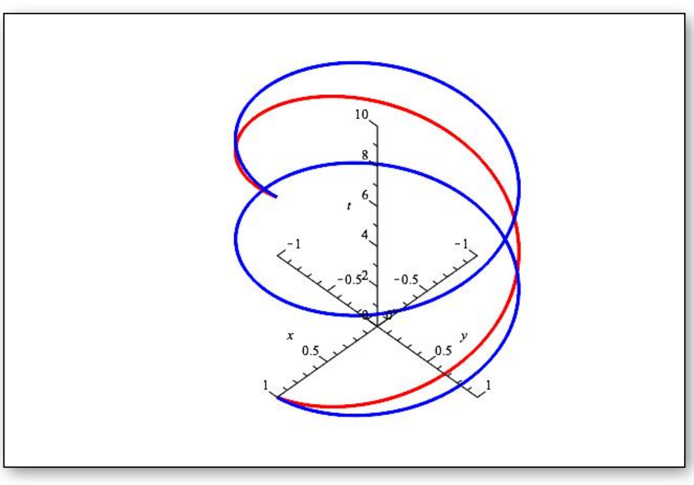

Fig. 8.1

2. See Fig. 8.2

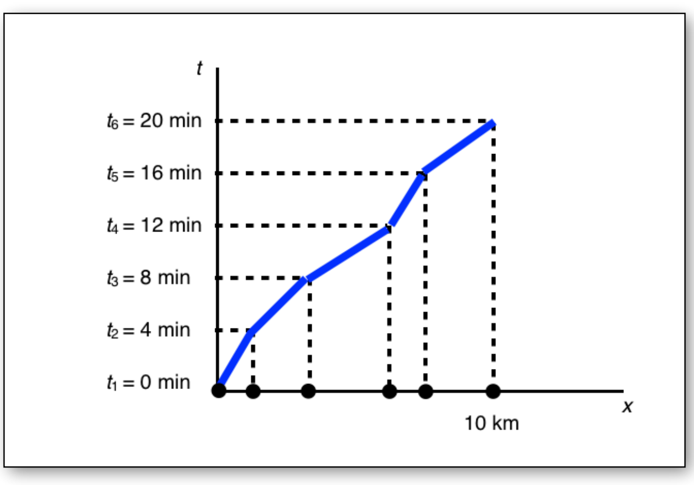

Fig. 8.2

3. (1) and (3) move with constant velocity, (1) is faster. (2) becomes faster, (4) becomes slower.

4. See Fig. 8.3

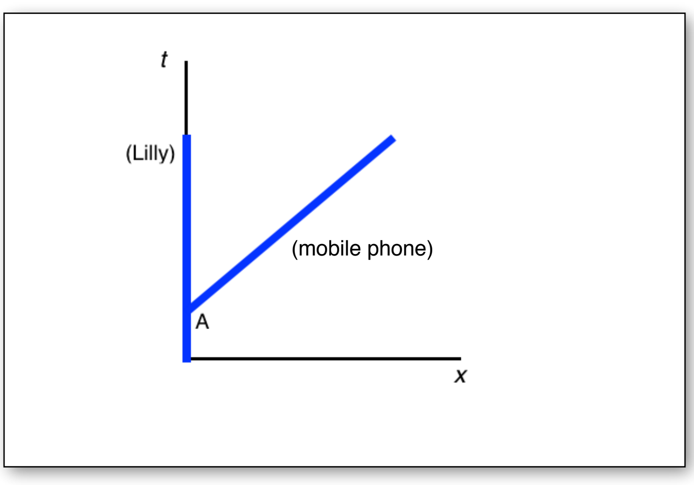

Fig. 8.3

5. See Fig. 8.4

Lilly turns back: $x = 7.5$ km; $t = 8{:}30$ h
Lilly meets Willy: $x = 0$ km; $t = 9{:}00$ h

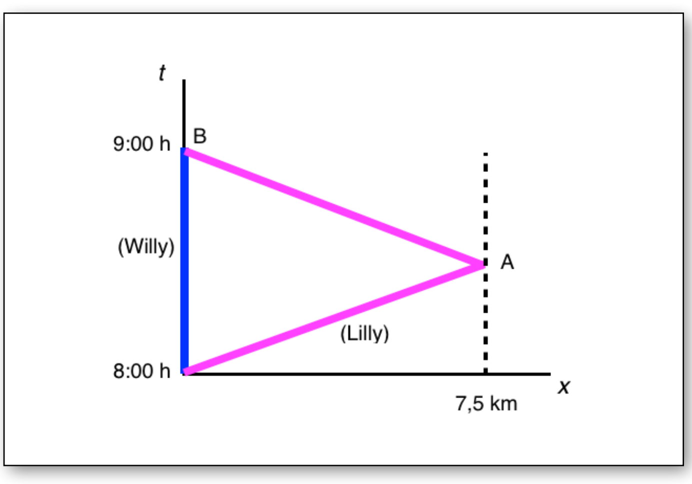

Fig. 8.4

8.2 The time interval between two spacetime points

1. (a) All trips except no. (3) are permitted. The travelers (3) would move with a velocity that is greater than the terminal velocity.

(b) Time (5) < Time (2) < Time (4) < Time (1) (Traveler (4) turns back before the time of the rendezvous and waits together with (1) until the other travelers are back.)

2. (a) Event A: Sending the laser pulse from the Earth;
Event B: Arrival of the laser pulse at the Earth.

(b) See Fig. 8.5

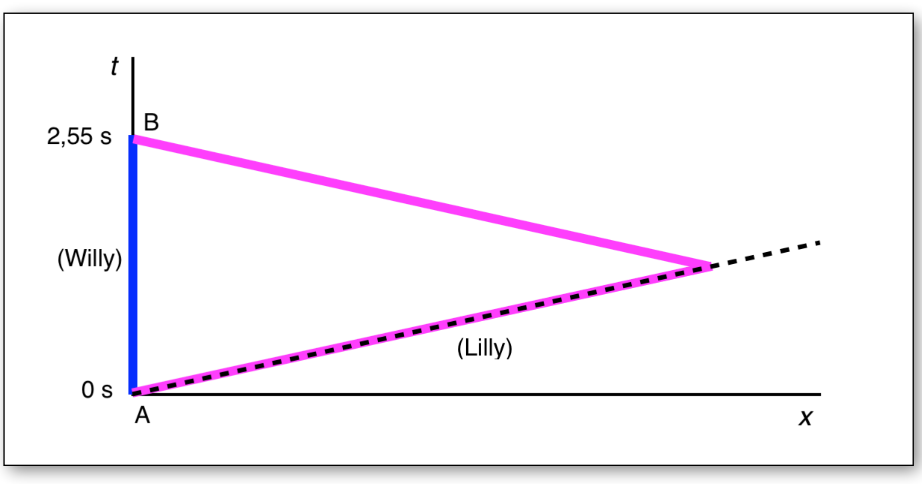

Fig. 8.5

(c) Lilly's watch indicates a travel duration of 0 s.

(d) Since $s = vt$ Lilly concludes that for her the distance Moon-Earth is 0 km.

3. (a) Willy has calculated, that Lilly must have covered a distance of $s = ct = 3 \cdot 10^8$ m/s $\cdot$ 1a $= 3 \cdot 10^8$ m/s $\cdot 3.15 \cdot 10^7$ s $= 9.5 \cdot 10^{12}$ km

(b) While Willy is celebrating his 16th anniversary, Lilly is still 16 years old.

8.4 Time travels – the twin paradox

1. $T_k = 0.5 \cdot T_g$ inserted in $T_k = T_g \cdot \sqrt{1 - v^2/c^2}$ gives us

$$v = \frac{\sqrt{3}}{2} c = 2.6 \cdot 10^8 \text{ m/s} = 87\,\%\, c$$

2. (a) Willy calculates the time that Lilly needs to arrive at her destination: $T_W = 0.98$ c/99 Ly $= 3.2 \cdot 10^9$ s $= 101$ years. So he should wait 202 years until Lilly is back.

(b) $T_L = T_W \cdot \sqrt{1 - \frac{v^2}{c^2}} = 20$ years

(c) $v = 99$ Ly/20 years $= 5\,c$. She is surprised, since she knows, that nobody can travel with a velocity greater than c.

(d) She concludes, that it was not correct to use 99 Ly in her calculation. The distance from the start to the star must have been much smaller for her.

8.5 Movement on a circular orbit

1.

$r_N = 4500$ million km $= 4{,}5 \cdot 10^{12}$ m; $T_N = 165$ a $= 5.2 \cdot 10^9$ s
$r_M = 58$ million km $= 5.8 \cdot 10^{10}$ m; $T_M = 88$ d $= 7.6 \cdot 10^6$ s

With $T_K = T \cdot \sqrt{1 - (4\pi^2 r^2)/(c^2 T^2)}$ we obtain

Neptun: $3.1535999994 \cdot 10^{15}$ s
Merkur: $3.1535999597 \cdot 10^{15}$ s

and therefore the difference becomes: $3.97 \cdot 10^7$ s $= 1.26$ a

2. Since $T_K = T \cdot \sqrt{1 - (4\pi^2 r_M^2)/(c^2 T_M^2)}$, we have $T_K < T$.

Clocks on the Moon run slower than clocks on the Earth. Since the decay time of a radioactive material is a property of the material, the same decay time is measured on the Moon as on the Earth. The clocks with which the decay time is measured on the Moon are slower than those on the Earth. That is why the material on the Moon decays more slowly. When half of it has decayed an the Earth, on the Moon there is still a little more than half.

9. Curved space

9.2 Mass curves space – geodesics

1. (a) The lines intersect although they are straight. The cause: the two-dimensional space to which they belong is curved.

(b) The lines do not intersect although they are not straight. The curvature of the lines is compensated by the curvature of the curved space.

2. No, the one-dimensional inhabitants are unable to detect any curvature of their world. No curved lines exist in this world. The people of the 2D world, in which the 1D world is embedded, pretend that the 1D world can be curved.

4. (a) For 2D people the curve MA is the radius. The actual radius, that has to be used in the formula to get the correct result, is the length of the line segment M'A, Fig. 9.1.

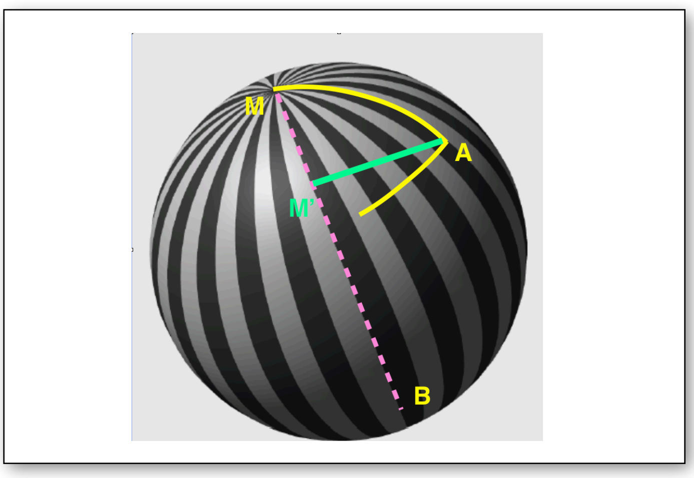

Fig. 9.1

This is shorter than the curved segment. In their world the 2D people have no access to this line segment. They can only calculate it by dividing the circumference (which they can measure) by 2π.

(b) As the distance of A from M increases, also the circumference increases, but not as expected by the 2D people who apply their formula. The error is becoming greater and greater. Finally, A is so far from M, that a further increase results in a decrease of the circumference. When A coincides with B, the circumference has shrunk to zero. Now A is placed at one of the ends of the diameter MB.

9.3 Space curvature in the environment of celestial bodies

1. A = surface area, V = volume

$$V = \frac{\pi}{6} \cdot \left(\frac{A}{\pi}\right)^{\frac{3}{2}}$$

2. Compared to a flat meadow, the surface area within a circle is greater when there is a hill inside, Fig. 9.2.

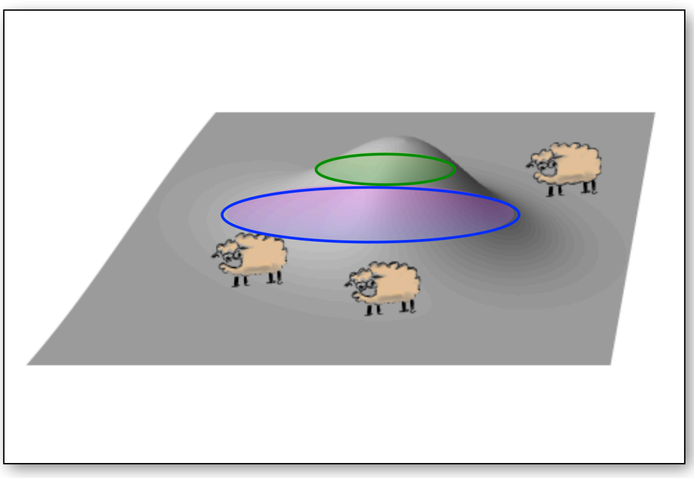

Fig. 9.2

9.4 Trajectories in the gravitational field

1. (a) Angle of deflection 0.016 deg

(b) Mass of the Earth $6 \cdot 10^{24}$ kg, radius 6350 km. Angle of deflection $2.8 \cdot 10^{-9}$ deg.

2. Light that comes form that part of the Sun's surface that is lighter in the left part of Fig. 9.3 does not reach the observer. The gravitational field of the neutron star is strong enough to bend the light so that the observer can see more than half of the surface (right image).

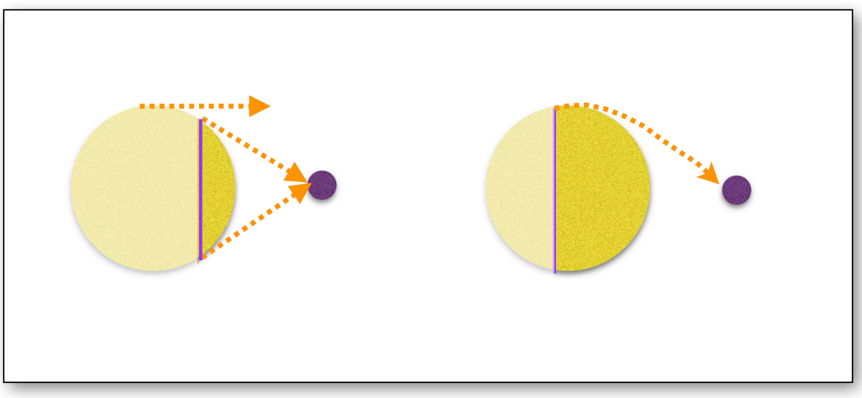

Fig. 9.3

9.8 Gravitational waves

1. The wavelength of the detected gravitational wave is $\lambda_{gr} = 10^6$ m and thus $1{,}4 \cdot 10^{14}$ times that of red light ($\lambda_{red} = 10^{-9}$ m).

2. In the instant of the snapshot the spacial extension is becoming greater in the horizontal direction and smaller in the vertical direction. If B is chosen as a reference point, both Lilly and Willy are moving away from B.

10. Cosmology

10.4 The expansion of the universe

Increase = x

$$H = \frac{2.1\ \text{m/s}}{100\ \text{Lj}} = \frac{\text{x/year}}{1\ \text{km}}$$

$$x = \frac{2.1\ \text{m/s} \cdot 1\ \text{km}}{100\ \text{Ly}} \cdot \text{year} = \frac{2.1\ \text{m/s} \cdot 1\ \text{km}}{100 \cdot c \cdot \text{year}} \cdot \text{year} = 7 \cdot 10^8\ \text{m}$$

10.6 What we see of the universe

We would see everything that was located at the instant of formation within a distance of 14 billion Ly.