Which way does the light go?

Thilo Wünscher, Holger Hauptmann, and Friedrich Herrmann[a)]
Abteilung für Didaktik der Physik, Universität, 76128 Karlsruhe, Germany

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Pictures of the energy density and the energy flow in distributions of incoherent light for various two-dimensional situations are shown and discussed. Rules are introduced that allow one to sketch and to interpret such pictures.
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I. INTRODUCTION

Consider the light in a room. The light is emitted by light sources, for example, the sun, lamps, or a television screen and is eventually absorbed by other bodies. Because light carries energy, energy flows from the sources to the absorbers. Now ask a student for the distribution of this energy flow. There is a good chance that he or she will be embarrassed.

This question is similar to questions about the nature of the electric field lines in electrostatics. In the latter case, physics students are expected to be able to draw a qualitative field line diagram, but they would have only a poor feeling for the energy flow in a standard light field. There are at least two reasons for this deficiency: they never have seen such diagrams, and they do not know the methods or rules for sketching them.

Our goal is to remedy this situation in both respects. Pictures of the energy and energy flow distributions will be shown and discussed for several common situations, and rules will be introduced that allow us to sketch and interpret such pictures.

The energy flow in a light field plays an important part in many technical applications and is of interest in nonimaging optics.[1] For example, nonimaging optics asks for the flow of light from area A to area B without requiring a point-to-point imaging of A on B. The objective may be to get the light from A onto the smallest area B that is allowed by the second law of thermodynamics.[2] The corresponding device is called a light concentrator, and is used, for example, in connection with solar collectors and the collection of the light from a Cerenkov counter.[3] Another application of nonimaging optics is to obtain a homogeneous distribution of the light on area B, as might be required for room or floodlight illumination. The quality of such a device can be judged by means of an energy flow diagram. Energy flow distributions are also needed for ultrahigh frequency applications. An example is channeling ultrahigh frequency electromagnetic radiation into a street or railway tunnel in order to communicate with the vehicles.[4] Finally, the energy flow is important for computer graphics in order to create photo-realistic pictures.[5]

Although these applications are sufficiently interesting to give some attention to the question of the energy flow, our object is provide insight into the nature of light. In our opinion, this consideration could be a natural complement to geometrical and wave optics.

We begin in Sec. II by introducing the radiance and two relations that enable us to calculate the energy density and the energy flow density when the radiance is known. In Sec. III, rules will be formulated for drawing qualitative energy flow diagrams. The most important part of the article is in Sec. IV, where diagrams of the energy flow in standard situations are shown and discussed. In Sec. V we discuss the question in the title of the article, Which way does the light go?

II. RADIANCE, ENERGY FLOW DENSITY, AND ENERGY DENSITY

To describe the distribution of incoherent light, an appropriate quantity is the radiance L.[6] The two quantities that are of interest, the energy flow density $\mathbf{j}$ and the energy density e, can be calculated from the radiance. The energy flow density is obtained by

$$\mathbf{j}=\int_{4\pi} L\, d\mathbf{\Omega}, \tag{1}$$

and the energy density by[7]

$$e=\frac{1}{c}\int_{4\pi} L\, d\Omega. \tag{2}$$

The vector quantity $d\mathbf{\Omega}$ is the differential of the solid angle, and $d\Omega$ is its magnitude.[8] The integration is over the full solid angle 4π in Eqs. (1) and (2).

To understand the meaning of L, consider Eq. (1), which can be considered to define the radiance. Equation (1) tells us that radiance is energy per unit time, area, and solid angle. It is a local quantity in two respects; L depends on the spatial coordinates x, y, and z and at every position, L depends on the chosen direction, which is defined by the polar and azimuthal angles ϑ and φ.[9] In other words, $L(x,y,z,\vartheta,\varphi)$ tells us how much light in the direction (ϑ, φ) is found at position (x, y, z).

The radiance is essentially what we perceive with our eyes as can be seen by considering a measuring device for L, a radiance meter (see Fig. 1). A cylindrical tube is terminated at one of its ends by a lens of diameter d and closed at the other end. The length of the cylinder is equal to the focal length f of the lens. The inner sides of the walls are completely absorbing. Located at the center of the closed end, that is, at the focus of the lens, is a small circular light detector of diameter d'. The device resembles a camera focused to infinity. In contrast to a real camera, we are interested only in the light that is parallel to the optical axis. The radiance meter detects light at a particular position and with a particular direction. The position is that of the entrance of the meter. Thus, the spatial resolving power is determined by the diameter of the lens. The direction is essentially that of the optical axis of the meter. The angular resolving power in

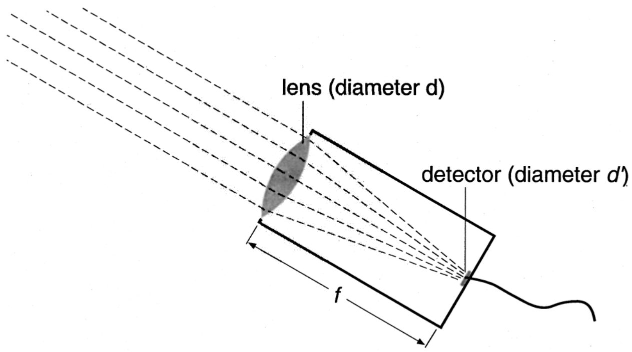

Fig. 1. Radiance meter. It detects that light at the *position* of the lens that has the *direction* of the optical axis of the lens.

the geometrical optics limit is equal to the ratio d'/f.[10] With such a meter we can verify an important theorem[11] (see Fig. 2):

> *On a straight line, the radiance in the direction of this line is the same everywhere, as long as the refractive index is constant, and as long as the line does not meet a region where light is diffused, absorbed, or emitted.*

This observation is equivalent to a very familiar experience: An object does not appear brighter to our eyes, when we go closer to it. We shall need this theorem to calculate energy flow and energy density distributions.

For Eqs. (1) and (2) to be valid, the light must be incoherent, because it has been assumed that the energy flow density and the energy density were obtained by adding up differential contributions of these quantities. This assumption is true only for incoherent light. For coherent light, the field strengths would have to be added instead of the energy flows. Incoherence will be assumed throughout this article.[12] Moreover, we have to assume that the light satisfies the geometrical optics limit. Otherwise, the concept of a light ray would not be applicable and the integrations in Eqs. (1) and (2) would not be possible.

Note that Eqs. (1) and (2) describe an averaging process. Because $Ld\boldsymbol{\Omega}$ stands for an elementary light bundle, Eq. (1) tells us that $\mathbf{j}$ is obtained by averaging all these bundles, taking into account their respective directions. Two light bundles of the same radiance, but opposite directions cancel each other. The integral in Eq. (2) stands for an averaging process, which results in the total amount of light, independent of the directions of the various contributions.

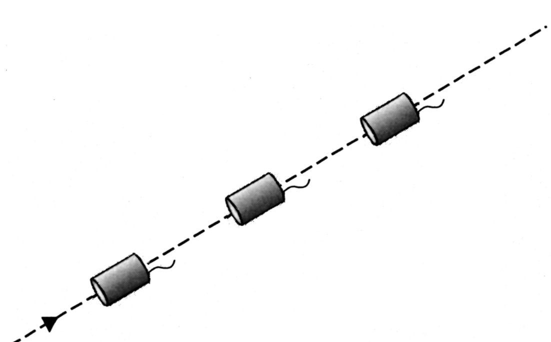

Fig. 2. On any light ray the radiance in the direction of that ray is constant.

Although most of the diagrams of Sec. IV could have been drawn qualitatively by using the rules of Sec. III, they were realized by means of a program called LIGHTLAB.[13] As input, LIGHTLAB asks the user to place objects on a drawing plane: light sources, mirrors, scatterers, and absorbers. These objects form what we call a scenery.

Because flow diagrams in a three-dimensional representation would be rather complex, we only consider cases where all of the flow lines remain in a plane. We thus either assume that all of our objects have infinite extension and translational symmetry in the direction perpendicular to the drawing plane, or we imagine the world to be two-dimensional. Then, L depends only on two positional coordinates x and y, and one directional angle φ: $L = L(x,y,\varphi)$. When calculating the energy density and the energy flow density from the radiance by means of Eqs. (2) and (1), respectively, we only have to integrate over φ. In a two-dimensional world, the surfaces of the light emitting, absorbing, reflecting, and scattering objects reduce to lines.

LIGHTLAB calculates the energy flow lines in two steps. First, the surfaces of all of the objects of the scenery are divided into many small surface elements; for every surface element, the directions are divided into many small angular elements. The energy flow in each of these surface/angular elements can be considered the quanta of the radiance at the objects' surfaces. LIGHTLAB calculates the radiance quanta for the surfaces of all of the objects by using the technique of ray tracing. The program chooses the first light source, on this light source the first surface element, and at this surface element the first angular element, and then calculates the path of the corresponding light ray. This ray may hit some of the other objects. The energy flow of this first radiance quantum contributes to one of the surface/angular elements of the objects that have been hit.

In the same way the contribution of the other radiance quanta of the source to the radiance of the other bodies is calculated and then that of the other sources. Now, the radiance is known for the surfaces of all the bodies of the scenery. A particular problem arises when there are bodies with a scattering surface. Then, at the point of impact, the ray-tracing process has to go in the directions of all of the angular elements, the energy flow per quantum being correspondingly weighted.

The last step is simple. According to Eqs. (1) and (2), the energy density and the energy flow is calculated at any given point by summing up the energy flow contributions of the radiance quanta going through this point. The method of ray tracing can be applied because of the validity of the theorem of Sec. II, according to which the radiance of a certain direction is constant on a straight line in this direction.

The results are displayed in the following way. The energy density is represented by small circles, their surface area being proportional to the energy density. The energy flow distribution can be represented either by vector arrows on a rectangular grid or by stream lines. We have opted for the more suggestive stream line representation, although it suffers from the same flaws as any other stream line or field line diagram.[14,15]

III. RULES FOR QUALITATIVELY DRAWING ENERGY FLOW DIAGRAMS OF LIGHT FIELDS

All physics students know the rules that allow them to sketch electric field lines and to interpret field line pictures. For comparison, let us recall some of these rules:

(1) Electric field lines begin on positive and end on negative charges.
(2) A field line has a kink only where the charge density or the dielectric constant has a discontinuity.
(3) Two field lines never cross one another.
(4) Field lines begin or end perpendicularly at the surface of electric conductors.
(5) A field line picture has the same symmetry as the boundary conditions (the system of sources, conductors, and dielectrics).

In the same way, rules can be formulated for creating qualitative pictures of energy flow lines in light fields or for interpreting such diagrams. We shall formulate some of these rules and make a short comment on each of them.

(1) *Energy flow lines never cross one another*. This rule is valid for every flow line diagram, because the corresponding vector quantity has a definite value at every position.

(2) *Energy flow lines begin on light sources and end on absorbers*. This rule is an expression of the continuity equation for the energy.

(3) *Any symmetry of the object, light sources, absorbers, mirrors, etc., is the cause of a similar symmetry of the energy density and the energy flow diagram*.

(4) *In the immediate vicinity of a reflecting surface or of a white diffusing surface, the energy flow lines run parallel to these surfaces*. This rule holds because no energy enters such surfaces. The flow vector is not allowed to have a component perpendicular to them.

(5) *At a great distance from a collection of sources, reflectors, scatterers, and absorbers, the energy flow lines run radially outwards*. To the integral of Eq. (1) there are only contributions from a small angular domain.

(6) *In the immediate vicinity of the surface of an isotropic radiator, the energy flow lines are perpendicular to this surface, as long as no other objects or other points of the same object contribute to the light field at these locations*. Isotropic means that at any surface point of the radiator, the radiance does not depend on the polar and azimuthal angles, ϑ and φ. To the integral in Eq. (1) there are contributions only from the half of the solid angle into which the surface is radiating. Because the integrand is constant, the resulting vector is perpendicular to the surface.

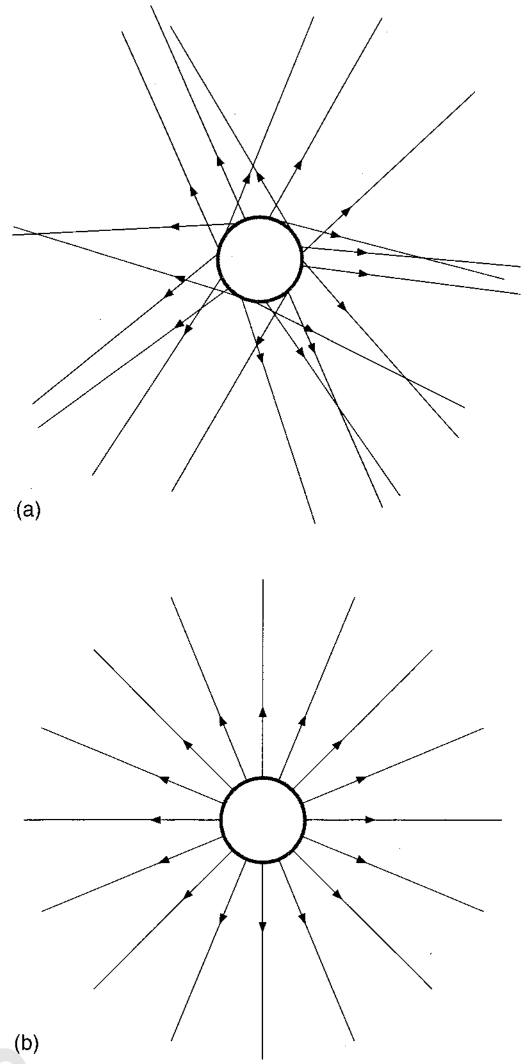

Fig. 3. The light in the vicinity of the sun: (a) light rays; (b) energy flow lines.

IV. EXAMPLES

Energy flow diagrams are shown for the following examples. For comparison we also sometimes show the familiar light ray representation. Where it is interesting, the energy density diagram is depicted.

A. A spherical light source

Two realizations are the sun and a spherical frosted lamp. In our two-dimensional world, such a source appears as a circle. Figure 3(a) shows some arbitrarily chosen light rays, and Fig. 3(b) is the energy flow diagram. The drawing is similar to how children sometimes draw the sun. Apparently, they are not so wrong.

B. Cloudy sky

Imagine the surface of the earth to be planar and completely absorbing. At a certain altitude, there is a continuous cloud cover (see Fig. 4). For a physicist who is accustomed to geometrical optics and wave optics, this situation is awkward: completely diffuse light in half of the full solid angle [Fig. 4(a)]. On the contrary, in the energy flow representation shown in Fig. 4(b), we have the physicist's favorite field: a homogeneous field. Note that from an energy flow diagram, the angular width of the radiance distribution, that is, the aperture of the light, cannot be read. The sun in the zenith would cause the same energy flow diagram as the clouds in Fig. 4.

C. Light beam incident on a plane mirror

Figure 5(a) shows the familiar diagram using the method of geometrical optics, and Fig. 5(b) is the energy flow diagram. Outside of the triangular domain of penetration, the light rays and the energy flow lines have the same shape. Not so, however, within the penetration region. As is to be expected, the energy flow lines run parallel to the mirror's surface according to rule (4).

D. Two mutually penetrating light beams

The beams have different energy flow densities. The representation of geometrical optics is shown in Fig. 6(a). The

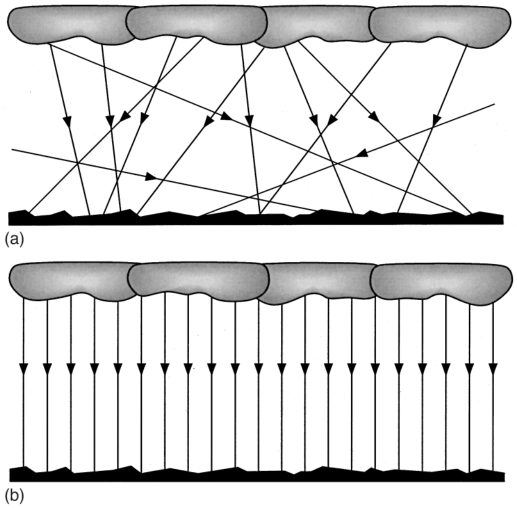

Fig. 4. The light when the sky is cloudy: (a) light rays; (b) energy flow lines.

energy flow diagram shows that in the penetration region, energy is changing from one beam (in the sense of geometrical optics) to the other one [see Fig. 6(b)].

E. The Lambertian radiator

If the radiance of the light emitted by a body is the same for all of the points of its surface and for all directions, the radiator is called Lambertian. It is the most uniform way a body can radiate.

According to rule (6), from a single Lambertian radiator whose surface is nowhere convex, the energy flow lines start perpendicularly in every point of the surface (see Fig. 7). There is a simple counterpart to this situation. Imagine the entire space to be filled with light that is completely homogeneous and completely diffuse (homogeneous in the space of directions). Thus, for every position and at every position for every direction, L has the same value. Now, immerse into this light field a body that has the same shape as the body at the center of Fig. 7, but which is absorbing instead of radiating. The corresponding energy flow diagram has the same shape as that of Fig. 7, the only difference being that the flow

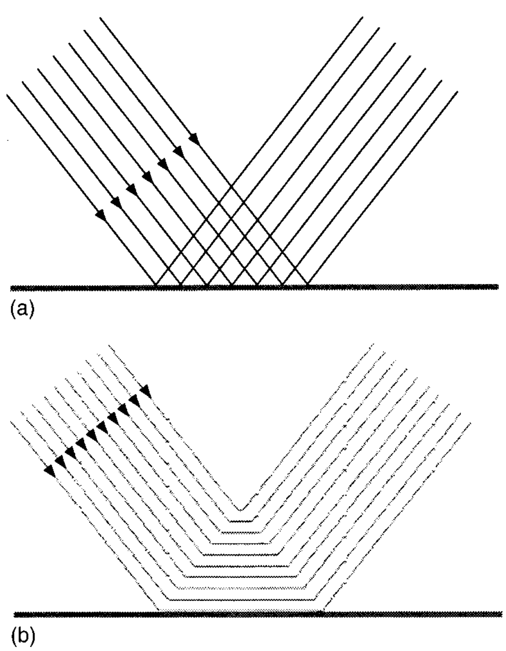

Fig. 5. Reflection of a light beam on a mirror: (a) light rays; (b) energy flow lines.

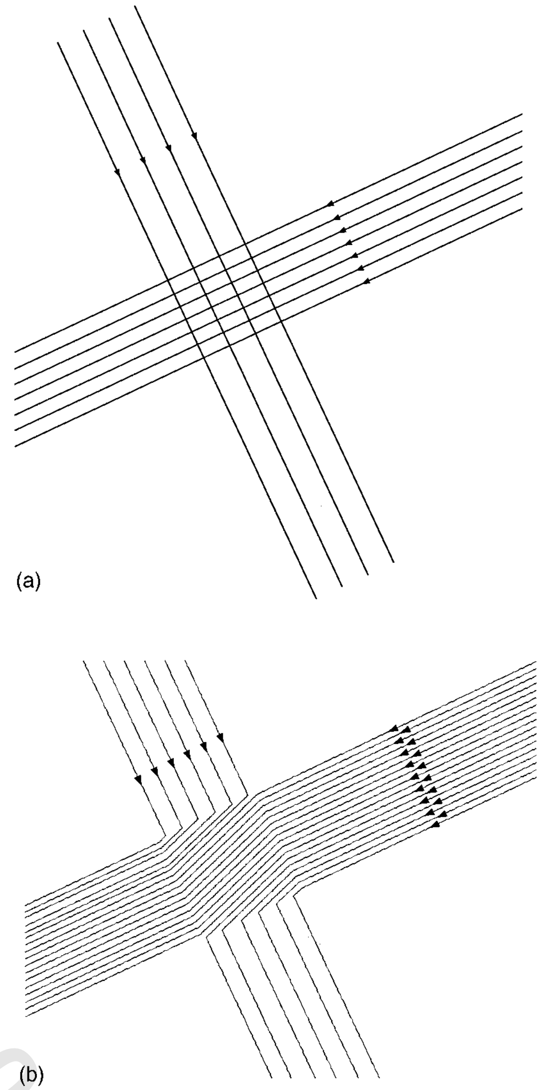

Fig. 6. One light beam ''penetrates'' another one. Do the beams cross or avoid one another? (a) Light rays; (b) energy flow lines.

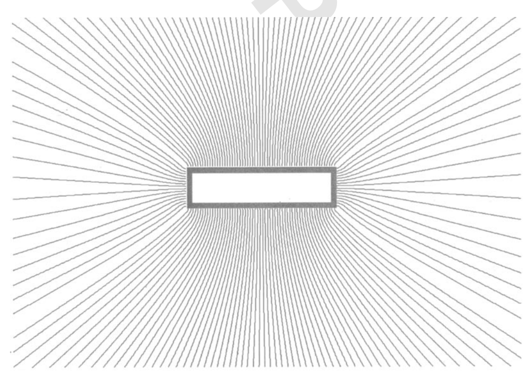

Fig. 7. Lambertian radiator or absorber.

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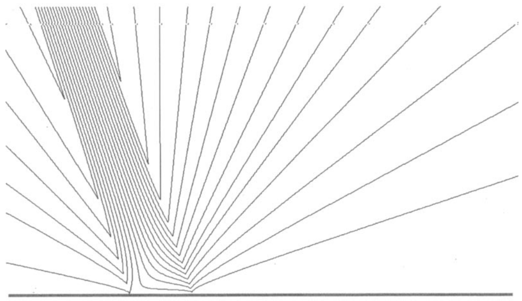

Fig. 8. Scattering of a light beam by a perfectly white surface.

directions are inverted. From now on we shall omit the arrows in the flow diagrams whenever the flow direction is obvious.

F. Light beam incident on a white surface and on a frosted glass surface

In Fig. 8 the radiance of the outgoing light at the scattering surface is assumed to be the same for every direction. For the mat glass plate in Fig. 9, the radiance is assumed to be the same for the forward and for the backward scattered light. The backscattering of light manifests itself in the backbending of the flow lines at the left side of the frosted glass. The effect is particularly pronounced for the two outermost lines.

G. Straight tube with reflecting or white walls

The end surface is a Lambertian radiator, the walls are perfect mirrors (Fig. 10) or perfectly white scatters (Fig. 11). In both cases, the energy flow is homogeneous within the ‘‘tube’’ [see Figs. 10(a) and 11(a)]. The flow is less for the scattering walls, although the radiators are identical in both cases. The reason for this difference becomes obvious in the energy density diagrams, Figs. 10(b) and 11(b). Although the energy density is low and constant in the reflecting-wall tube when going from left to right, it decreases within the tube with the scattering walls. In the language of geometrical optics, in the latter case light is backscattered and reabsorbed by the radiation source. (Like any thermal radiator, our light source is supposed to be a perfect absorber.) The backscattered light contributes to the energy density, but compensates part of the energy flow from left to right. Imagine now that we make both tubes longer and longer. The diagrams of the

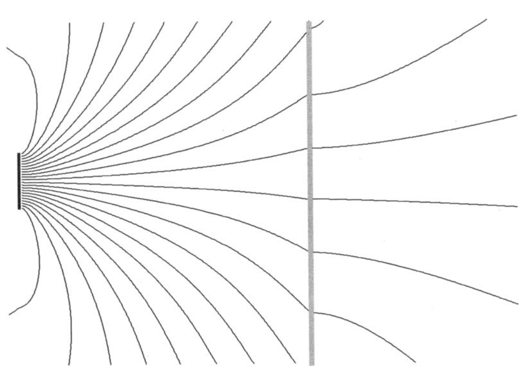

Fig. 9. Scattering of light from a flat Lambertian source by a mat glass plate.

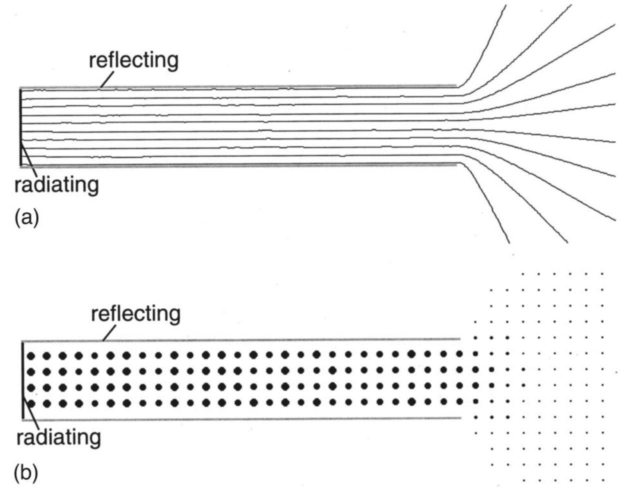

Fig. 10. Lambertian radiator and ‘‘tube’’ with reflecting walls: (a) energy flow density; (b) energy density.

tube with the reflecting walls will not change significantly, whereas from the tube with the white walls the energy leak becomes smaller and smaller.

H. Bent tube with reflecting walls

The situation of Fig. 12 is not very different from the straight reflecting tube of Fig. 10. Again we observe an energy flow that reminds us of a laminar flow of a liquid. The irregularities of the energy density are even more pronounced than in Fig. 11(b).

I. Closed and spiraling stream-lines

Figure 13 shows a phenomenon that is well-known from Poynting vector fields: closed energy flow lines. They appear in many situations. The example of Fig. 13(a) is a very simple case. In Fig. 13(b), we have modified the scenery only slightly: In the center of the region with closed flow lines in Fig. 13(a), a small absorber is placed. Now, one of the incoming flow lines is spiraling toward the absorber instead of bending back together with the other incoming lines. Here, a phenomenon is observed that we had already dis-

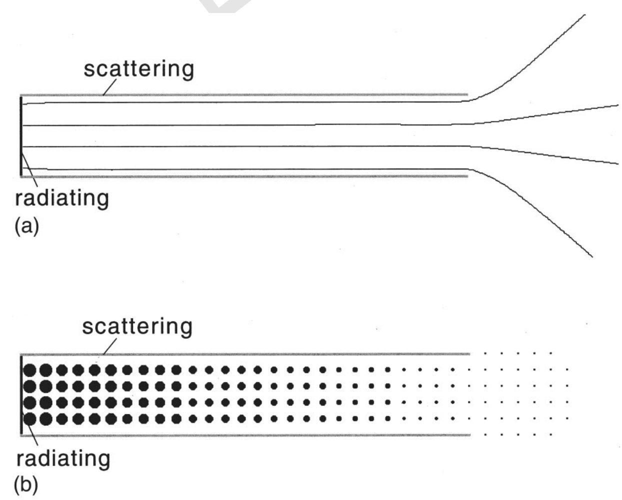

Fig. 11. Lambertian radiator and tube with white walls: (a) energy flow density; (b) energy density.

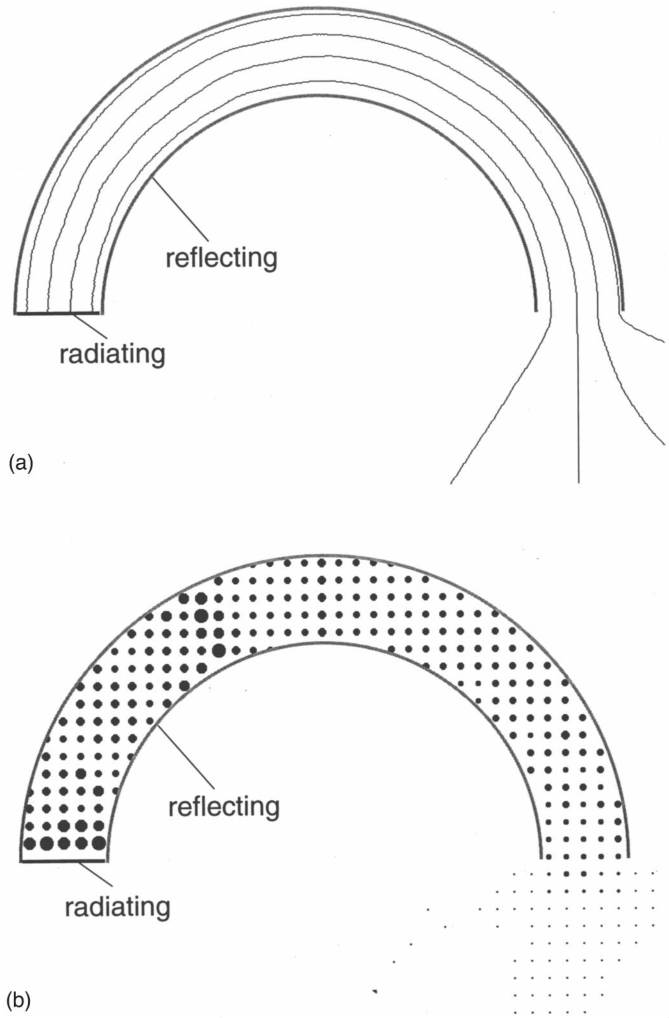

Fig. 12. Lambertian radiator and a bent tube. The inner sides of the walls are reflecting: (a) energy flow density; (b) energy density.

cussed in the context of electric field lines:[15] The line density does not even approximately reflect the magnitude of the energy flow vector. If the absorber is made sufficiently small, the line density in the region of the spiral can be made arbitrarily high.

J. Light concentrator

Figure 14 shows a simple version of a light concentrator. The light that enters opening A at the left-hand side of the mirrors is channeled to the smaller opening B at the right. A three-dimensional analog of this concentrator would be a conical mirror. The maximum value of the concentration factor c, defined as $c = A/B$, is determined by the second law of thermodynamics. Indeed, the concentrator deforms the phase space region occupied by the light: the light entering the concentrator has a great extension in space and occupies only a small angular interval, that is, its extension in momentum space is small, whereas the light reaching the exit of the concentrator has a great angular and a small spatial spreading. The trade-off between position and momentum is an expression of the reversibility of the process: No entropy is produced as the light proceeds through the concentrator.

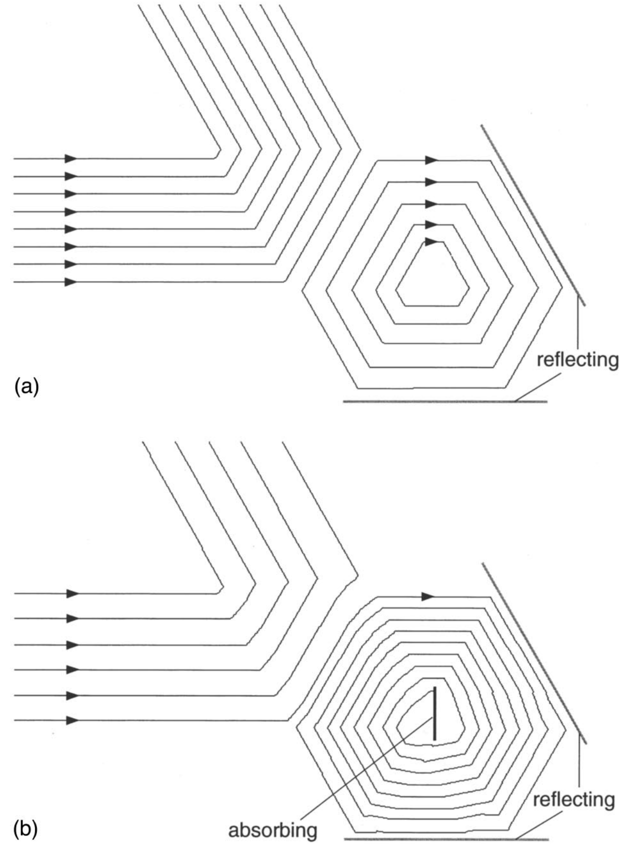

Fig. 13. (a) With two mirrors closed energy flow lines can be created. (b) The energy flow spirals around a small absorbing object. It is obvious that the line density does not correspond to the amount of the flow density vector.

K. Illumination of a surface

In Fig. 15, an arrangement of two mirrors is used to illuminate a distant surface by means of a flat Lambertian radiator. The difference between Figs. 15(a), 15(b), and 15(c) is the angle extended by the mirrors. If the mirrors are parallel, Fig. 15(a), there is no angular concentration at all. In Fig. 15(b), the bundling effect is weak, because the angle between the mirrors is too great. With the mirrors of Fig. 15(c), a floodlight effect is obtained. The problem of obtaining an optimum bundling effect is analogous to finding the maximum concentration factor of a light concentrator.

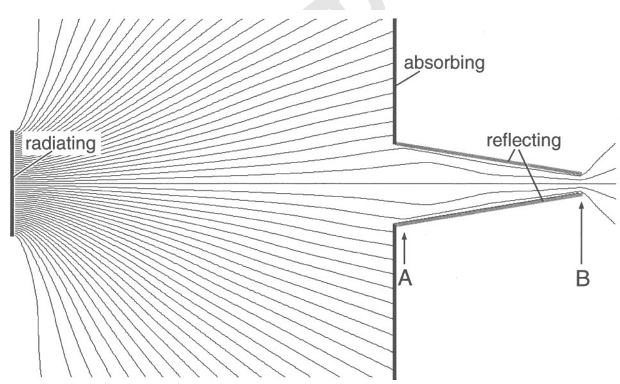

Fig. 14. Simple light concentrator.

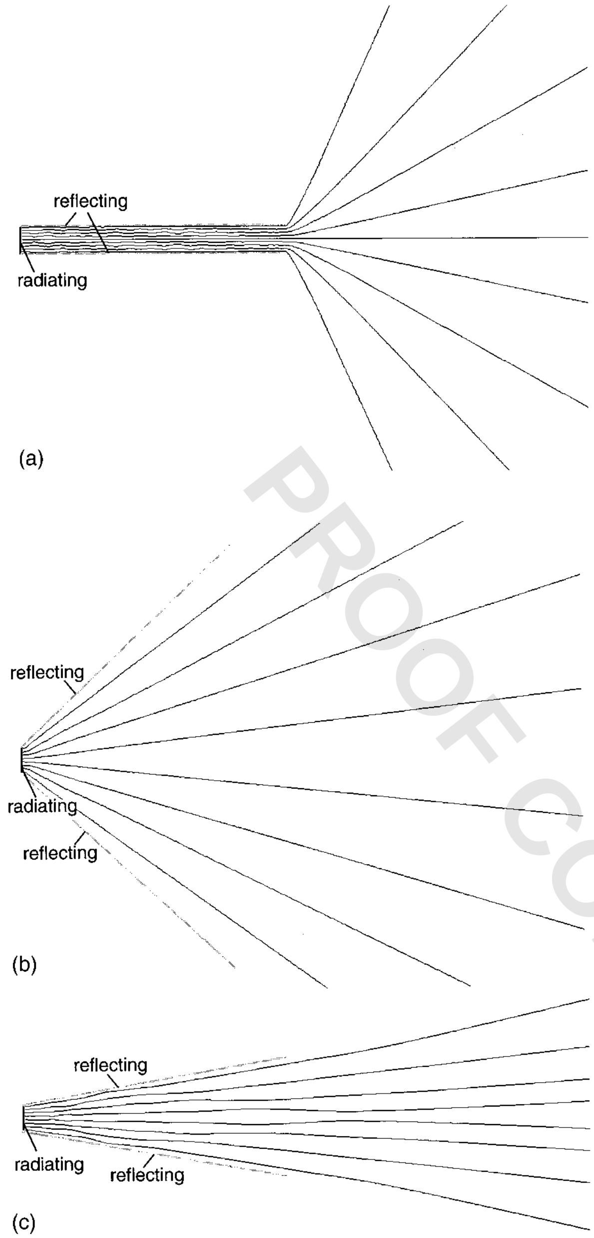

Fig. 15. Flat Lambertian light source and two mirrors: (a) no angular concentration; (b) weak angular concentration; (c) good angular concentration.

L. Sunshine and a room with a window

Figure 16 shows a more involved situation: a room with a window. The walls and the ceiling of the room scatter 80% of the incoming light, the floor only 30%. The sun is 15° above the horizon. But there is also diffuse light from the sky. The direction of the sun can be read from the figure in several ways: It is identical with that of the large light beam going from the window to the mirror, and it can also be read from the row of kinks in the energy flow lines on the left-hand side of the wall outside of the house.

V. WHICH WAY DOES THE LIGHT GO?

There is no doubt that light goes in some way from a source to an absorber. However, if we are asked for the path of the light from the source to the absorber, we can easily get into trouble. The question of the trajectory or the path of

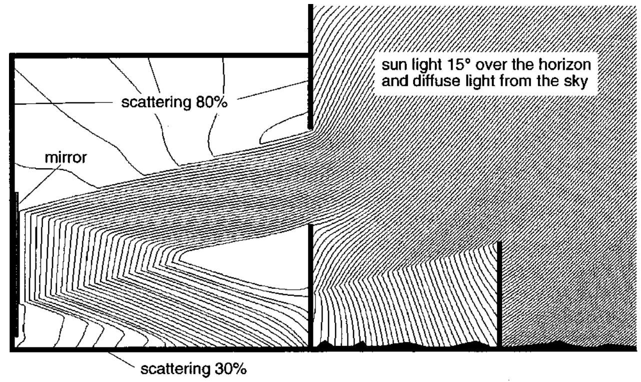

Fig. 16. The left-hand side shows a room. Light is coming from outside: The sun is shining, and there is diffuse light from the sky.

something has a clear meaning only if this ''something'' can be tracked in space and time. But photons, the quanta of light, cannot be tracked. Therefore, quantum electrodynamics cannot tell us which way the light goes.

There are, however, ersatz concepts that can give us the trajectory in certain circumstances. Such concepts are based on a particular model of light. One model interprets the light rays of geometrical optics as the path of the light. Only when this model is used, does it make sense to say that light propagates rectilinearly or to formulate the law of reflection in the usual way. A generalization of this model is used when interpreting Fermat's principle: The light takes the trajectory that minimizes the ''optical path length.''

We should not be surprised that other models give different answers for the path of the light. That is what happens when interpreting the energy flow lines as the trajectories of light. The question of where the light is and its path becomes equivalent to the distribution of the energy density and of the energy flow density.[16] Note that this description is neither false nor correct, just as the statement of geometrical optics that light bounces back from a mirror.

Let us consider again the example of Fig. 6. By interpreting the energy flow lines as the trajectories of the light, we are led to say that part of the light of one beam is deviated into the other, whereas geometrical optics tells us that the two light beams penetrate each other undisturbed. Neither of the two statements is more correct than the other. Each of them is a convenient interpretation in the framework of a particular mathematical description.

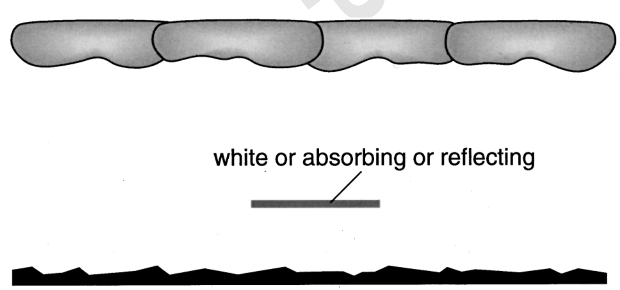

Fig. 17. Cloudy sky, the earth is completely absorbing. What are the energy flow lines? The flat object can be black, white, or reflecting. Next, assume that the sky is not cloudy and the sun is in the zenith. (For simplicity, assume that the sky is dark instead of blue.)

VI. CONCLUSION

Our objective was to gain a feeling for the shape of energy flow distributions in typical light fields, in the same spirit as we have a feeling for the electric field distribution in standard situations. We hope that you are now able to solve the problem of Fig. 17. Perhaps you will find an opportunity to propose it to one of your colleagues.

[a]Electronic mail: Friedrich.herrmann@physik.uni.karlsruhe.de

[1]W. T. Welford and R. Winston, *High Collection Nonimaging Optics* (Academic, New York, 1989).

[2]P. Würfel, *Physik der Solarzellen* (Spektrum, Akademischer Verlag, Heidelberg, 2000), p. 34.

[3]H. Hinterberger and R. Winston, ''Efficient light coupler for threshold Cerenkov counters,'' Rev. Sci. Instrum. **37**, 1094–1095 (1966).

[4]D. Didascalou, T. M. Schäfer, F. Weinmann, and W. Wiesbeck, ''Ray-density normalization for ray-optical wave propagation modeling in arbitrarily shaped tunnels,'' IEEE Trans. Antennas Propag. **48**, 1316–1325 (2000).

[5]A. S. Glassner, *An Introduction to Ray Tracing* (Academic, New York, 1989).

[6]To most of the ''radiometric'' quantities (energy, energy flow, energy flow density, etc.), a corresponding ''photometric'' quantity can be defined (quantity of light, luminous flux, illuminance, etc.). The photometric quantities are obtained by multiplying the spectral energetic quantities by a wavelength dependent weighting factor. Thus, all of our energy flow diagrams can also be interpreted as the flow lines of the (photometric) ''quantity of light.''

[7]Max Planck, *Vorlesungen über die Theorie der Wärmestrahlung* (Verlag Johann Ambrosius Barth, Leipzig, 1913), pp. 22–23.

[8]A cosine appears in Eq. (1) when written with the magnitudes of the vector quantities **j** and $d\mathbf{\Omega}$. See for example, Max Born and Emil Wolf, *Principles of Optics* (Pergamon, Oxford, 1964), 2nd ed., p. 182, Eq. (5).

[9]In effect we have a six-dimensional phase space with the three space coordinates x, y and z, and three Cartesian momentum coordinates (k_x, k_y, and k_z) equivalent to the spherical bookkeeping (k, ϑ, and φ), where k is the magnitude of the momentum.

[10]Note that the angular resolution cannot be increased arbitrarily, because for $d'/f<\lambda/d$, the resolution is diffraction limited. (λ is the wavelength of the radiation.)

[11]The radiance meter will indicate the correct value of L only as long as it ''sees'' the light source under an angle that is greater than its angular resolving power. Thus, if the source is too small and/or too far away, as for example a star other than the sun, the meter will no longer measure correctly.

[12]The energy flow of Eq. (1) can be interpreted as the time average of the Poynting vector. Note, however, that this remark is not of much use. For the kind of fields that we are considering, the distributions of the electric and magnetic field strength are so complicated that there is no possibility to know them and no interest in knowing them. Knowing the electric and magnetic fields of, say, thermal radiation would be a task similar to knowing the positions and momenta of all of the molecules of a material gas in thermodynamic equilibrium.

[13]A free download of LIGHTLAB 2.0 is available at http://www.physikdidaktik.uni-karlsruhe.de.

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