The Karlsruhe Physics Course

for the lower secondary school

Volume 1

Note to the reader

We have chosen a *one-section-one-page* layout. The advantage is that figures, tables and equations stay where they are supposed to be. Moreover, it is easy to update the text. For reading we recommend the Adobe reader or the GoodReader.

Friedrich Herrmann

The Karlsruhe Physics Course

A Physics Text Book for the Lower Secondary School
Volume 1

With the collaboration of Karen Haas, Matthias Laukenmann, Lorenzo Mingirulli, Petra Morawietz, Dieter Plappert and Peter Schmälzle

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Preface

Detours due to the historical development of physics – Knowledge that got lost

Today's science curriculum is the result of a process of evolution. It reflects the process of the development in great details. Those who are learning science have to follow a path that is very similar to the course of the historical development. They have to take detours, to overcome unnecessary obstacles and to reproduce historical errors. They have to learn inappropriate concepts and employ outdated methods. When developing the *Karlsruhe Physics Course* we have tried to eliminate such obsolete concepts and methods.

F. Herrmann and G. Job:

Series of articles "Historical burdens on physics"

F. Herrmann and G. Job:

The historical burden on scientific knowledge, Eur. J. Phys. **17** (1996), p. 159

In the history of science it happened time and again that important works and results were not accepted by the scientific community: When they arrived it was too late. A change, – although it might have been extremely useful – had become too tedious. Here are three examples:

1. The physical quantity entropy had three chances to become a quantity that would be easy to grasp, even for a beginner; the first chance was after the works of Joseph Black and Sadi Carnot, the second after the work of H. L. Callendar and the third through the book *A new concept of thermodynamics* by Georg Job. All of these chances were missed. The corresponding ideas had been incorrectly interpreted or simply ignored.

2. The physical quantity force with the corresponding terminology – a sophisticated construction of Newton – turned out to be the strength of the current of momentum. The corresponding publication from 1908 by Max Planck remained virtually unnoticed.

3. The first 50 years after the introduction of the energy into physics it was not clear if energy obeys a local conservation principle. It was expected but not proven. For that reason a terminology came in use that took these doubts into account. The publication of 1898 by Gustav Mie, in which it is shown that energy obeys a continuity equation did not lead to a more appropriate and simple language. We still speak about energy as if we had to be prepared that one day actions at a distance might be discovered.

The Karlsruhe Physics Course takes these buried findings into account. Since the original literature is not easily accessible, here some links:

J. Black: Lectures on the elements of chemistry, Mundell and Son, Edinburgh (1803)

S. Carnot: Réflexions sur la puissance motrice du feu, Librairie scientifique et technique, A. Blanchard, Paris (1953)

H.L. Callendar: The caloric theory of heat and Carnot's principle, Proc. Phys. Soc. London **23** (1911), S. 153

G. Mie: Entwurf einer allgemeinen Theorie der Energieübertragung, Sitzungsberichte der Kaiserlichen Akademie der Wissenschaften. CVII. Band VIII. Heft (1898), S. 1113

M. Planck: Bemerkungen zum Prinzip der Aktion und Reaktion in der allgemeinen Dynamik, Physikalische Zeitschrift, 9. Jahrgang, Nr. 23 (1908), S. 828

G. Job: Neudarstellung der Wärmelehre – Die Entropie als Wärme, Akademische Verlagsgesellschaft Frankfurt (1972)

Substance-like quantities in the Karlsruher Physics Course (KPC)

In the KPC the substance-like quantities are the basic quantities. A substance-like or extensive quantity is a quantity, whose value refers to a region of space:

mass
energy
momentum
electric charge
entropy
amount of substance

To each of these quantities a density can be defined, and to each of them corresponds a current with a current strength and a current density. To each of them a simple and direct intuitive representation can be formed: It is convenient to imagine them as a measure of the amount of a substance, i.e. to apply the *substance model.*

The use of the substance model in the KPC has two consequences:

In mechanics momentum and momentum currents are treated right from the beginning.

In thermodynamics entropy and entropy currents are treated right from the beginning.

Analogies in the Karlsruhe Physics Course

The KPC takes advantage of several analogies:

1. The analogy between the substance-like quantities allows for a simple mental representation of many physical processes as a flow.

Energy E	Momentum $\mathbf{p}$	Electric charge Q	Entropy S	Amount of substance n
Energy current P	Momentum current $\mathbf{F}$	Electric current I	Entropy current I_S	Substance current I_n

2. The analogy between momentum, electric charge, entropy and amount of substance allows for the use of a model, where a difference of the values of the intensive variable appears as the cause of a current of the corresponding extensive quantity.

Momentum $\mathbf{p}$	Electric charge Q	Entropy S	Amount of substance n
Momentum current $\mathbf{F}$	Electric current I	Entropy current I_S	Substance current I_n
Velocity $\mathbf{v}$	Electric potential ϕ	Temperature T	Chemical potential μ

3. The analogy between momentum and angular momentum allows for a simple description of rotational dynamics.

Momentum $\mathbf{p}$	Angular momentum $\mathbf{L}$
Momentum current $\mathbf{F}$	Angular momentum current $\mathbf{M}$
Velocity $\mathbf{v}$	Angular velocity $\boldsymbol{\omega}$
Mass m	Moment of inertia $\mathbf{J}$

4. The analogy between electric and magnetic field strength results in a simplified presentation of magnetostatic phenomena.

Electric charge Q	Magnetic charge Q_m
Electric field strength $\mathbf{E}$	Magnetic field strength $\mathbf{H}$

5. The analogy between energy and amount of data (information) indicates similarities between phenomena and devices of the modern energy and information techniques.

Energy E	Amount of data H
Energy current P	Data current I_H

The genesis of the Karlsruhe Physics Course

Before beginning with the development of a school version there was always the work on a version for the University. In this way it was assured, that the course could serve as a solid basis for any follow-up course. The first of a newly developed teaching unit took always place at the Europa-Gymnasium at Wörth.

1988-1992: Test of the KPC at 20 selected schools in the Federal State of Baden-Württemberg, under the supervision of the Ministry for Culture and Sport.

1994: Thanks to a special clause in the official curriculum the KPC can be used at all High Schools in Baden-Württemberg. The course is marketed by the *Landesinstitut für Erziehung und Unterricht* (Federal Institute for Education and Instruction) at Stuttgart.

1998: Printing and marketing is taken over by the AULIS publishing house.

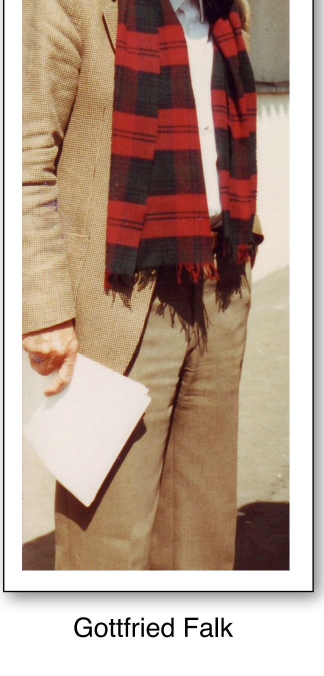
1996-2001: The KPC is evaluated in a PhD theses at the IPN (Leibniz Institute for Science and Mathematics Education).

2004: Accreditation of the lower secondary KPC books in Baden-Württemberg; ideas of the KPC enter the new education standards; text books of other authors take over KPC ideas.

The developers of the KPC

Author: *Friedrich Herrmann*, Karlsruhe Institute of Technology
Numerous basic ideas stem from books and other publications of, and from personal contacts with *Gottfried Falk* (until his death at the University of Karlsruhe) and *Georg Job* (University of Hamburg).

The following PhD students and other collaborators at the Institut für Didaktik der Physik participated in the development of the KPC:
Karen Haas-Albrecht (Nuclear physics)
Holger Hauptmann (Electrodynamics, oscillations and waves, rotational mechanics, thermodynamics)
Matthias Laukenmann (Atomic and solid state physics)
Lorenzo Mingirulli (Mechanics)
Petra Morawietz (Thermodynamics)
Dieter Plappert (Energy and energy carriers)
Peter Schmälzle (Electricity, data physics)



Gottfried Falk

Friedrich Herrmann und Georg Job

TABLE OF CONTENTS

1

Energy and energy carriers

1.1 Energy

Automobiles need gasoline, diesel locomotives need diesel oil, and electric locomotives need electricity. Every vehicle needs fuel, but not only vehicles need it. If you walk or ride a bicycle, you also use up fuel. A person walking or riding a bicycle uses up food. All these ‘fuels’ have something in common: A vehicle or a person gets energy through them. Energy is what is really needed for transportation.

Energy has something to do with effort. If we pull a wagon, we make an effort. We need energy to pull the wagon. While we pull we are sending energy into the wagon.

> Energy is needed in order to move something.

Motion is not the only thing energy is needed for. Many other processes take place only when energy is constantly being supplied.

Some kind of fuel is always needed for heat production, such as wood, coal, natural gas, heating oil, and electricity. Again, energy is what counts, and it is supplied together with the “fuel”.

> Energy is needed for producing heat.

The propellants or fuels by which energy gets into a motor or an oven are called *energy carriers*. Wood, coal, gasoline, diesel oil, natural gas, and electricity are energy carriers.

If one wishes to move something or to heat something, only the energy is important. Which carrier is used is often of no importance. But if it is the energy which really matters, why then don’t we use energy without a carrier? You might think that this would be easier. Unfortunately, it is impossible because energy without a carrier does not exist.

> Fuels, propellants, food, and electricity are energy carriers. There is no energy without a carrier.

Energy is a physical quantity. What does this mean? It is possible to give it a number just as we do with a length, a time span, or a temperature. It has a unit of measurement just as length, time span, or temperature do. The unit of measurement of energy is the Joule, abbreviated to J. Large amounts of energy are measured in Kilo-Joule (kJ) or Mega-Joule (MJ):

1 kJ = 1000 J

1 MJ = 1000 kJ

Energy has a symbol just like other physical quantities do. Just as the letter l is used for length, and time is shortened to t, the letter E is used for energy. If a car’s fuel tank contains an amount of energy of 800 Mega-Joule, one can briefly write:

E = 800 MJ.

Don’t mix up the symbol E for energy with the symbol J for its unit.

One can say that a kilogram of fuel contains a certain amount of Joules, Table 1.1. The energy content for food is often printed on the package. A completely charged 4.5-volt battery contains about 10 kJ. A fully charged auto battery has around 2000 kJ, approximately the same as a bar of chocolate. The engine of a freight train uses up about 10,000 MJ per hour, a wristwatch with a digital display uses 0.1 J.

coal	30 000 kJ pro kg
briquets	20 000 kJ pro kg
freshly cut wood	8 000 kJ pro kg
propane gas	46 000 kJ pro kg
heating oil	42 000 kJ pro kg
gasoline	43 000 kJ pro kg

Table 1.1
Energy contents of some fuels

To measure the amount of energy, different methods are used for different energy carriers. To determine the energy consumption of a car, we simply multiply the quantity of gasoline used (measured in kg) by the corresponding value in Table 1.1. The energy delivered to a household by means of electricity is measured by an electricity meter.

1.2 Energy sources and energy receivers

Fig. 1.1 shows a section of a central heating unit. Water is heated in the boiler which is usually in the cellar. The heated water is pumped through pipes to the individual radiators. Only a single radiator is shown in Fig. 1.1. We call the boiler the *energy source* and the radiator, the *energy receiver*. The car engine in Fig. 1.2 obtains its energy from the gas tank with the help of the energy carrier gasoline. In this case, the gas tank is the energy source and the engine is the energy receiver. The energy for the light bulb in Fig. 1.3 comes from the power plant via the energy carrier electricity. The power plant is the energy source, the light bulb is the energy receiver.

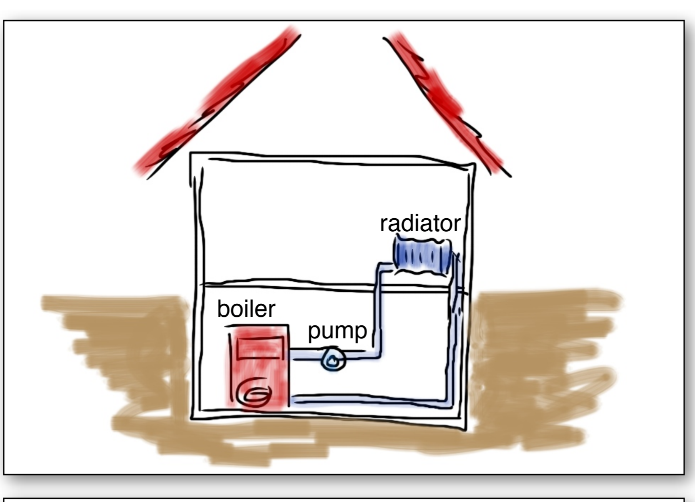


Fig. 1.1
Energy flows from the boiler with the energy carrier "warm water" to the radiators.

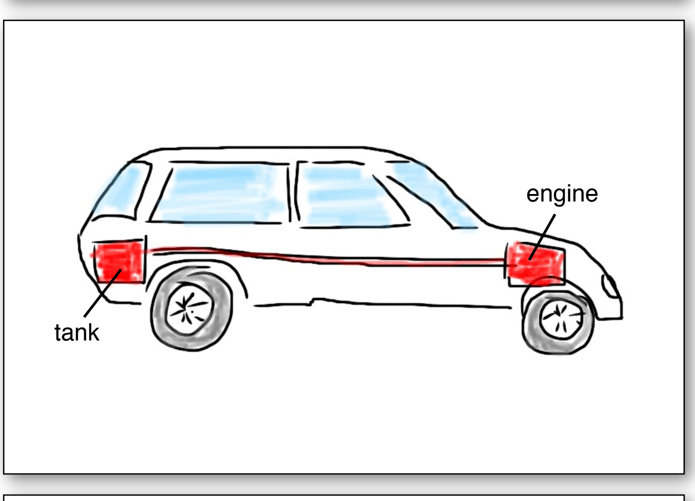


Fig. 1.2
The energy carrier gasoline carries energy from the tank to the engine.

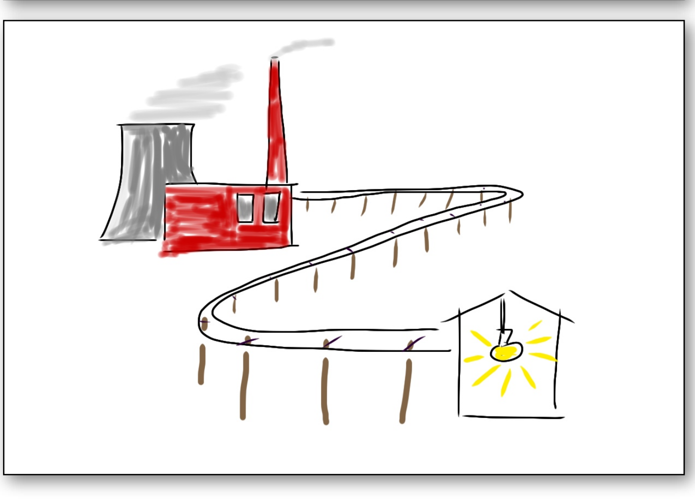


Fig. 1.3
The energy carrier electricity transports energy from the power plant to the lamp.

When energy flows somewhere (with a carrier, of course), what the source is and what the receiver is can be found out. By following the path of the energy carrier backwards to its beginning, one finds the energy source. If the path is followed forward to its end, one finds the energy receiver.

The processes in the figures 1.1 to 1.3 have something in common: In each case energy is flowing together with its carrier from a source to a receiver. If details are unimportant to us and when we wish to express the similarity of the devices and systems shown, then it is convenient to represent the processes symbolically, as in the figures 1.4 to 1.6. Energy source and energy receiver are each represented by a box. The boxes are connected by a thick arrow representing the energy, and by a thin arrow representing the energy carrier. We call these symbolic figures *energy flow diagrams*.

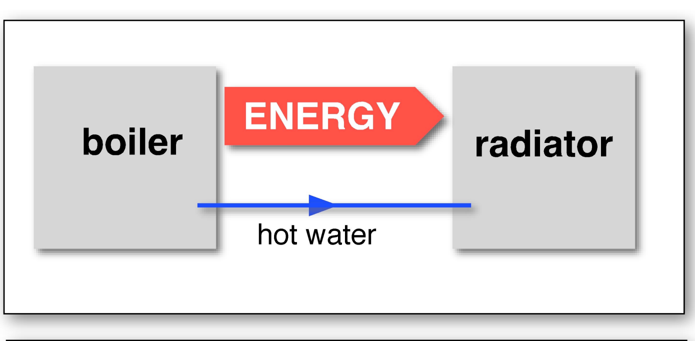


Fig. 1.4
Energy flow diagram for Fig. 1.1

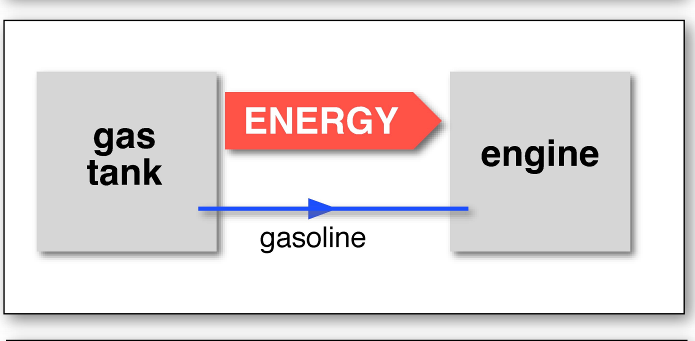


Fig. 1.5
Energy flow diagram for Fig. 1.2

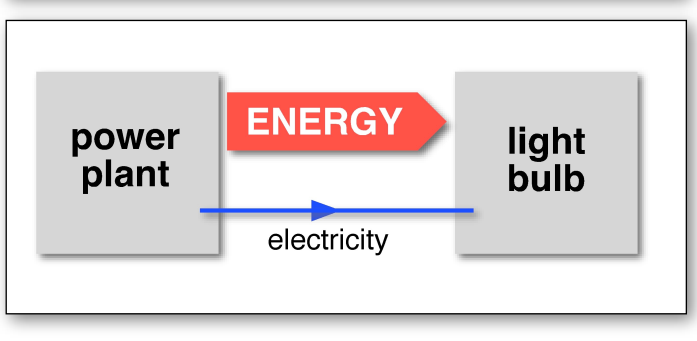


Fig. 1.6
Energy flow diagram for Fig. 1.3

We wish to complete Figures 1.4 to 1.6 by adding another aspect to them. The energy carrier, along with the energy, goes from the source to the receiver. After it has discharged its energy, it generally leaves the receiver. This is represented in Figures 1.7 to 1.9. You can see that different things can happen to the energy carrier after it has left the receiver.

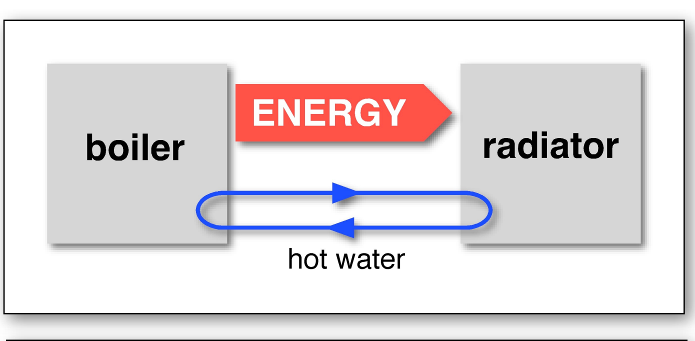


Fig. 1.7
Complete energy flow diagram for Fig. 1.1

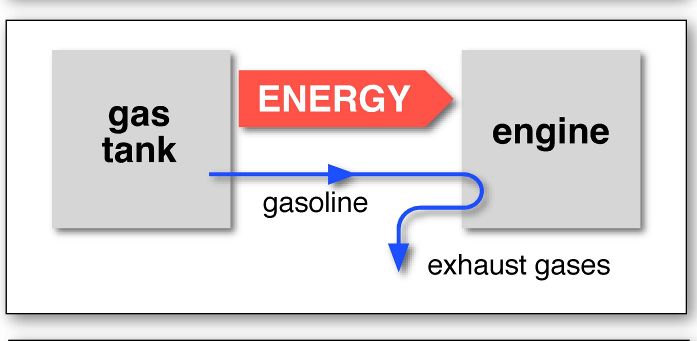


Fig. 1.8
Complete energy flow diagram for Fig. 1.2

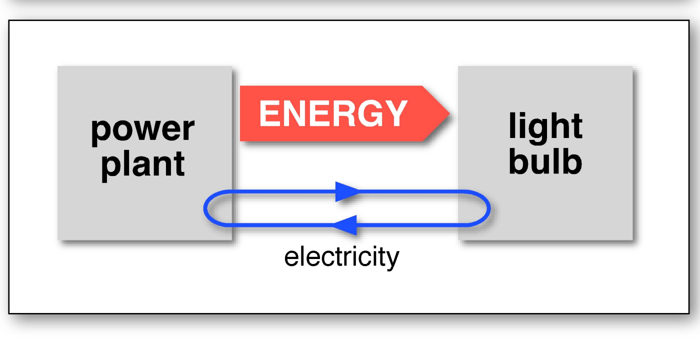


Fig. 1.9
Complete energy flow diagram for Fig. 1.3

In the case of a central heating unit, it is returned to the source. The water flows through the feed pipe to the radiator. It gives energy from there to the room being heated, and in the process, it cools down. It then flows back to the boiler through the return pipe where it is reheated and then recirculated. This is similar to a drink filled into bottles with refundable deposits. After they have been emptied, the bottles are returned to the company where they came from. We will call the water in the central heating unit a *'refundable-deposit-bottle energy carrier'*.

It is a different story with the energy transport in Fig. 1.8. The gasoline burns in the motor and transforms into exhaust. Naturally, this exhaust gas is not returned to the gas tank. It goes out the exhaust pipe, it is 'thrown away'. This is similar to one-way bottles, so we call gasoline a *'one-way-bottle energy carrier'*.

It is easy to distinguish between the two kinds of energy carriers. Refundable-deposit-bottle energy carriers flow in a closed circuit. Energy source and energy receiver are always connected to each other by *two* conduits (pipes, wires, etc.). The one-way-bottle energy carrier, on the other hand, has only *one* conduit from the source to the receiver.

Electricity must be a refundable-deposit-bottle energy carrier because an electric cable has two wires, Fig. 1.9.

Sometimes it is not easy to decide if one is dealing with a one-way or a refundable-deposit energy carrier.

An energy carrier we have not mentioned yet is light. It carries the energy from the Sun to the Earth, Fig.1.10. The energy receiver has no outlet for the light. Therefore light is a one-way-bottle energy carrier. We will go into this in more detail later.

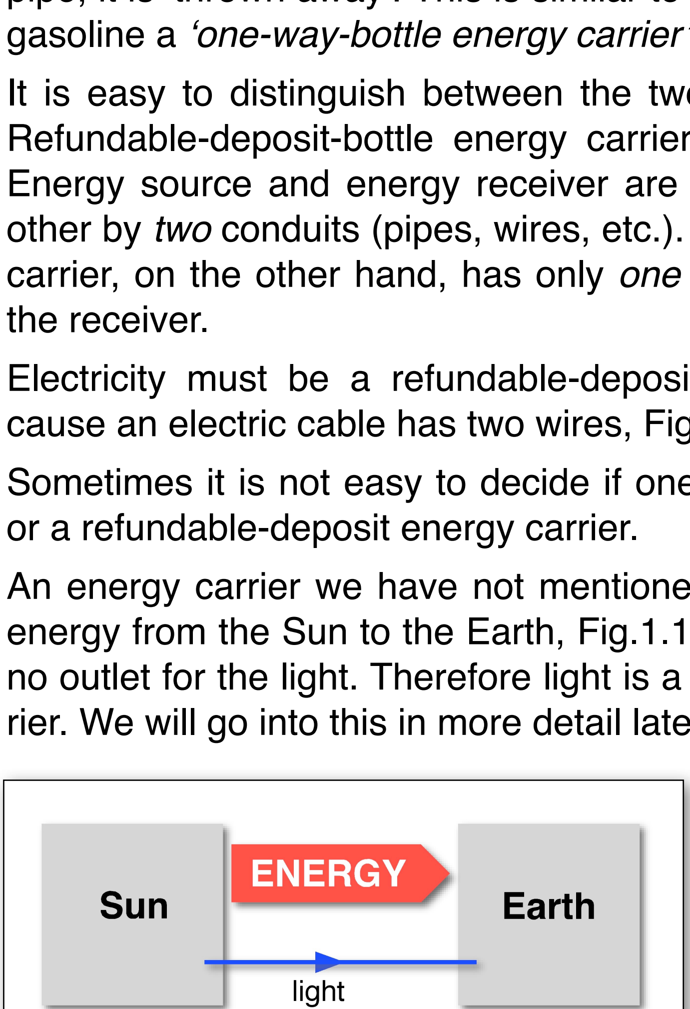


Fig. 1.10
Some energy flow diagrams

Hot air transports energy in exactly the same way that warm water in central heating units transports energy from the boiler to the radiators. This is put to use in heating automobiles.

A jackhammer (pneumatic hammer) needs to be connected to a compressor in order to work. It receives its energy from the compressor. The energy carrier is the compressed air. It is a one-way-bottle energy carrier.

Liquids under pressure are also used as energy carriers. A water turbine obtains its energy by water under high pressure. The shovel and arms of a shovel excavator get their energy from hydraulic oil under high pressure.

Air and water can serve as energy carriers without being warm or under pressure. It suffices that they are moving fast. A windmill, for example, gets its energy through the energy carrier 'moving air'.

If the motor of a device (maybe a water pump), is driven by a rotating shaft, then energy flows from the motor through the shaft and to the machine. The carrier by which the energy gets through the shaft is called *angular momentum*. There is always a return path for angular momentum: It flows from the driven device back to the motor through the foundation upon which the device and motor are installed. It is therefore a return-deposit-bottle energy carrier. This, too, we will go into later.

In Table 1.2, all these energy carriers are listed.

Propellants, fuels, food
electricity
light
angular momentum
hot water, hot air
water and air under pressure
moving water, moving air

Table 1.2
Energy carriers

Exercises

1. Name three different energy receivers which obtain energy by means of the energy carrier electricity.

2. Name three different energy sources which emit energy with the energy carrier angular momentum.

3. Name three return-deposit-bottle energy carriers and three one-way-bottle energy carriers.

1.3 Energy exchangers

Some of the energy sources we have listed are made so that they never become empty: they continuously obtain new energy, but with a different carrier than the one they use for emitting it. They are sources of energy with a carrier A and receivers for energy with another carrier B. An example would be a central heating boiler which receives energy with the carrier "heating oil" and gives it up with the carrier "hot water". We say that in the boiler energy is transferred from the carrier "heating oil" to the carrier "hot water". There are still other ways of saying this, such as: the energy carrier "heating oil" releases energy which is picked up by the energy carrier "hot water." The boiler is then a so-called *"energy exchanger"*.

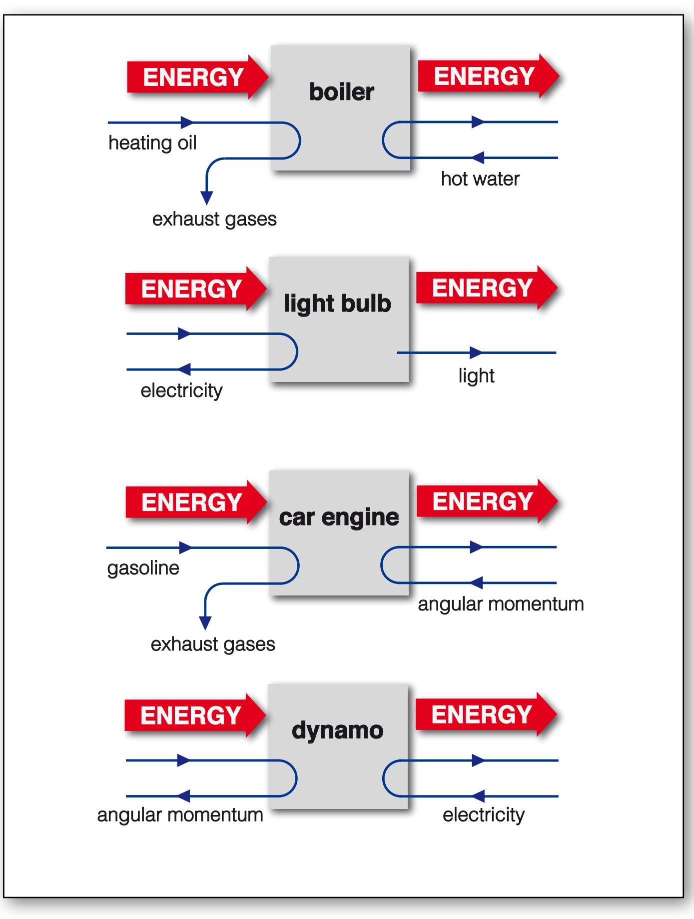


Fig. 1.11
Energy exchangers

Correspondingly, in a car engine energy is transferred from gasoline to angular momentum, and in a light bulb, from electricity onto light. In Fig. 1.11, some energy exchangers are represented symbolically, and Table 1.3 contains a longer list of energy exchangers with the corresponding carriers at inlets and outlets.

Table 1.3
Energy exchangers with the corresponding carriers at the inlet and outlet

energy exchanger	energy carrier *inlet*	energy carrier *outlet*
electric motor	electricity	angular momentum
light bulb	"	light
electric oven	"	hot air
hot water heater	"	hot water
electric pump	"	water under pressure
electric fan	"	moving air
Diesel compressor	fuel	compressed air
coal-fired power plant	"	electricity
gasoline engine	"	angular momentum
petroleum lamp	"	light
oil stove	"	hot air
boiler	"	hot water
solar cells	light	electricity
Crookes radiometer	"	angular momentum
solar panel	"	hot water
forest	"	wood
compressor	angular momentum	compressed air
water pump	"	water under pressure
dynamo, generator	"	electricity
propeller	"	moving air
water turbine	water under pressure	angular momentum

One finds that for every device where energy is transferred from a carrier A to a carrier B, there is one that does just the opposite, meaning it transfers energy from B to A. For instance, in an electric motor, energy is transferred from the carrier electricity to the carrier angular momentum. In the generator it goes from angular momentum to electricity. Similarly, light bulbs go together with solar cells, or water turbines with water pumps.

Energy is often transferred several times in succession from one carrier to another. Fig. 1.12 shows a light bulb powered by a hydroelectric power plant.

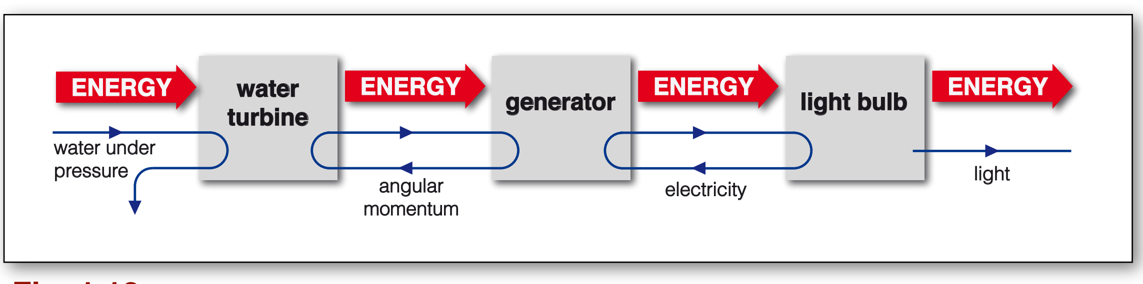


Fig. 1.12
Energy being exchanged three times during transport

If two energy exchangers are connected, the energy carrier at the exit of the first one must match the one at the inlet of the second. The rule for chaining energy exchangers is the same as the rule for playing dominos.

Exercises

1. In Fig. 1.13, the names of the energy carriers at the inlets and outlets of both exchangers are missing. Complete the figure.

2. Enter the names of the energy exchangers in Fig. 1.14.

3. Sketch a chain of exchangers in which at least three different energy exchangers participate.

4. Some devices can be represented in different ways by exchanger symbols. For instance, a vacuum cleaner can be understood as a single exchanger, and can be represented by a single symbol; or one can represent it by two connected symbols. Demonstrate both of these possibilities.

5. In order to power a light bulb with the help of a windmill, an additional piece of equipment is necessary. What is it? Sketch the flow diagram.

6. Device 1 transfers energy from carrier A to carrier B. Device 2 does just the opposite, it transfers energy from carrier B to carrier A. Give three pairs of energy exchangers that are related in this manner.

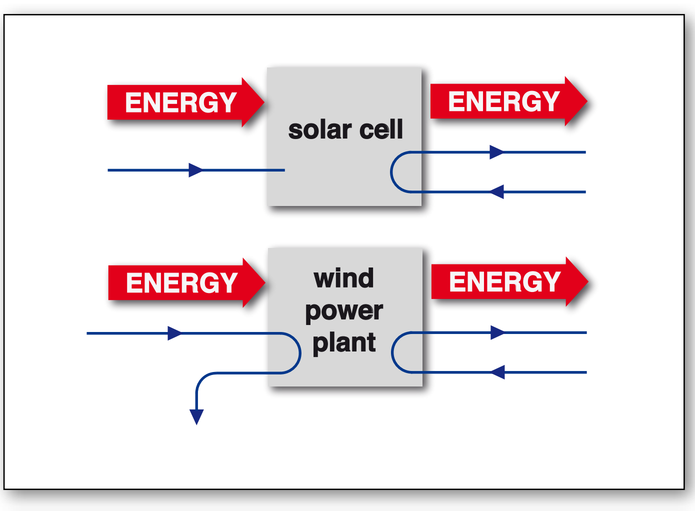


Fig. 1.13
Which ones are the energy carriers at the inlets and outlets?

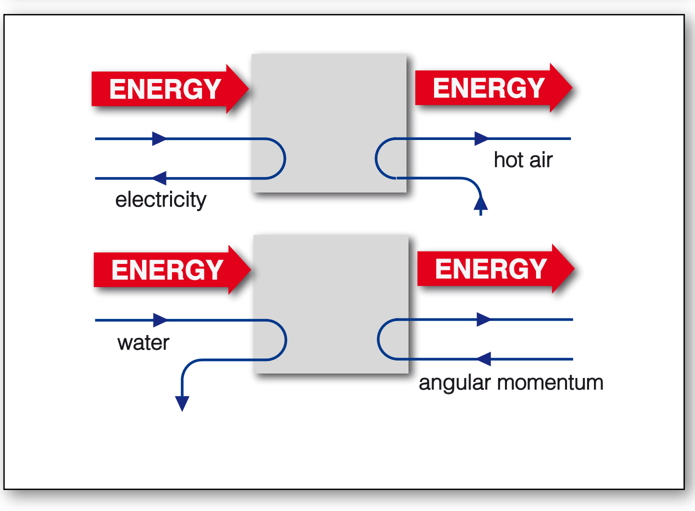


Fig. 1.14
What are the energy exchangers here?

1.4 The energy current

In order to judge the energy consumption of a device, one must find out how much energy flows into the device during a determined time span. (The energy must naturally flow out again.) A device with a flow of 1000 J per second ‘consumes’ more energy than one with a flow of 500 J per second.

Imagine a device that consumes 25,000 Joules in 50 seconds. How can the energy consumption per second be calculated from this? By dividing the total energy of 25,000 J by 50 s. The device therefore consumes 25,000 J/50 s = 500 J/s.

The amount of energy divided by the time span is called the energy current.

Energy current = Energy/time span

Using the abbreviations E for energy, t for time span, and P for energy current, we have

$$P = \frac{E}{t} .$$

Sometimes, the quantity P is also called “power”. The unit of the energy current is Joule/second. A Joule/second is called a Watt. In short:

W = J/s.

An energy current of 15 Watts flows into a standard light bulb (with the carrier electricity). Therefore

P = 15 W.

About 50 kW flow from the motor to the wheels of a car (with the carrier angular momentum). A large power plant delivers an energy current of 1000 MW carried by electricity.

The fraction of the energy current from the Sun that reaches the earth is $1.7 \cdot 10^{11}$ MW; this is as much as one hundred and seventy million large power plants could deliver. When a person eats, he or she receives energy, so an energy current flows through the person. On average it measures about 100 W.

There are energy sources which can become empty. Examples are car batteries, single-cell batteries, or gas tanks. Energy can be stored in these, so we call them *energy storage units.* Other energy storage units would be a motor with a windup mechanism, a fly wheel, a pumped hydraulic plant, a night storage heater, a heating oil tank, fluorescent paint, and the Sun.

Exercise

A hair dryer with two settings has the inscription:

Setting 1: 500 W

Setting 2: 1000 W

What does this mean?

2

Flows of liquids and gases

Most excavation shovels, some building cranes and many other machines are powered *hydraulically*. This is easy to see by the pipes and hoses leading from a central pump to various places in these machines where something is to be set in motion.

There are also machines and equipment that are driven *pneumatically*, for example a jackhammer. Powering equipment pneumatically functions very similarly to the hydraulic case, only the energy carrier here is compressed air.

In this chapter we will investigate liquid and gas flows as they are used in these machines. In so doing, we will discover some simple rules. It is worth it to take note of these rules. They are valid not only here, meaning not only for flowing air and for water currents. When the rules are slightly altered, they also hold for totally different currents: for electric currents, heat flows, and for co-called momentum currents.

2.1 Pressure

If a water faucet is turned on all the way, a strong flow of water shoots out, Fig. 2.1. This happens because the water in the pipes is under high pressure. When the valve of a pumped up bicycle tire or car tire is opened, the air comes rushing out. This is because the air in the tires is under high pressure.

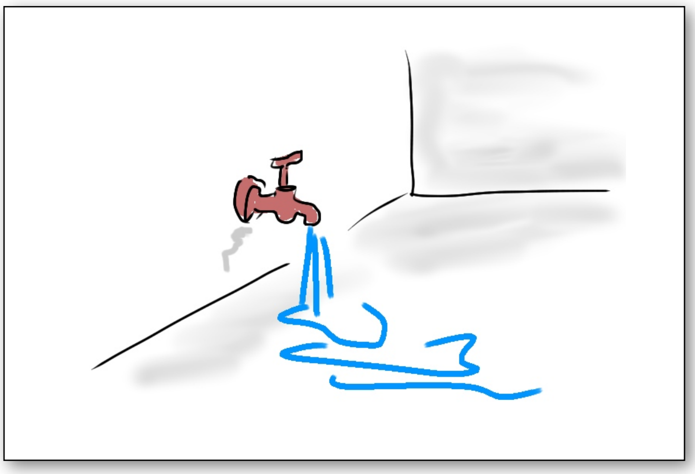


Fig. 2.1
The water in the pipes is under pressure.

Pressure is a physical quantity just as weight, length and energy are. The unit of pressure is called a *bar*. The devices used for measuring pressure are *manometers*, Fig. 2.2. Table 2.1 contains some typical pressure values.

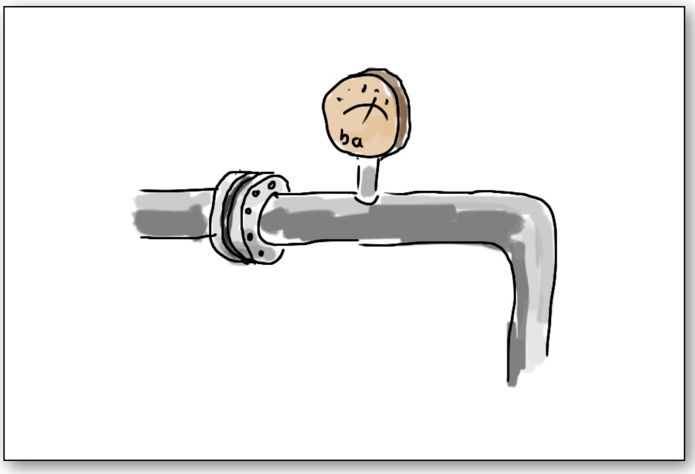


Fig. 2.2
Pressure is measured with a pressure gauge (manometer).

Another unit for pressure is the Pascal, shortened to Pa. Here is the relation between these two units:

1 bar = 100,000 Pa.

The bar is more practical and handy than the Pascal. The smaller unit Pascal has advantages for physicists. The relation between pressure and other physical quantities becomes simpler when the Pascal is used. You will understand this better later.

Water pipe	2 – 5 bar
Automobile tire	4 bar
Steam in the boiler of a power plant	150 bar
Hydraulic fluid of a power shovel	150 bar
At the deepest part of the ocean	1000 bar
In a filled oxygen bottle	150 bar
In a full propane gas tank	8 bar
To create diamonds artificially from graphite, the graphite is put under pressure of at least	15 000 bar
Inside the sun	221 000 000 000 bar

Table 2.1
Typical pressure values

2.2 Air pressure, excess pressure, vacuum

The air around us has a pressure of almost exactly one bar. This pressure is called standard pressure. Air pressure is the result of the weight of the air above pressing down on the air below it.

Because of this, air pressure decreases as altitude increases in the atmosphere. It decreases quickly at the beginning, and more slowly higher up. Fig. 2.3 shows the air pressure as a function of altitude above sea level. At 4000 m, i.e., in high mountain areas, the air pressure is about 0.6 bar.

Fig. 2.3
Air pressure as a function of altitude above sea level.

Air pressure is not the same all the time. Its value changes according to the weather. The instrument we use to measure the pressure of the air around us has a special name. It is called a *barometer.*

We don't feel air pressure because it is pressing our bodies from all sides. Even in the cavities of our bodies, e.g., in our lungs or ears, the air is at the same pressure as the air outside. Most manometers do not show absolute pressure, i.e., real pressure, but the so-called *excess pressure*. For example, when tire pressure is checked, the manometer shows the pressure difference between the air inside the tire and the air outside.

It is possible to have a container of air that has lower pressure than the outside air pressure. If there are no air and no other substances in the container, then the pressure is 0 bar. A space with no matter in it is called a vacuum.

Exercises

1. If you have a barometer at home: Read the air pressure in the morning and in the evening for 7 days. Plot it as a function of time.

2. A driver checks the air in her spare tire. The manometer shows 0 bar excess pressure. How high is the actual pressure in the spare tire?

2.3 Pressure difference as driving force for a gas or a liquid current

When the valve of a pumped up car tire is opened, the air flows out. It flows out of the tire where the pressure is high to the outside where the pressure is lower. When a package of "vacuum-packed" peanuts is opened, it hisses. Air flows in since the pressure is lower inside than outside. In both cases air flows from a place of higher pressure to a place of lower pressure. When we attach a long thin hose to a faucet and turn the faucet on, water flows out. In the pipes the water is under high pressure. At the end of the hose there is low pressure, namely standard pressure. Water flows from places of higher pressure to places of lower pressure. This is true for other gases and liquids as well.

Liquids and gases flow by themselves from places of higher to places of lower pressure. The pressure difference is the driving force for liquid and gas currents.

A car tire is pumped up and connected by a hose to another, empty tire, Fig. 2.4. One can hear the air flowing through the hose but after a while, it stops. We remove the hose and measure the pressure in both tires. Result: the pressure in both tires is the same. In the pumped up tire, where the pressure had a higher value at the beginning, it has gone down. In the other tire, the pressure has increased. What happened? The air flowed from the tire with higher pressure into the one with lower pressure until the pressure difference, meaning the driving force for the current, disappeared. The end state in which the air no longer flows (although the connection is still there) is called *pressure equilibrium*.

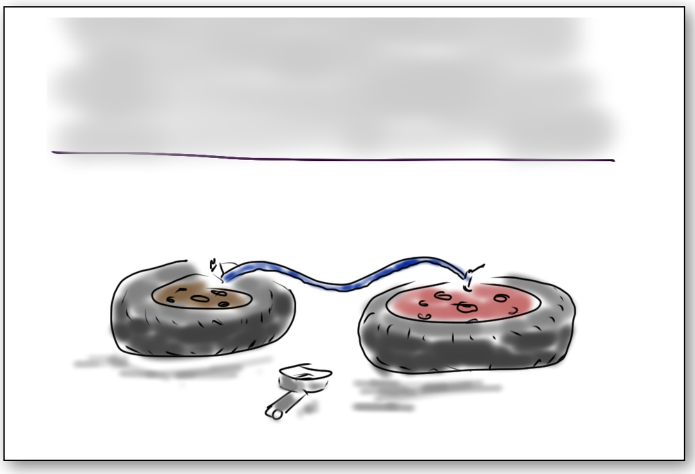


Fig. 2.4
Air flows from the tire with higher pressure to the one with lower pressure.

Notice that the amount of air in the final state of both tires is not identical. There is more air in the larger tire.

It is obvious that not pressure, but pressure *difference* is the driving force for the air flow. At equilibrium, there is no longer any air flow even when the pressure itself is very high.

Exercise

The air pressure in a large tire is 1 bar and in a small one, 4 bar. The two tires are connected to each other by a hose, so that the air in one can flow into the other.

(a) What happens?

(b) Is the final pressure nearer to 1 bar or to 4 bar?

(c) Which tire has more air at the end?

2.4 Pumps

Often it is necessary to transport a liquid or a gas from a place of lower pressure to a place of higher pressure. This is achieved by use of a pump. At the outlet of the water pump in Fig. 2.5, the water has a higher pressure than at the inlet.

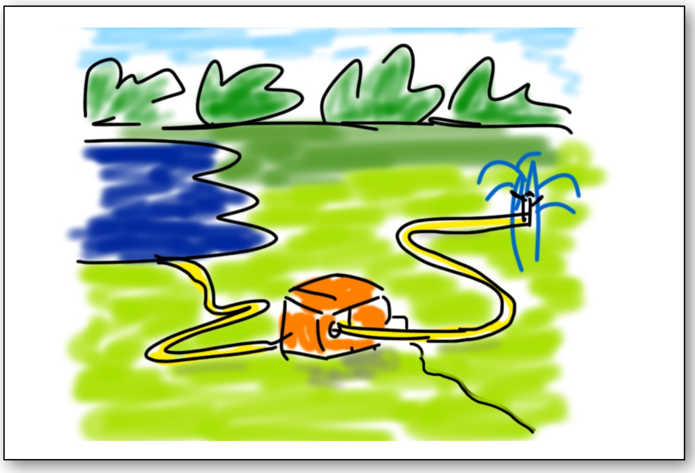


Fig. 2.5
The water has higher pressure at the pump's outlet than at its inlet.

There are various kinds of pumps. Fig. 2.6 shows a *centrifugal pump.* The incoming water flows into the middle between the blades of the propeller wheel. Because the wheel is turning, the water must turn with it. It is forced outward (like a passenger in a car going around a curve) and is pressed out of the outlet. Centrifugal pumps are used for pumping water out of the washing machine, for example.

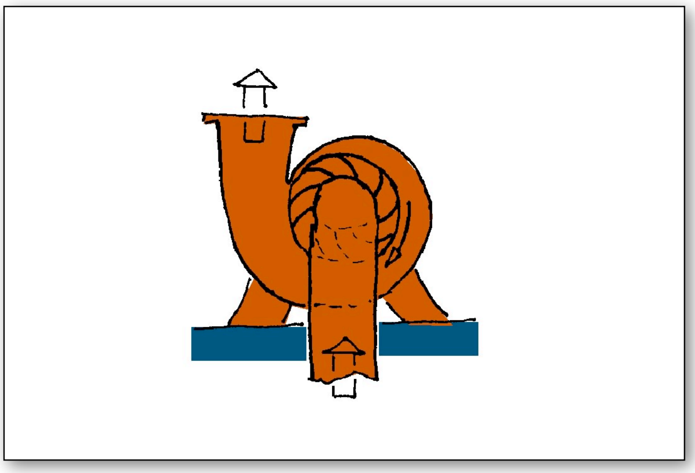


Fig. 2.6
A centrifugal pump

In Fig. 2.7, you can see how a gear pump works. Gear pumps are good for creating large pressure differences. A somewhat different version of this pump is used as a hydraulic pump in power excavators. Pumps with which gases are brought to high pressure are called compressors.

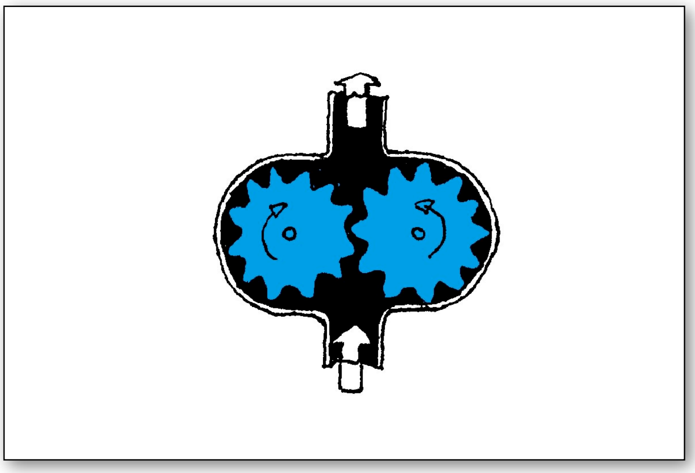


Fig. 2.7
A gear pump

Pumps transport gases and liquids from places of lower pressure to places of higher pressure.

2.5 Current intensity

There are occasions when two currents, for example two water currents, are to be compared. One could ask “Which of them is wider?” or “Which of them is faster?” but often we are not interested in the width or the speed, but rather in the *current intensity* or *current* in short. The water current intensity is the amount of water that flows by a certain point within a determined time span, divided by the time span:

$$\text{water current} = \frac{\text{amount of water}}{\text{time span}}$$

The amount of water can be measured in liters or kilograms. The unit of a current is therefore either l/s or kg/s. In the Rhine river, about 1,500,000 liters flow each second beneath the bridge at Karlsruhe. The current of the Rhine there is 1,500,000 l/s.

In the last chapter we got to know energy currents. They tell us how many Joules per second flow by a certain point.

It is easy to mistake a current for its speed. The river in Fig. 2.8 has the same current everywhere. However, the speed at the narrow place is greater than at the wider one.

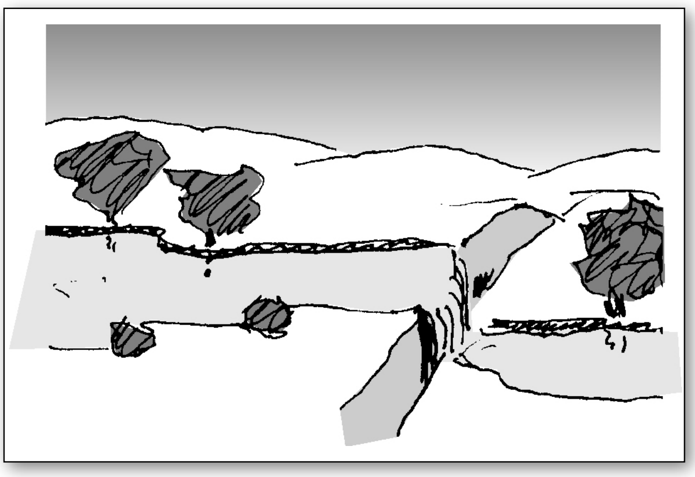


Fig. 2.8
The current is the same at every section of the river.

In Fig. 2.9, a water current of 1 l/s flows from the left through pipe A to the intersection or junction. Through pipe B and pipe C, currents of 0.5 l/s and 0.2 l/s, respectively, are flowing away from the intersection. What is the current in Pipe D, and in which direction is the water flowing?

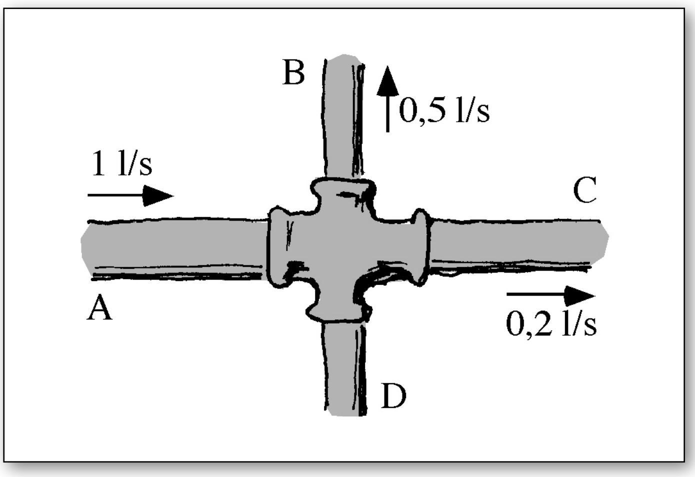


Fig. 2.9
Every second, exactly the same amount of water is flowing through pipe A to the intersection as flows away from it through pipes B, C, and D.

Since water can neither disappear nor be created in the junction, the quantity of water flowing toward the intersection every second must be the same as that flowing away from it. For the balance of the amount of water to be correct, 0.3 l/s of water must flow away from the intersection through Pipe D:

Flowing into the junction 1 l/s

Flowing away from the junction 0,5 l/s + 0,2 l/s + 0,3 l/s = 1 l/s.

A place where several currents meet is called a *junction*. We have used the *junction rule* in calculating the current of water in Pipe D.

The currents flowing into a junction are, in total, equal to the ones flowing away from it.

Exercises

1. A bathtub with a capacity of 120 l, fills up in 20 minutes. What is the current of water flowing into the tub?

2. The current in a water pipe is 2 l/s. In another one it is 3 l/s. Is it possible, using this information, to determine in which pipe the water flows faster? Give your reasoning.

3. Three rivers with currents of 5m³/s, 2 m³/s, and 3 m³/s flow together at a certain point. What is the current after the confluence?

2.6 Current and driving force

You have turned on the water faucet full force but not as much water is coming out as usual. What could be causing this? Of course it is the pressure in the pipes. The difference between the pressure in the water pipes and the pressure outside (standard pressure of 1 bar), is the driving force of the water current coming out of the faucet. The higher the pressure in the pipes, the greater the pressure difference and therefore the greater the current.

We fill a plastic bag with air coming out of a pumped up automobile tire, Fig. 2.10. We do the experiment using a tire with 2 bar excess pressure, and then again with one at 0.5 excess pressure. We notice that in the first case, the plastic bag fills more quickly with air than in the second. In this case, as well, a higher pressure difference causes a greater current.

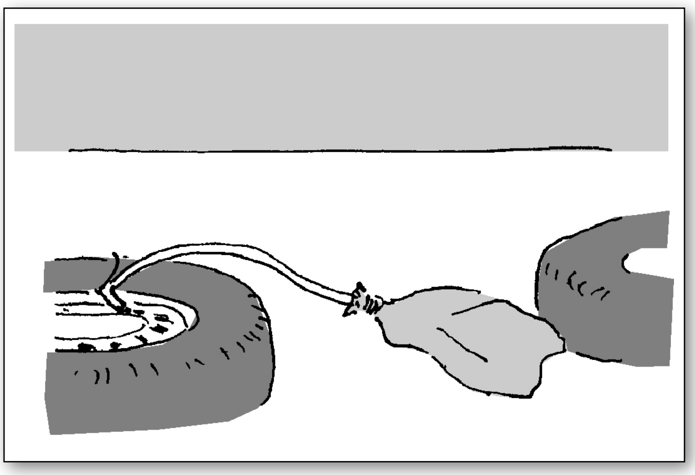


Fig. 2.10
The greater the pressure in the tire, the faster the plastic bag fills up.

The greater the pressure difference between two points (the greater the driving force), the greater the current flowing from one to the other.

Exercise

Water flows through the pipe in Fig. 2.11.

(a) The current at the left end of the pipe is 10 l/s. What is the current at the right end of the pipe? Give reasons for your answer.

(b) The pressure difference between the left end and the narrowing is 2 bar. Is the difference between the right end and the narrowing greater or smaller than 2 bar? Give reasons for your answer.

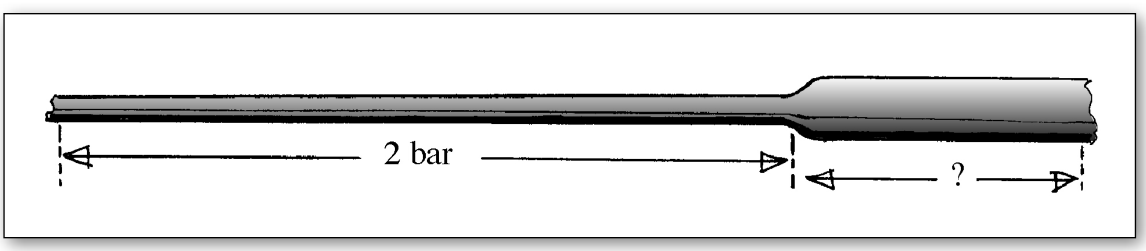


Fig. 2.11
Is the pressure difference between the narrowing and the right end of the pipe greater or smaller than 2 bar?

2.7 Current and resistance

A 50 meter long garden hose is attached to a faucet out of which a strong jet usually flows. If the faucet is turned fully, the water jet at the outlet of the hose is considerably weaker. The water current with the hose is less than without it, Fig. 2.12. Why is this? It cannot be because of the driving force, because that is the same in both cases. The hose is responsible for the reduction of current: It obstructs the current of water. It puts up a *resistance* to it.

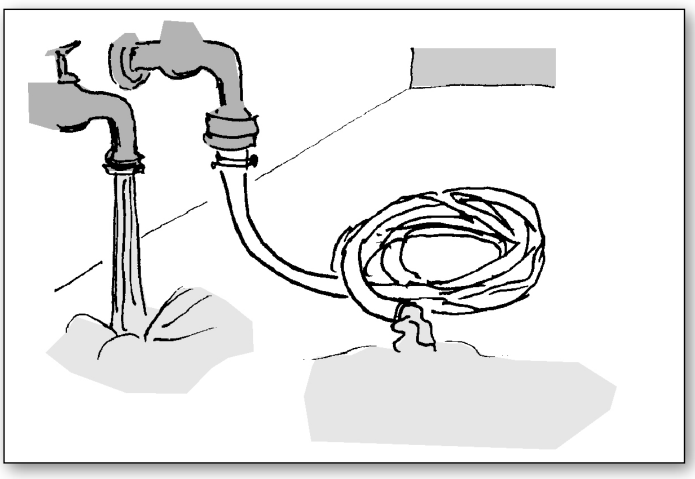


Fig. 2.12
The hose puts up a resistance to the water current.

Again, we will do an experiment with a car tire and a plastic bag. We fill the plastic bag twice, with the air out of the same tire. For the first filling, we use the shortest hose possible and for the second, a very long one. Both hoses have the same diameter. In the first case, the plastic bag fills up more quickly than in the second one. The air current is stronger in the first case than in the second. The long hose puts up a greater resistance to the air than the short one does. One could say that the long hose "has" a greater resistance.

We now compare the resistance of three hoses or pipes that are of the same length, but have different cross sections. We realize that the greater the cross-sectional area of the hose, the smaller the resistance will be.

> Every conduit sets up a resistance to the gas or liquid current flowing through it. The smaller the cross-sectional area of the pipe is, and the longer it is, the greater the resistance is.

The current is therefore not only dependent upon the driving force, but also upon the characteristics of the pipe through which it flows.

> The current of a gas or liquid in a pipe is greater if the pressure difference is larger between the two ends of the pipe, and if the resistance of the pipe is smaller.

The relationships between current, pressure difference, and resistance, as well as between resistance, length, and cross-sectional area of the pipe are summarized in Fig. 2.13.

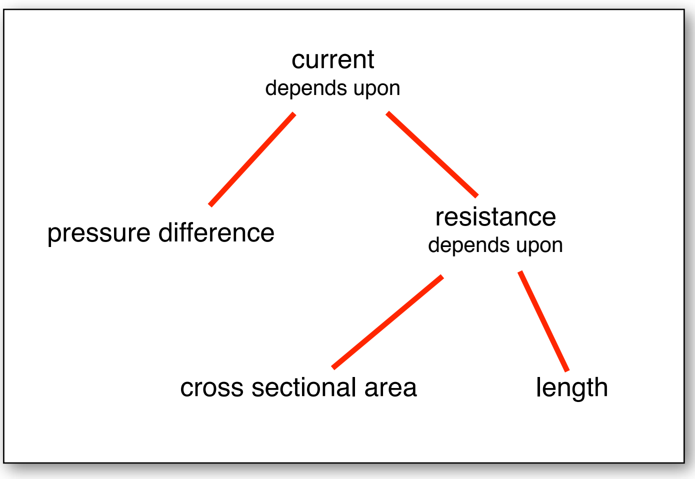


Fig. 2.13
The relation between current, pressure difference and the characteristics of the conduit

Fig. 2.14 shows a longer water pipe upon which pressure gauges (manometers) have been mounted at even intervals. We want to understand the values shown by these gauges. We can conclude from the higher value shown by the manometer on the left, that the water flows from left to right, meaning from higher pressure to lower pressure. However, we see that already between the first and second manometers the pressure drops because the water needs a driving force to move even this small distance. This is also true for the distance between the second and third manometers, and so forth. We realize that the pressure differences between two neighboring pressure gauges in the thin part of the pipe are the same, namely 0.6 bar. The pressure differences between two neighboring manometers on the thick part of the pipe are equal as well, namely 0.2 bar. The pressure difference between two neighboring manometers on the thin part of the pipe are not the same as on the thick part. This is easy to understand, though. In order to press the same amount of water through the thin pipe, a greater pressure difference is necessary than to press it through the one with a larger diameter.

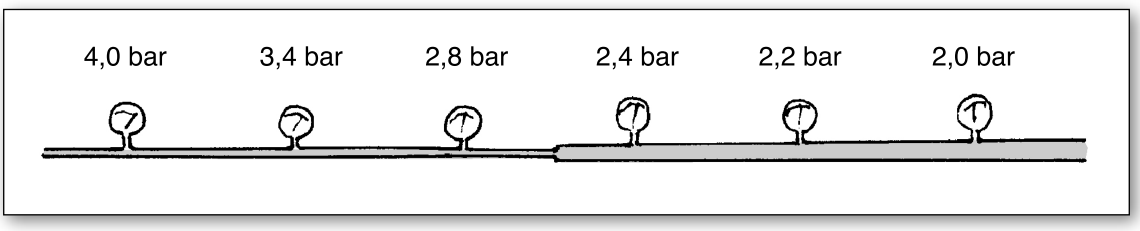


Fig. 2.14
The decrease of pressure in the narrow part of the pipe is greater than in the wider part.

2.8. Hydraulic energy transfer

A power shovel is a versatile machine. It can move, it can turn its upper section, it can swing and bend its arm, and it can tip the shovel at the end of its arm, Fig. 2.15. A system of hydraulic circuits makes all these manipulations possible.

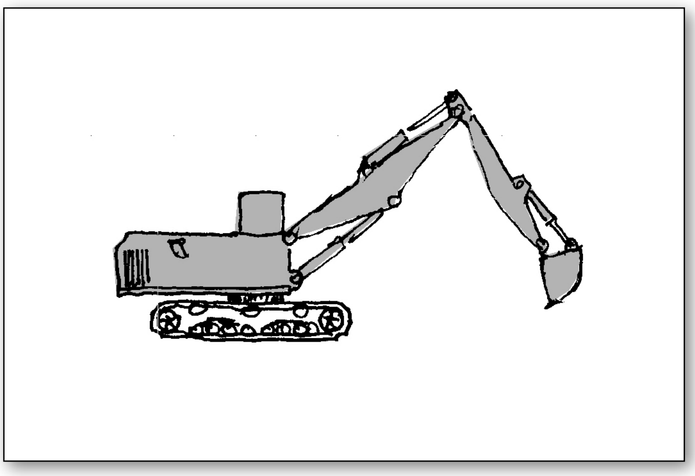


Fig. 2.15
The power shovel can roll, rotate its upper section, swing and bend the arm, and tip its shovel.

A diesel engine drives a pump. The pump presses hydraulic oil through pipes and hoses to the various places where something is to be moved. The oil flows back to the pump through a second pipe. A hydraulic motor is located where something is to be rotated such as the wheels of the power shovel or its upper section. A hydraulic cylinder is used where something is to be moved back and forth, Fig. 2.16.

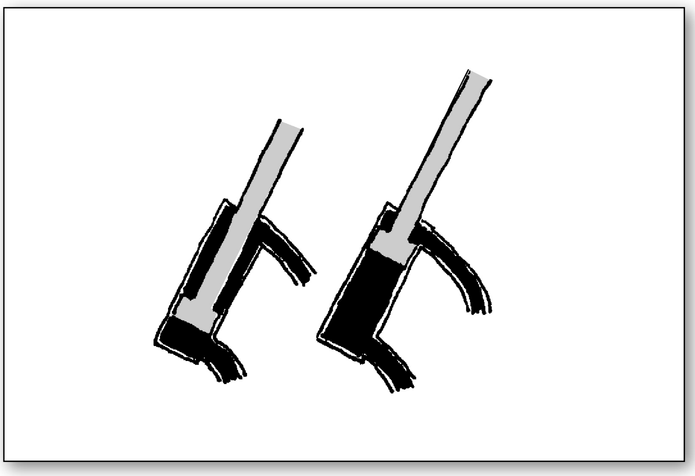


Fig. 2.16
Hydraulic cylinder

Fig. 2.17 shows a part of the hydraulic system of the power shovel: the pump and one of the hydraulic motors. One sees that the hydraulic oil flows in a closed circuit. It is under high pressure on its way to the motor. On its way back to the pump, the pressure is low.

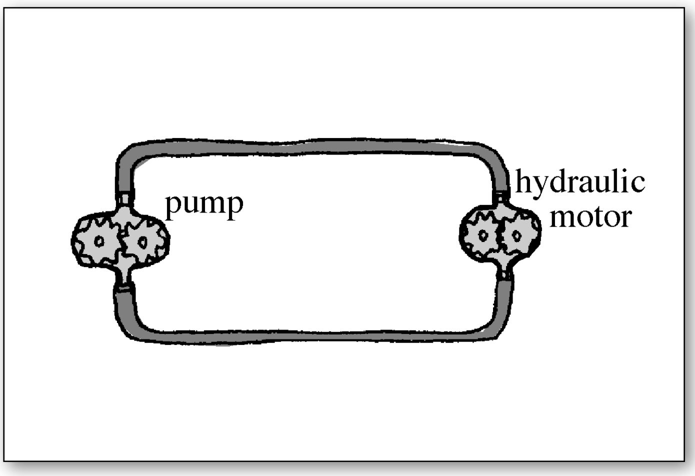


Fig. 2.17
Hydraulic circuit

Let us describe the processes from the point of view of energy. The energy flows from the diesel engine to the pump with the energy carrier angular momentum. In the pump it changes its carrier. It is transferred over to the hydraulic oil. Along with the oil in the high-pressure pipes, it reaches the hydraulic motor where it is again transferred to angular momentum. After it has given up its energy, the oil flows back to the pump. Fig. 2.18 shows the corresponding flow diagram.

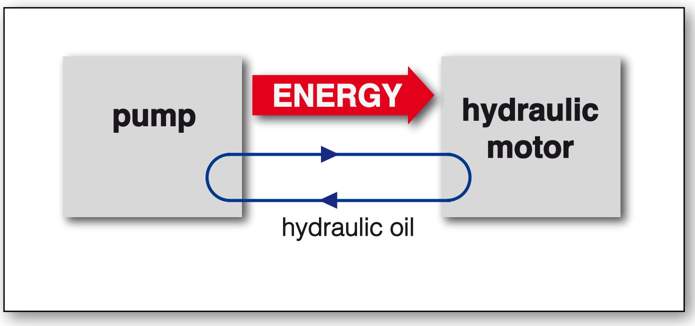


Fig. 2.18
Energy flow diagram of a hydraulic engine

3

Momentum and momentum currents

We begin with an extensive area of physics called mechanics. Here is a definition of it for a start: Mechanics deals with the motion of objects. With time we will realize that this definition is too limited, but for the moment it suffices.

Mechanics is the oldest field of physics. Its most important laws have been known for 200 years. For a long while it was the stated goal of physics to interpret everything happening in nature mechanically, to base not only the processes representing obvious movement upon mechanics, but thermal, optical, electrical, and chemical processes as well. This viewpoint saw the world as nothing more than a huge and very complicated „mechanism“.

It has been known since the beginning of the 20th century that this conception is unsound. Other parts of physics are fully equal to mechanics, for example electricity and thermodynamics. In a given process, mechanics as well as electricity and heat normally play a role and other phenomena of physics do as well. In the following, when we deal with mechanics we will look at only one aspect of processes, the mechanical one. When we investigate an object, we will be interested in whether and how it moves. We will not worry about what temperature it has, if it is electrically charged, or what color it is and we will naturally not be interested in problems having nothing to do with physics, such as how expensive the object is or whether it is beautiful or ugly.

Before we begin with mechanics we must first learn about one of the most important tools of a physicist: the physical quantity. We will do this in the next section.

3.1 Physical quantities

A characteristic of physics is that it describes nature quantitatively. "Quantitative" means that statements are expressed in numbers. A physicist is not satisfied to know that an object has a high temperature, a small mass or a low speed. He wants to determine the *values* of temperature, mass or speed. His goal might be to calculate or determine by measuring, that the object has a temperature of 1530˚C, a mass of 5.3 milligrams or a speed of 882 meters per second.

Temperature, mass and speed are called *physical quantities*. There are also many other physical quantities. Many of them are already known to you, and others you will learn about during your physics classes.

Physical quantities are some of the most important tools of physicists.

We want to remind ourselves of some basic rules for dealing with physical quantities. These are rules that you already know well, but maybe you are not aware of them and do not always follow them.

Each physical quantity is shortened to a letter. These abbreviations have been established internationally. Table 3.1 shows some examples.

Name of the quantity	Symbol
mass	*m*
velocity	*v*
time	*t*
volume	*V*
energy	*E*
pressure	*p*

Table 3.1
Names and abbreviations of some physical quantities

Notice that it is important whether the abbreviated symbol is a lower or upper case letter. A certain lower case letter often stands for a different physical quantity than the corresponding upper case letter. For example, *v* means velocity (speed) and *V* stands for volume. Sometimes more than one symbol is used for a quantity. In the case of energy this is so. The symbol *E* is used for energy as well as *W*.

As you already know, every quantity has a unit of measurement. The unit for time is the second, for energy it is the Joule and for pressure, the bar. Table 3.2 gives some examples of units of measurement.

Name of the quantity	Unit
mass	kilogram
velocity	meters per second
time	seconds
volume	cubic meters
energy	Joule
pressure	bar

Table 3.2
Names and units of some physical quantities

A unit of measurement represents a determined amount of the quantity. The value of a quantity is always given in multiples or fractions of its unit. When one says, "the energy content of an object is 1000 Joules", one means that the object has 1000 times the determined unit of "1 Joule".

We abbreviate the name of a unit of measurement in exactly the same way we have done with quantities. Hence, "meter" is shortened to "m", "Joule" to "J", and "second" to "s". In order not to mix up the symbols for physical quantities with those of units of measurement, the symbols for quantities are written in italics. This means that *m* is the physical quantity mass but m is the measurement of the unit of measurement meter. Units of measurement have also been defined internationally. The quantities we have discussed so far are in table 3.3. The table contains the names of some quantities, the abbreviations of these names, the corresponding units and their abbreviations.

Name of the quantity (Symbol)	Unit (Symbol)
mass *(m)*	kilogram (kg)
velocity *(v)*	meters per second (m/s)
time *(t)*	seconds (s)
volume *(V)*	cubic meters (m^3)
energy *(E)*	Joule (J)
pressure *(p)*	bar (bar)

Table 3.3
Names and units for some physical quantities and their abbreviations

Thanks to abbreviations of names and units, it is possible to write physical statements very compactly. Instead of "the speed is one hundred meters per second", we can just write

v = 100 m/s.

In place of "the energy is forty thousand Joules", we can write

E = 40,000 J.

Prefix	Abbreviation	Meaning
kilo	k	thousand
mega	M	million
giga	G	billion
tera	T	trillion
milli	m	thousandth
micro	µ	millionth
nano	n	billionth
pico	p	trillionth

Table 3.4
Prefixes that are used to denote multiples and fractions of units

Important: 1. Do not confuse the names of quantities and units! 2. Do not mix up the symbols for quantities and units!

We often have to deal with very large or very small absolute values of physical quantities. In these cases, it is possible to use a determined multiple or fraction of the usual unit. One labels these multiples and fractions by putting in an identifying word before the usual name of the unit. The definitions of these determinatives are listed in Table 3.4. Every determinative has an abbreviation. These are contained in Table 3.4 as well. We have for example:

40 000 Joule = 40 kJ = 0.04 MJ,

or

0.000 002 m = 0.002 mm = 2 µm.

Exercises

1. Name four quantities (other than those in Table 3.1), their units of measurement, as well as the symbols for the quantities and the units.

2. Write a shorter form of the following expressions by using the determinatives in Table 3.4:

E = 12,000,000 J

v = 1,500 m/s

p = 110,000 Pa.

3. Give the speed *v* = 72 km/h in units m/s.

4. Name some units of measurement of quantities that are no longer used today.

3.2 Momentum and velocity

According to our present definition, mechanics deals with the motion of objects or, as we sometimes say, *bodies*.

In order to begin our physical description of motion we need to create the tools necessary for doing so. You remember that our most important tools are physical quantities. Gradually we will get to know quite a few of these quantities. For the moment though, two quantities are enough for us. These characterize the state of motion of a body. One of these is already well known to you, velocity, which is abbreviated by *v*. Many units of measurement are used for velocity: kilometers per hour, knots, millimeters per day, etc. In physics the unit of measurement used is meters per second, abbreviated by m/s.

The second quantity we need as a physical quantity that can be given a numerical value, is certainly unknown to you at this point although you are already well aware of it. You will quickly get to be so comfortable with it that you will be able to determine its values. Again, it is a quantity by which motion can be described. For example, it can be used to describe the difference between a vehicle standing still and one that moves. It has a special feature that velocity does not have: it represents what is *contained* in a body when it is in motion and what is not in it anymore when it comes to rest. Everybody knows an expression for describing exactly this property. Every one of us says that a heavy rolling car has "impetus". The same car has no impetus when it is no longer moving. The characteristics described by this word is very similar to the characteristics of the physical quantity we are looking for. Actually, we could use this name for this quantity, i.e., "impetus". Certain expressions have become standard, though. This quantity is called "momentum". Its symbol is *p*. (Attention: this symbol is the same as the one used for pressure).

> A body in motion contains momentum. If it is moving quickly or is heavy, it has a lot of momentum. When it is not in motion, it contains no momentum.

Later on we will discuss how to quantitatively determine how much momentum (impetus) a body contains. Right now we will get to know the unit of momentum. It is called a *Huygens*, abbreviated to Hy and named for the physicist Christiaan Huygens (1629-1695) who made important contributions toward creating the quantity called momentum.

In the following, we will develop the most important characteristics of the quantity *p*. In doing so, it is enough to always keep in mind that momentum is essentially what we usually call impetus.

Two identically built cars are driving down the street, one is faster than the other, Fig. 3.1. Which car contains more momentum? (Which car has more impetus?) The faster moving car, which has a higher speed.

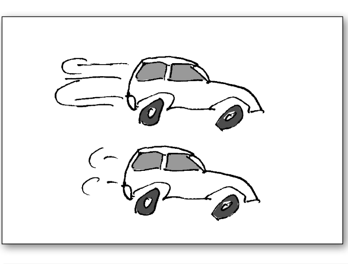


Fig. 3.1
The two cars are built exactly the same. The one moving faster has more momentum.

> The greater the velocity of a body, the more momentum it contains.

A truck and a car are moving at the same speed, say 60 km/h, side-by-side. The truck's weight is 8,000 kg, and the car weighs 1,200 kg, Fig. 3.2. Which of these two vehicles has more momentum? Of course, it is the truck. The quantity we measure in kg, the one that is usually call "weight", is called *mass* in the natural sciences. Therefore:

> The greater the mass of a body, the more momentum it contains.

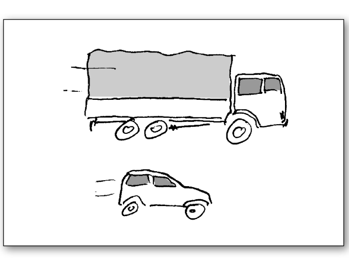


Fig. 3.2
The two vehicles are traveling at the same speed. The heavier one has more momentum than the lighter one.

We are now able to give the definition of the unit *Huygens* for momentum:

> A body with a mass of 1 kg and a speed of 1 m/s contains 1 Hy.

In the following we will do several experiments in which *friction* would interfere. We will therefore use cars that have good bearings. A bearing with very little friction can be achieved by using air cushions instead of wheels. Fig. 3.3 shows an air track, the kind often used in experiments. The track has four rows of very fine holes out of which air flows. The gliders do not touch the track but float upon an air cushion.

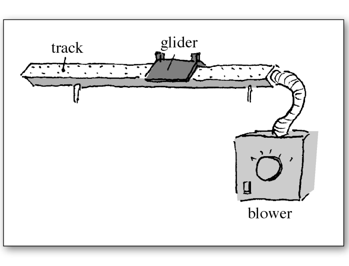


Fig. 3.3
An air track. The glider moves almost without friction.

A vehicle unimpeded by friction moves upon a horizontal track. This vehicle could be an air glider or possibly a train car (without an engine) rolling on horizontal tracks. We observe the vehicle at three different points in time, Fig. 3.4. At the first point, Fig. 3.4a, the glider is moving at a certain speed; it therefore contains a certain amount of momentum. At the second point, Fig. 3.4b, the speed is the same and, finally, at the third point Fig. 3.4c, it is also the same. The momentum contained in the glider at the first point is still there at the second and third points. The momentum is still in the vehicle, similarly to a load that it might carry without losing any of it.

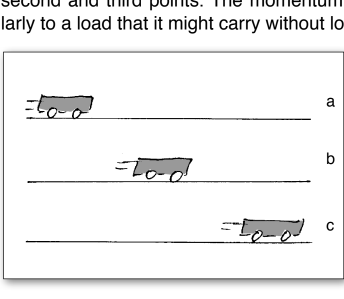


Fig. 3.4
The wagon has very good bearings. It doesn't lose any momentum.

If a vehicle has bad bearings, its "charge" of momentum decreases with time. Later we will investigate what happens to the momentum in this case, and where it goes. At the moment, we will only experiment with vehicles that have good bearings (or air cushions) where no, or almost no friction occurs.

Fig. 3.5a shows two identical gliders. The one on the left, glider A, moves to the right. Glider B is at rest. A little later A bumps into B and one can observe that after the collision, A stays still and B moves to the right, Fig. 3.5b. We will explain this process by stating what happens to the momentum. At the beginning, meaning before the collision, A had a certain amount of momentum, say 12 Hy, and B had none. During the collision, the entire amount of momentum transferred from A to B. The entire 12 Huygens was transferred from A to B so that after the impact, glider A had no more momentum.

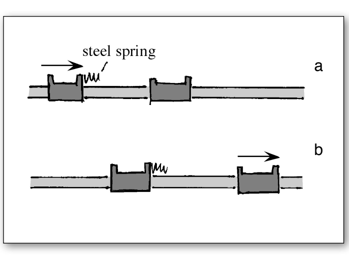


Fig. 3.5
Before the collision (a), the glider on the left moves and the one on the right is at rest. Afterwards (b), the right one moves and the left one is at rest.

> Momentum can go from one body to another.

In the experiment in Fig. 3.5, there was an elastic spring buffer between the two gliders. We will now repeat the experiment once again. However, this time by use of a totally inelastic buffer. We replace the spring buffer with a piece of putty, Fig. 3.6. The experiment now proceeds very differently. At first, glider A moves, and B is at rest. After the collision, though, both gliders move at the same speed to the right. This speed is smaller than the speed of the glider on the left before the collision. What is the explanation? This time, not all of the momentum is transferred from the left glider into the one on the right. The 12 Hy are distributed half and half over A and B so that in the end, each has 6 Hy.

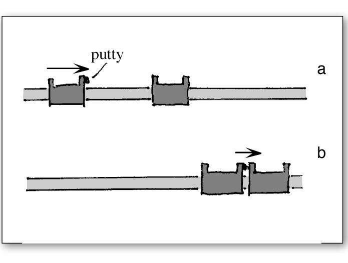


Fig. 3.6
Before colliding (a), the left hand glider moves and the right hand one is at rest. Afterwards (b), they both move but at a lower speed.

> Momentum can be distributed over several bodies.

What happens to the momentum in Fig. 3.6 is comparable to what happens to the water in Fig. 3.7. In Fig. 3.7a, all of the water is in the left hand container. After the faucet is opened, half of the water flows into the container on the right. The water is distributed over both containers exactly like the momentum in Fig. 3.6. It distributes over both gliders after the collision.

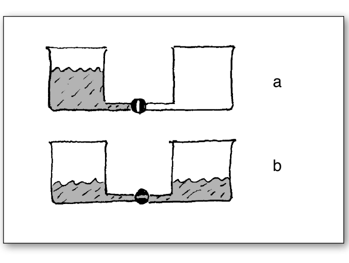


Fig. 3.7
The water distributes over both containers similarly to how the momentum in Fig. 3.6 distributes over the two gliders.

We now let glider A (with the inelastic buffer) collide with two coupled gliders B and C, Fig. 3.8. This time the momentum that A had at the beginning is distributed evenly over A, B and C. Each one has 1/3 of the amount of momentum that A had at the beginning. If A had 12 Hy to start with, then each glider would have 4 Hy after the collision.

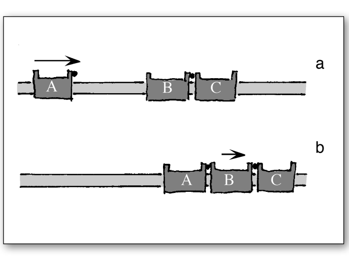


Fig. 3.8
At impact the momentum of A distributes over all three gliders, A, B, and C.

If A collides with 3, 4 or 5 gliders at rest, its momentum is distributed over 4, 5 or 6 gliders at impact. The longer the "train" of gliders, the less momentum each individual glider receives and the slower the train moves.

Instead of with a train of gliders, we let A simply collide with a buffer at the end of the air track, Fig. 3.9. Glider A naturally comes to an immediate stop. Where did the momentum go in this case? What is actually the collision partner of A? Initially, the collision partner is the track. The momentum distributes over A and the track. Now the track is firmly attached to the table. The momentum, therefore, distributes over A, the track and the table. Ultimately, the table stands upon the ground so that the momentum is further distributed into the ground. In other words, the momentum flows into the Earth, where it is so widely distributed or "diluted" that it isn't noticeable anymore.

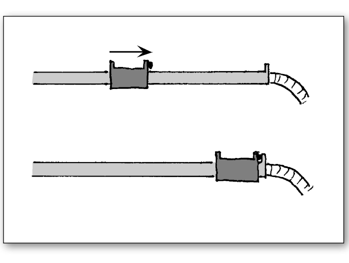


Fig. 3.9
At impact, the glider's momentum flows into the ground.

Another version of the experiment looks like this: We set a glider in motion and before it reaches the end of the track, we turn off the air flow. The air cushion disappears and the glider comes to rest upon the track. Again, its momentum has flowed into the Earth. As long as the air cushion was available, the glider's motion was frictionless. By removing the air cushion, we have activated friction. We can conclude that:

> If a vehicle has bad bearings so that it comes to a stop by itself, its momentum is flowing into the Earth.

A comparison of momentum with water is useful here as well. A vehicle with bad bearings that rolls to a stop, meaning that it gives its momentum to the Earth, is comparable to a leaky pail, Fig. 3.10. The water gradually dissipates into the environment so that eventually no one is aware of it anymore.

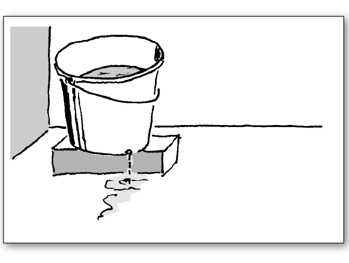


Fig. 3.10
A pail with a leak. The water distributes into the environment so that there is nothing left of it to see.

Bad bearings, i.e., friction, represent a leak of momentum. A vehicle with good bearings is comparable to a sealed pail.

We will do another experiment with two cars (or gliders on an air track). The two vehicles have an inelastic buffer and are propelled so that they move toward each other at the same speed. They collide and stop, Fig. 311. Again we ask the question: where did the momentum go? Because the cars have good bearings, it cannot have flowed into the Earth. By the way: two objects in outer space that collide in this way would also come to rest. In outer space there is no Earth to have taken up the momentum. The answer to our question must be that the momenta of our cars have somehow cancelled each other out, but how is this possible?

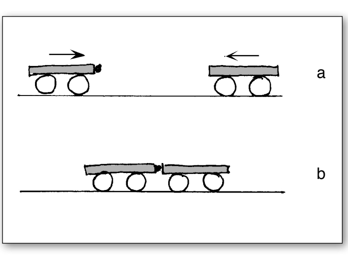


Fig. 3.11
Two cars move toward each other at equal speeds. When they collide, they come to rest.

The explanation is very simple if the momentum of one body is considered positive and the other negative. If one car had +20 Hy and the other one –20 Hy before the collision, then the total momentum already before the collision is 0 Hy. As the experiment shows, it is 0 Hy after the collision, and the balance is achieved. We conclude that:

> Momentum can take positive and negative values.

Which of the two bodies in Fig. 3.11a has the positive and which the negative momentum? This can be decided at will. You know from math class that the positive *x*-axis is pointing to the right. We do exactly the same in the case of momentum. We establish that:

> The momentum of a body is positive when the body moves to the right and negative when it moves to the left.

Exercises

1. A vehicle containing 1,500 Hy of momentum, bumps into four vehicles at rest. All of them are built identically and are coupled after colliding. What is the total amount of momentum of all five vehicles after the collision? How much is contained in each one?

2. Two coupled wagons with a total momentum of 12,000 Hy, collide with a third one which initially does not move. All the wagons are constructed identically and are coupled after the collision. How much momentum does each wagon have before colliding? How much momentum is contained in each wagon after the collision?

3. Two identical gliders move toward each other at the same speed. They are equipped with elastic buffers. The glider on the left contains +5 Hy of momentum and the one on the right contains –5 Hy. What happens with the momentum of the gliders during the collision?

4. Two cars with a total of 500 Hy roll to the right and bump into a third one coming the other way. The third one has –200 Hy of momentum. (All three cars are built the same and are connected after the collision). How much momentum does each car have after the collision? In which direction do the cars move?

5. A ball flies horizontally against a wall and bounces off of it so that it flies off at equal but opposite speed. Its momentum before the collision was 1 Hy. What is the momentum after the bounce? What is the difference of the momentum before and afterwards? Where did the missing momentum go?

3.3 Momentum pumps

We were just dealing with the question of where the momentum goes from a body with decreasing speed. We found that the momentum flows into the ground. We now ask the opposite question: Where does the vehicle get its momentum from when it speeds up?

A wagon is set in motion as a result of someone pulling on a rope attached to it, Fig. 3.12. While the person pulls, the wagon picks up speed, meaning that its momentum increases. Where does the momentum come from? From the person pulling? The momentum of the person would then have to decrease, which it doesn't. The person is at rest at the beginning and at the end. The momentum was and is 0 Hy.

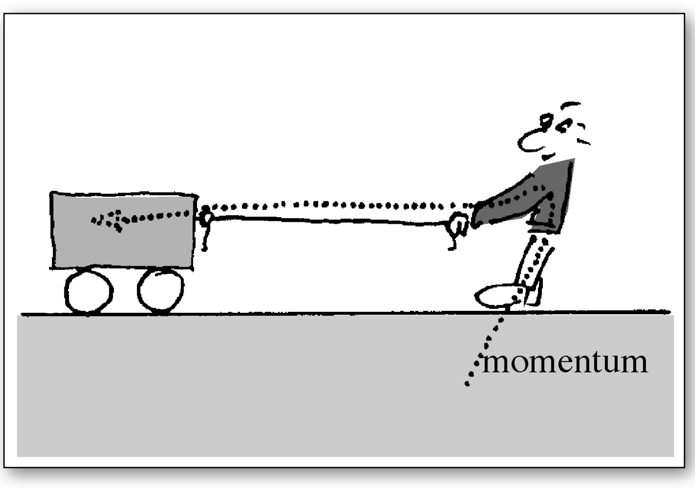


Fig. 3.12
The momentum of the wagon increases while the person is pulling.

We can change the experiment slightly so that the momentum actually does come from the person, Fig. 3.13. When the person here pulls on the rope, the momentum of the wagon on the left increases. The wagon on the right, including the person, is put into motion, but to the left. Wagon and person are receiving negative momentum, or in other words: their momentum decreases. As long as one pulls, momentum flows from the wagon on the right (plus person) through the rope into the one on the left. The person used her muscles to make sure that momentum flowed from right to left. She used herself as a "momentum pump".

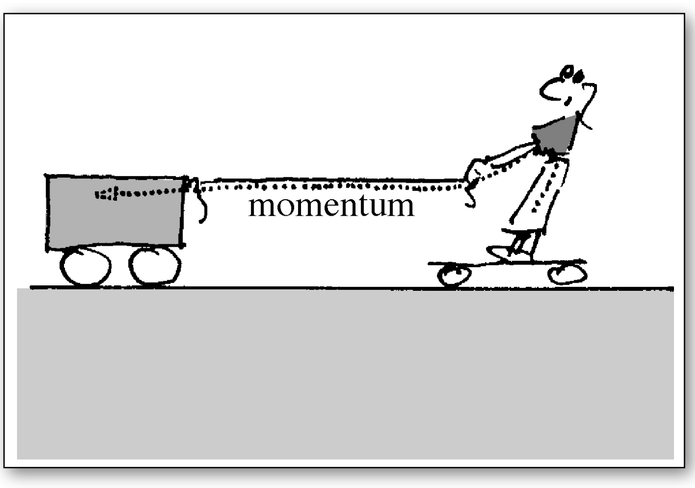


Fig. 3.13
The person sends momentum through the rope from right to left.

We also see what must have happened in the case of Fig. 3.12: The person pumped momentum out of the Earth, through the rope and into the wagon. It is just as impossible to see that the momentum of the Earth become negative as it is to see the increase of momentum of the Earth when a vehicle rolls to a standstill (thereby giving momentum to the ground).

We consider some other examples of momentum being pumped from one body into another.

The person in Fig.3.14 pulls the wagons A and B towards himself so that they become faster. The momentum of A increases and the momentum of B takes increasingly negative values, (meaning that it decreases). The momentum of the person at the center is equal to 0 Hy and it does not change. Therefore, the person transports momentum from the right to the left wagon. The person stands on a skateboard to ensure that there is no momentum coming from or flowing into the Earth.

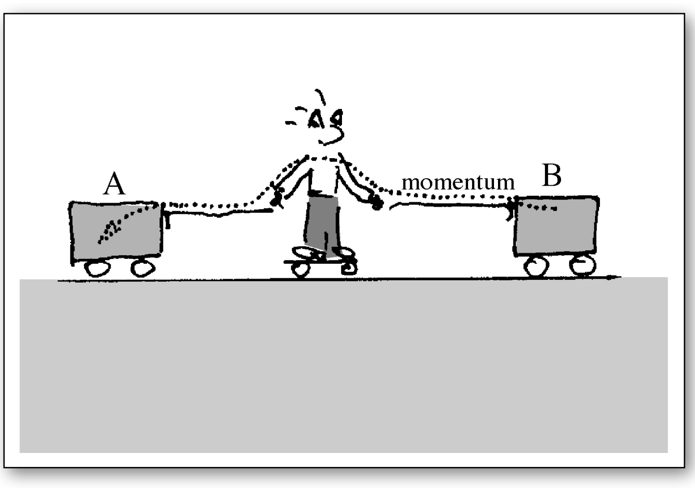


Fig. 3.14
The person pumps momentum out of the wagon on the right into the wagon on the left.

A car is traveling with increasing speed, meaning that its momentum increases. In this case, the motor acts as the momentum pump. It transports momentum out of the ground over the drive wheels (in cars these are mostly the front wheels) into the car, Fig. 3.15.

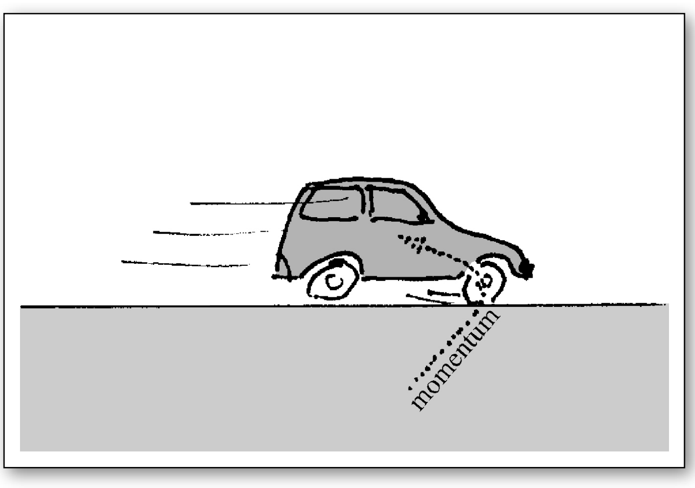


Fig. 3.15
The car's motor pumps momentum out of the Earth, over the drive wheels into the car.

A toy car with remote control is standing upon a piece of cardboard under which rollers, say drinking straws or pencils, are placed, Fig. 3.16. The car is set in motion so that it moves to the right. Its momentum increases. At the same time, the cardboard rolls to the left, meaning that its momentum becomes negative; it decreases. The car's motor pumped momentum out of the piece of cardboard into the car.

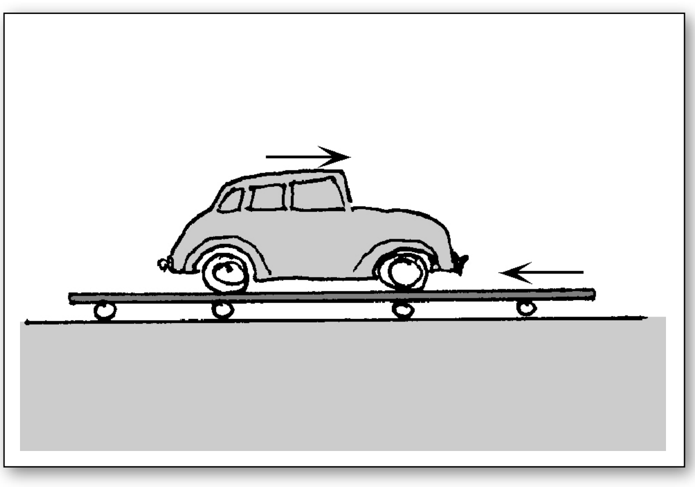


Fig. 3.16
The motor of the toy car pumps momentum out of the cardboard into the car.

Two cars (or gliders on an air track) are connected by a thread, Fig. 3.17. One of the cars has a spring buffer attached to it. The thread is so short that the spring is compressed. When the thread is cut, the cars are set in motion. The one on the right moves to the right, and the one on the left moves left. The right hand car has received (positive) momentum, the left hand one has lost (positive) momentum. The spring worked here as the momentum pump. As long as it was expanding it transferred momentum from the wagon on the left to the wagon on the right.

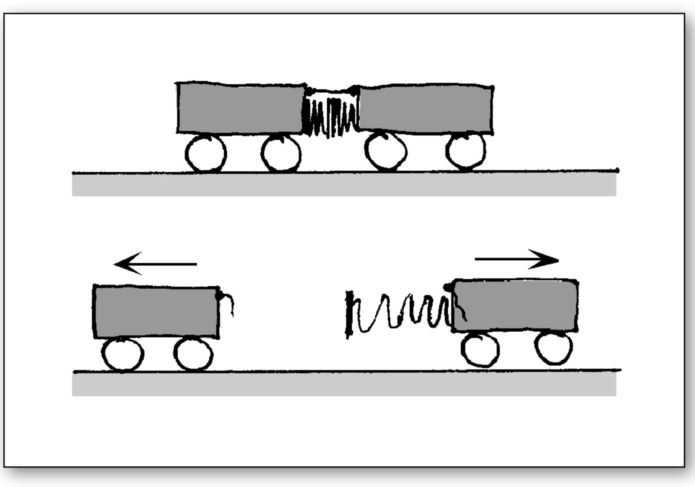


Fig. 3.17
The spring pumps momentum from the wagon on the left into the one on the right.

3.4 Momentum conductors and insulators

We saw that momentum can get from one body A into another body B. We also say that momentum *flows* from A to B, or we say that a *momentum current* flows between bodies A and B.

A necessary requirement for momentum to flow from A to B is that there be a connection between A and B. Not just any connection is enough. It must be able to transmit momentum. It must be a "momentum conducting" connection. How do such momentum conductors look? What kinds of objects conduct momentum? What kind do not?

In Fig. 3.18a, a person is pressing a rod against a wagon. The wagon becomes faster, its momentum increases. The person is pumping momentum out of the ground and into the wagon. Momentum is flowing from left to right through the rod. In Fig. 3.18b, a wagon is being charged with momentum but this time the person pulls the car with the rod. In this case, momentum is flowing through the rod from right to left. In both cases it can be seen that the rod is a momentum conductor. It is also clear that the exact form of the rod makes no difference, nor does the material it is made of, which must only be solid. We conclude:

Solid materials conduct momentum.

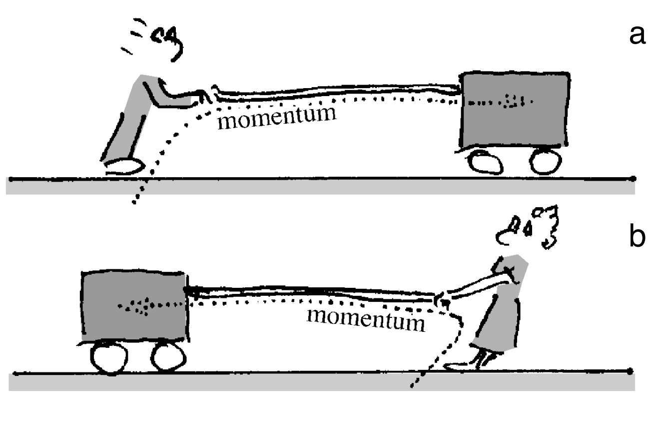


Fig. 3.18
Momentum flows through the rod from the Earth into the wagon. (a) The momentum flows through the rod to the right. (b) The momentum flows through the rod to the left.

Fig. 3.19 shows someone who believes in miracles. She is trying to set the wagon in motion by pressing on the air around it in the hopes that the air will conduct momentum to the wagon. Eventually she is convinced that:

Air does not conduct momentum.

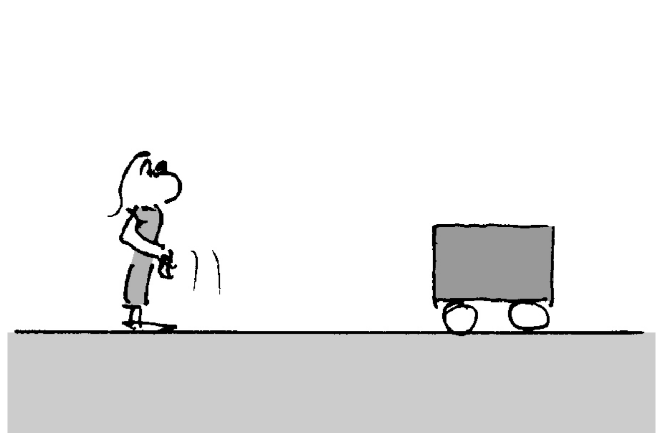


Fig. 3.19
The person tries unsuccessfully to send momentum through the air.

Later on we will see that this statement has only limited validity. In the case of air cushions it is useful: The air between the track and the gliders prevents momentum from flowing from the gliders into the track.

In Fig. 3.20, someone is investigating the momentum conductivity of a rope and finds that momentum flows easily from right to left, Fig. 3.20a, but absolutely not from left to right, Fig. 3.20b.

Ropes conduct momentum in only one direction.

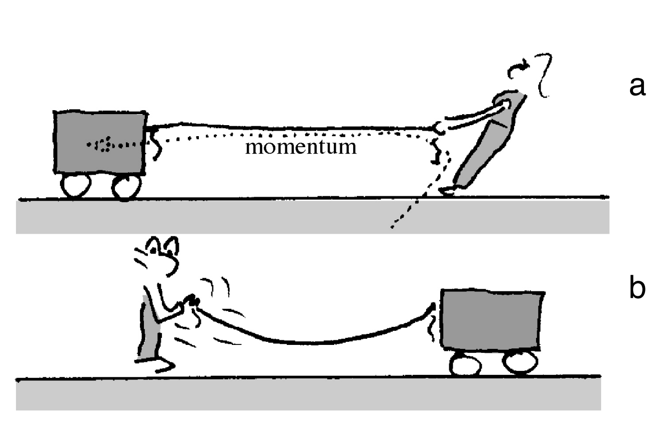


Fig. 3.20
In a rope, momentum can flow from right to left (a) but not from left to right (b).

We will carry out an experiment that is not quite as easy to interpret as those we just did. A magnet A is attached to a small wagon, Fig 3.21. One holds another magnet B up to it so that the poles with the same sign face each other: north pole facing north pole and south pole facing south pole. If magnet B is held close enough to magnet A, the wagon is set in motion, its momentum increases. We have pumped momentum out of the ground, through magnets B and A, and into the wagon.

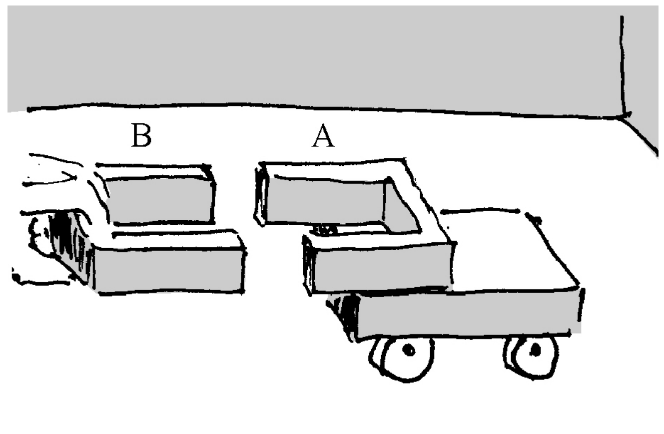


Fig. 3.21
There is a magnetic field between the magnets. The field is a momentum conductor.

The question is, how did the momentum get from A to B? From our observations we assume that there must be a connection between the magnets. There must be an invisible entity between them which can conduct momentum. This entity, that surrounds every magnetic pole, is called a *magnetic field*.

Magnetic fields conduct momentum.

Fig. 3.22 shows a person who is transferring momentum into a wagon by pushing a rod across the top of it. The rod slides over the top of the wagon because it is not attached to it. It is actually possible to get momentum into the wagon this way, but not very effectively. One sees that transfer of momentum is the better the greater the *friction* between rod and wagon. If the rod slides very lightly over the wagon, the momentum current from the rod to the wagon is very small. If the friction is strong, for example if the rod and the wagon both have rough surfaces, the transfer of momentum works well. We conclude:

If two objects rub against each other, momentum flows from one to the other: The stronger the friction, the stronger the flow.

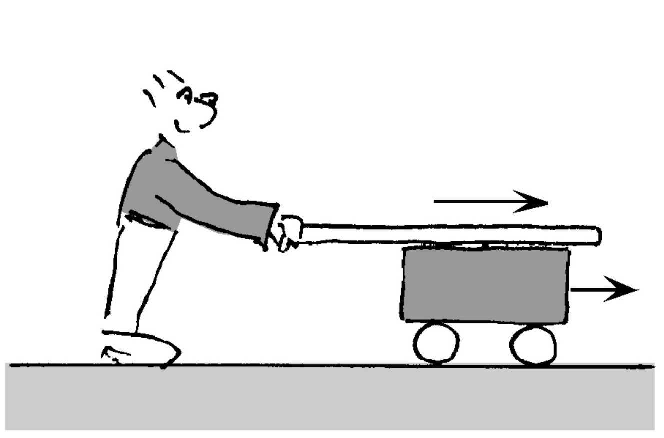


Fig. 3.22
Transfer of momentum by friction.

Basically we have always assumed the validity of this rule: In order to make sure that the momentum of an object does not flow into the ground, one must make sure that there is no momentum conducting connection between the object and the ground. This means that one must ensure that there is only very slight friction.

The most important apparatus we have for keeping friction between an object and the Earth to a minimum is the wheel.

Wheels help to insulate momentum.

There are definitely other methods to do this: air in airtrack gliders, airplanes and helicopters, the runners of sleds and blades of skates, and water in the case of river boats and ships.

Exercises

1. Ropes do not conduct momentum to the right, but only to the left. Invent a device that conducts the momentum only to the right and not to the left.
2. A car driver brakes sharply on ice. What happens? Momentum conduction plays an important role in the process of braking. What can you say about this in the case of ice on the street?
3. A driver attempts to drive off quickly on ice, what happens?

3.5 Drives and brakes

Wheels are insulators for momentum only when they spin freely. The propelled wheels of a car are not insulators for momentum. They are connected through the engine to the chassis and the body of the car so that the engine can pump momentum out of the ground and into the car.

It is often desirable to get rid of the momentum stored in a car as quickly as possible, Fig 3.23. Vehicles have brakes for this. In the process of braking, the wheels' friction is greatly increased; the wheels are transformed into good momentum conductors so that the momentum of the vehicle quickly flows into the ground. A brake is therefore a pipeline for momentum that can be "turned on and off", meaning it is a kind of valve or switch for the momentum current.

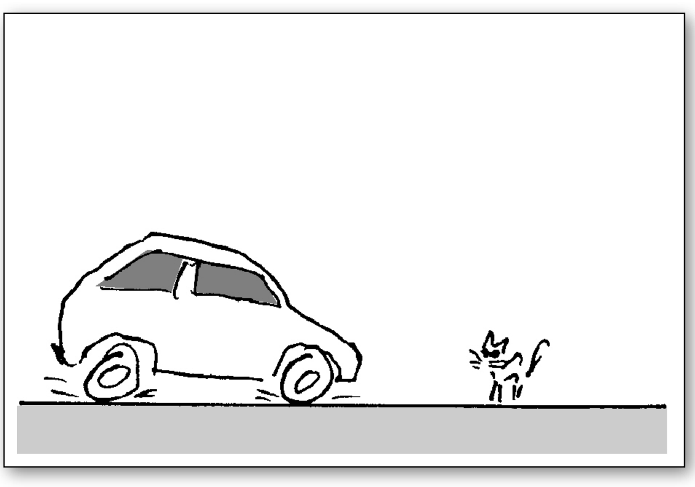


Fig. 3.23
This car should get rid of its momentum quickly.

A fast moving car not only loses momentum through the friction of its wheels but also through friction between its surface and the air. At speeds above 80 km/h, this is actually the source of greater loss. In this process, the car's momentum first flows into the air. The fact that the air actually contains the momentum can be seen in that it is still strongly moving shortly after the car has gone by. Gradually it gives the momentum to the ground, again by friction.

The experiment represented in Fig. 3.24 shows us that momentum can be contained in air. A toy balloon is mounted upon a small wagon. If the balloon is opened and the wagon is let go, it will start to move. The skin of the balloon presses the air out to the left. The air flowing out of the balloon gets negative momentum and the wagon positive momentum.

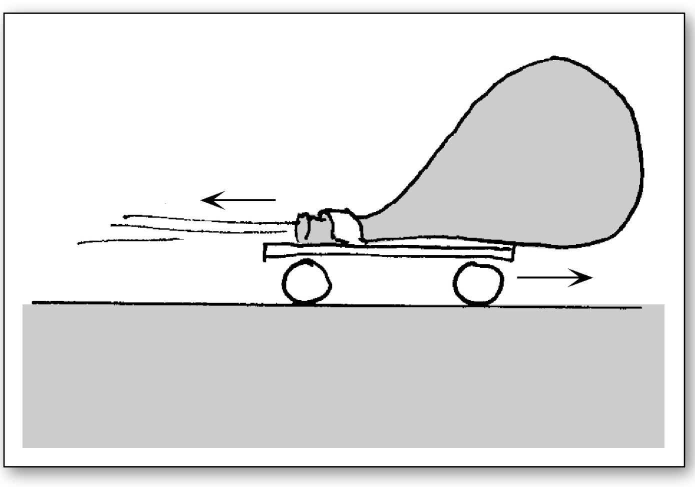


Fig. 3.24
The air flowing out gets negative momentum, the wagon gets positive momentum.

In Fig. 3.24, the wagon's propulsion basically functions like that of a rocket. A rocket also gets momentum resulting from a gas being ejected with great speed out of the rear of it. The space inside the rocket is taken up mostly by two tanks, Fig. 3.25. One contains the fuel, for example liquid hydrogen. The other tank contains liquid oxygen. In the process of burning hydrogen, water vapor at very high pressure is created. This vapor flows out the back of the rocket with great speed taking momentum with it. In this way the rocket gets momentum having the opposite sign.

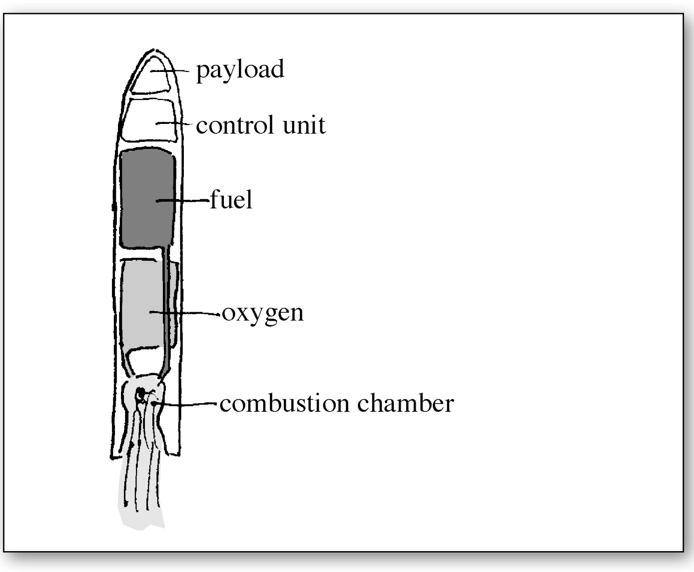


Fig. 3.25
How a rocket is constructed.

Ships are moved by propellers, Fig. 3.26, which turn under water and are driven by the ship's engine. A propeller sets the water in motion backwards, charging it with negative momentum. The corresponding positive momentum is given to the ship. In other words, the propeller pumps positive momentum out of the water and into the ship. If the ship brakes, the propeller motion is reversed, and momentum is pumped from the ship into the water.

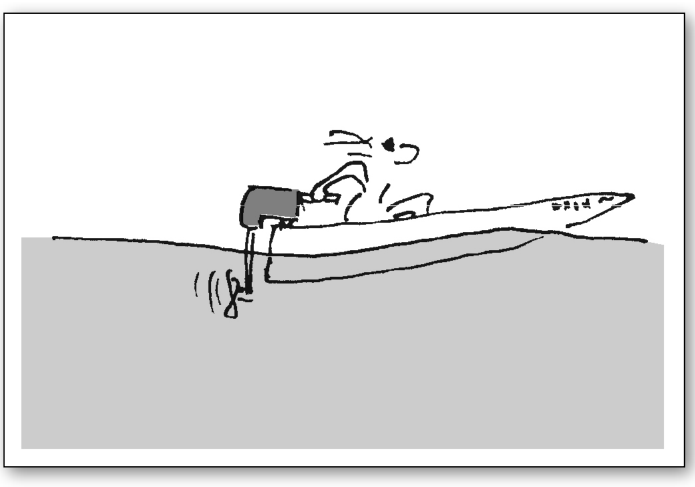


Fig. 3.26
The propeller pumps momentum out of the water into the boat.

Propulsion systems of airplanes work similarly to those of ships, only here the momentum is taken from the air instead of from the water.

An airplane with propellers uses them to pump momentum out of the air and into the airplane. In a jet, the jet engine does this. A jet engine is actually nothing more than a very strong fan hidden inside the casing and driven by a turbine. The turbine receives its energy with kerosene, a fuel similar to gasoline (petrol).

Airplanes must brake very quickly after landing. This means they must get rid of a lot of momentum very quickly. One way to do this is through the wheels into the ground, as an automobile does. A more efficient method, though, is something called thrust reversal. In some airplanes one can observe this well from inside through a window. Two flaps are extended from each engine. These divert the air blown out of the rear, forward, Fig. 3.27. In the process, the air receives positive momentum from the airplane, meaning that the airplane's momentum decreases (we have assumed the airplane moves to the right).

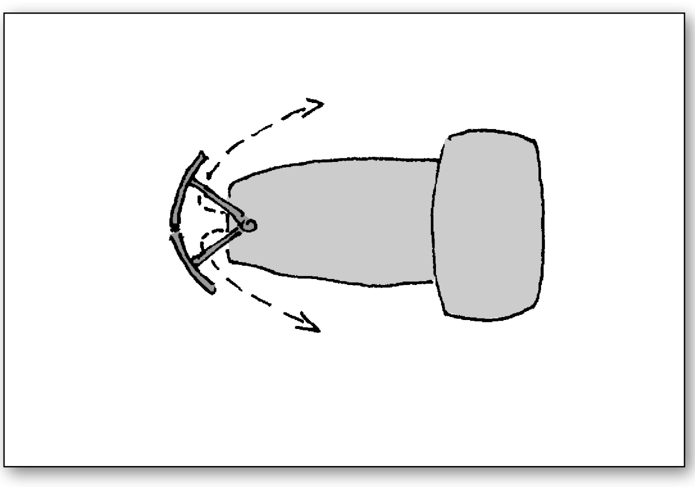


Fig. 3.27
A jet engine with the thrust reverser working: The airplane gives momentum to the air.

Exercises

1. Where does a sailboat get its momentum from?

2. A ship is moving at a constant speed, meaning its momentum doesn't change. Where does the momentum go which the motor constantly pumps into the ship?

3.6 Steady state

An automobile accelerates: The engine continuously pumps momentum out of the ground and into the car. The faster the car travels, the greater is the air friction and the more momentum is lost. At a certain speed, exactly the same amount of momentum is pumped into the car as flows off by friction. As a result, nothing is left to be stored and the car's momentum no longer increases, Fig. 3.28.

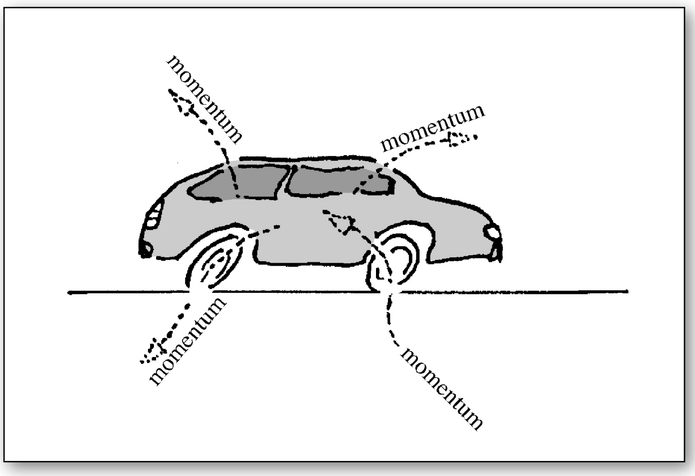


Fig. 3.28
A car traveling at constant speed. Due to friction, all of the momentum that the motor pumps into the car flows back out into the environment.

This situation always occurs when a car travels along a level road at constant speed. The inflow of momentum equals the outflow.

This situation is again comparable to another one where water plays the role of momentum, Fig.3.29. The pail with the hole is equivalent to the car. The pail has a leak for the water just as a car has a leak for momentum. There is a continuous flow of water into the pail, but exactly the same amount of water is flowing out of the hole so that the amount of water in it never changes.

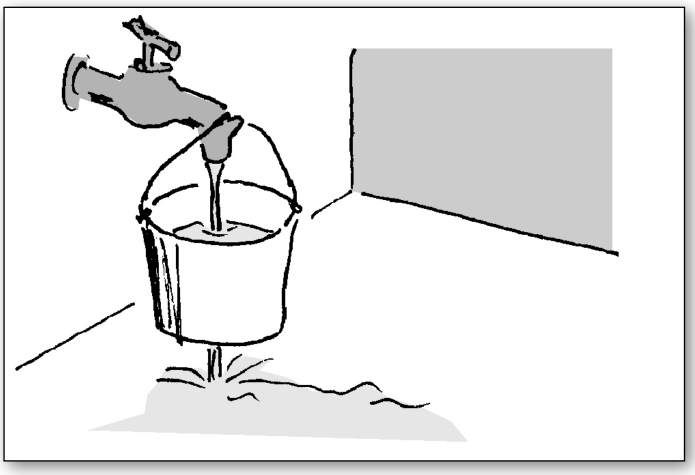


Fig. 3.29
The same amount of water flowing into the pail is flowing out of the hole in it. The amount of water in the pail stays constant.

A process where the current flowing out is adjusted so that it is exactly as strong as the current flowing in, is called *steady state*.

Steady state: The outflow is adjusted to equal the inflow.

We often have a steady state when something moves at a steady (constant) speed.

A bicyclist pumps momentum into a bicycle (and into the person himself). A current of equal strength flows off, due to friction, over the air and wheels. This works correspondingly for airplanes and ships.

Exercises

1. Describe the following situations in the movement of a car by showing what happens with momentum.

(a) The car starts moving.

(b) The car rolls slowly in neutral.

(c) The car brakes.

(d) The car moves at a high and constant speed.

2. Earlier we found out about a process by which a body moves at constant speed although there is no steady state. Why does the momentum stay constant in this case?

3.7 Direction of momentum currents

We can do the following experiment only in our minds because we need a moving train for it.

An object G is thrown onto the floor of a train car W traveling to the right, Fig. 3.30. It is thrown so that it slides across the floor to the right. The speed of G immediately after hitting the floor is greater than that of the train. The object comes quickly to “rest”, though. By rest, we mean that it no longer moves relative to the train. In other words: it moves exactly a fast as the train. While it was sliding, the momentum of G decreased. The momentum flowed from G to W.

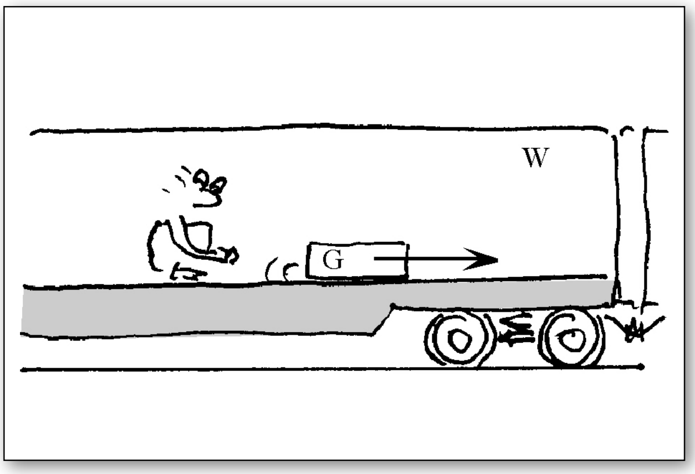


Fig. 3.30
The object slides across the floor of a train car.

Again, we toss G onto the floor, but this time so that it slides to the left. At first, its speed is lower than that of the train. Again, the two speeds quickly even out. This time, the momentum of G increases during sliding. Momentum flows from train car W to object G.

Have you noticed that there is a simple rule for the direction of the momentum current? In both cases, momentum flows from the body with the greater velocity to the body with the lower velocity: In the first case, from G to W and in the second, from W to G. This rule always holds when momentum flows because of friction. In the example of a car rolling to a stop, Fig. 3.31, momentum flows from the body with the greater velocity (the car) into the body with lower velocity (into the ground which has a velocity of 0 km/h).

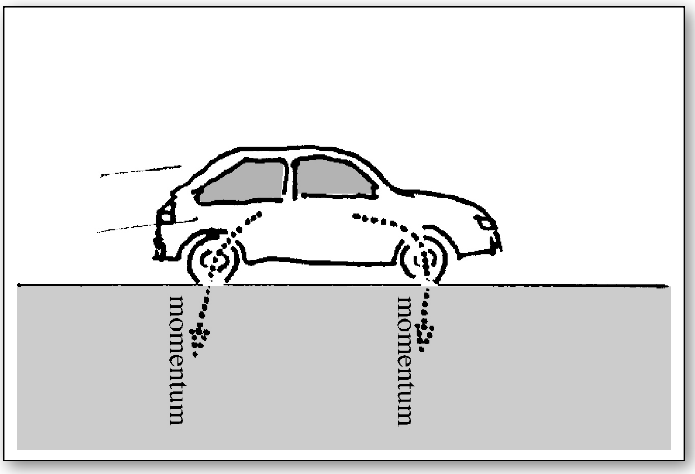


Fig. 3.31
A car slows down. The momentum flows from the body with higher velocity into the one with lower velocity.

Whenever momentum should flow in the opposite direction, meaning from a body with lower to a body with higher velocity, a momentum pump is necessary.

We therefore have the rule:

> Momentum flows by itself from a body of higher velocity into a body of lower velocity. A “momentum pump” (engine, person) transfers it in the opposite direction.

3.8 Compressive and tensile stress

In Fig.3.32a, someone sets a wagon in motion. Momentum flows through the rod from left to right. In Fig. 3.32b, the wagon rolls on alone. Its momentum remains unchanged (except for loss through friction). Therefore, no momentum is flowing through the rod in Fig. 3.32b. In Fig.32c, momentum flows through a rod from right to left. Put yourself in the position of the rod. Would you feel any difference in these three situations? Of course. One can actually consider the person's arms as extensions of the rod. One can feel with one's arms the differences in the three situations. In the first, there is compressive stress, in the third one feels tensile stress, but in the second there is neither pressure nor tension.

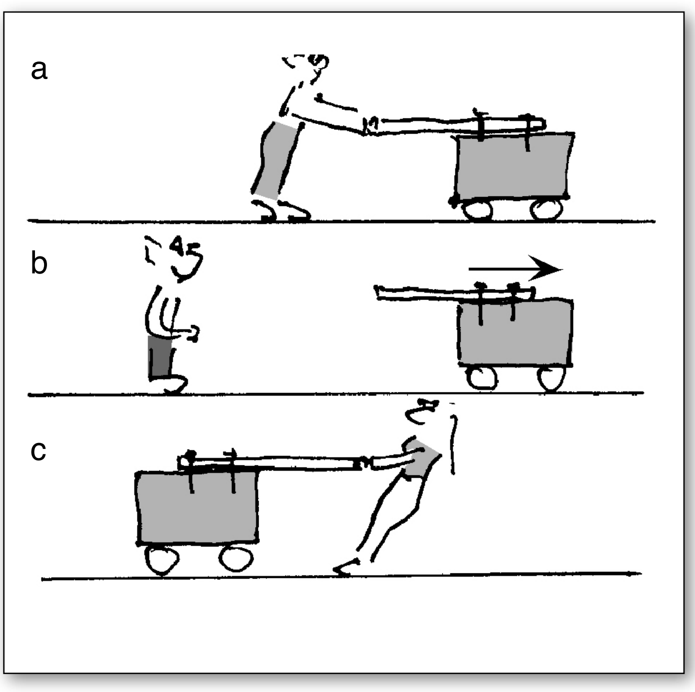


Fig. 3.32
(a) Momentum flows to the right in the rod. (b) No momentum flows in the rod. (c) Momentum flows left in the rod.

These statements can be used for the rod as well. In the first case, the rod is under compressive stress, in the second, it is under no stress at all, and in the third, it is under tensile stress. We then have the following rule:

Momentum flow to the right: compressive stress
Momentum flow to the left: tensile stress

We will convince ourselves of the validity of this rule by using a further example. Fig. 3.33a shows a truck just driving off. The truck's engine pumps momentum out of the ground into the truck, and across the coupling to the left into the trailer. We know that the coupling rod is under tensile stress in agreement with our rule.

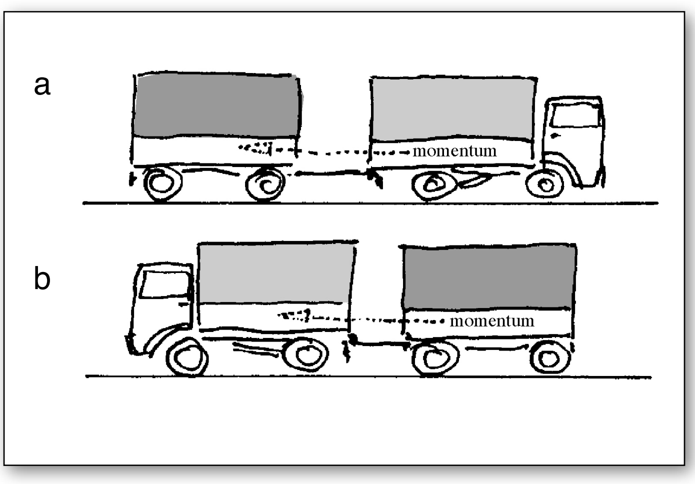


Fig. 3.33
A tractor-trailer starts off to the right (a) and then to the left (b). In both cases, the coupling rod is under tensile stress, and both times the momentum current is flowing to the left.

We now consider a truck starting to move to the left, Fig. 3.33b. In this case, the motor pumps negative momentum into the truck, meaning it pumps positive momentum out of it. Therefore, (positive) momentum flows through the coupling rod to the left. The coupling rod is, naturally, under tensile stress. Our rule also holds here.

A rod does not show whether it is under compressive or tensile stress or even whether or not it is under any stress at all. One also does not actually see whether or not, nor in which direction, momentum flows in it. However, there are objects that show very clearly their stress states. They all deform elastically. Examples of these would be rubber bands or steel springs.

Such objects lengthen by applying tensile stress and shorten by applying compressive stress. One sees whether or not and in which direction a momentum current flows through it, Fig. 3.34 and 3.35.

Lengthening: Tensile stress;
momentum current flows to the left
Shortening: Compressive stress;
momentum current flows to the right

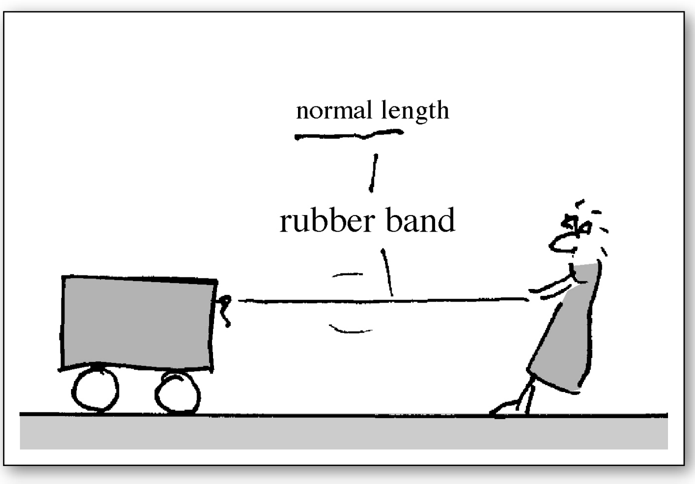


Fig. 3.34
A momentum current is flowing to the left through the rubber cord. The cord is under tensile stress and it stretches.

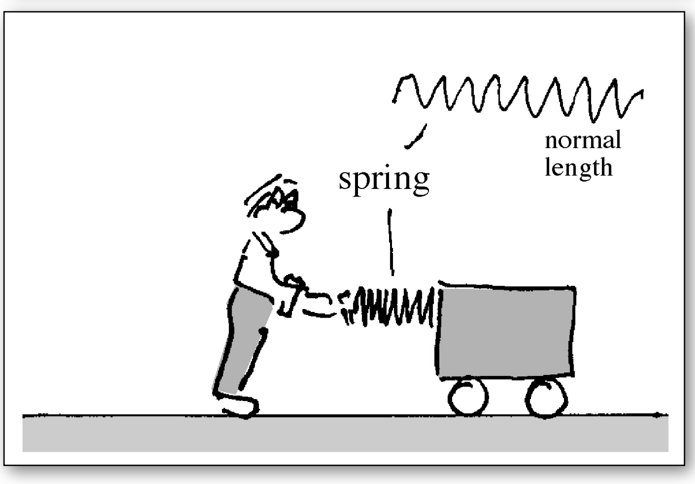


Fig. 3.35
A momentum current flows to the right through the spring. The spring shortens.

Exercises

1. A car traveling to the left, suddenly brakes. From where to where does the momentum flow? In this case, is the rule obeyed that states that momentum flows by itself from a body with greater velocity to a body with smaller velocity?

2. A person accelerates a car to the left by pushing it. In the process, her arms are under compressive stress. In what direction does the momentum current flow in her arms?

3. A truck travels to the right at a constant high speed. Under what kind of stress (compressive or tensile) is the trailer coupling? Sketch the path of the momentum.

3.9 Momentum current circuits

It can happen that a momentum current flows, and in spite of this, the amount of momentum does not change anywhere. Fig. 3.36 shows an example: A person pulls a crate across a floor at a constant speed.

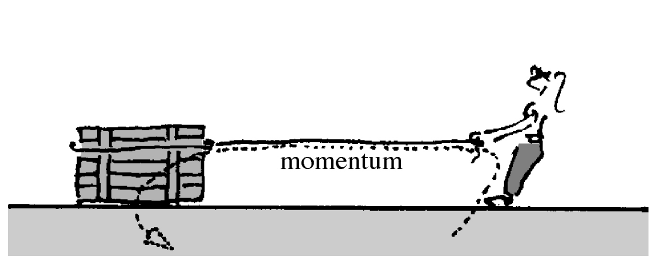


Fig. 3.36
Although a momentum current flows, momentum isn't building up anywhere.

The person could just as well be pulling a car at a constant speed instead of the crate. For our purposes, the crate has the advantage of showing very clearly the place where friction occurs, on the surface between crate and floor. In the case of wheels, friction not only occurs in the bearings but in the rubber tire and the point of contact between the tire and the ground as well.

We again ask the old question: What is the path taken by the momentum? Hopefully, the answer isn't too difficult. The person pumps momentum out of the ground, through the rope and into the crate. It flows out of the crate and back into the ground because of friction between the bottom of the crate and the ground. In this case, we can say that the momentum flows "in a circuit", even if we don't exactly know the path it takes back through the ground.

Again, a water current gives us a good image of this situation. Do you know how?

Fig. 3.37 shows a variation of the experiment in Fig. 3.36: Here, the crate is pulled, not across the ground, but across a board mounted upon rollers. The path of the momentum is even simpler. Because the board is mounted upon rollers, the momentum cannot flow back into the ground, and the person cannot pump any momentum out of the ground. The person pumps momentum out of the board. The momentum then flows through the rope into the crate. It then flows out of the crate back into the board, from where it continues to flow to the right to the person. Here again, the current flows in a closed circuit, and this time the path is clearly visible everywhere. One could say that the momentum current creates a *circuit* in this case.

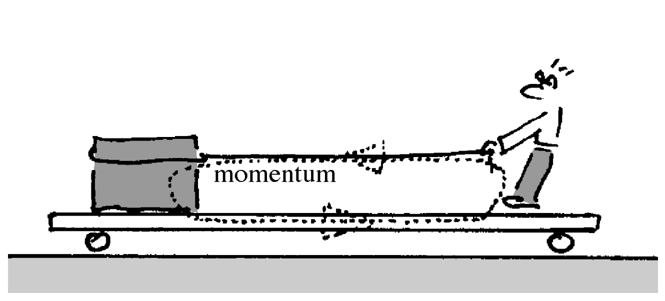


Fig. 3.37
A closed momentum current circuit

The fact that momentum really flows to the left in the rope and to the right in the board, can be demonstrated in a further variation of the experiment, Fig. 3.38. The rope and the board are each split by a spring. The springs show the direction in which momentum is flowing. The spring in the rope is stretched, it is under tensile stress. This means that momentum is flowing to the left. The spring between the two halves of the board is compressed, it is under compressive stress. This means that momentum is flowing to the right.

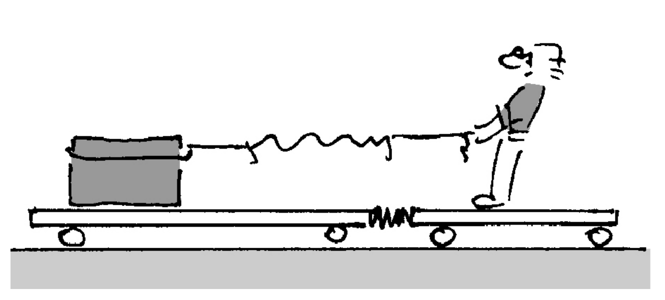


Fig. 3.38
The springs show the direction of the momentum current.

Momentum can flow in a closed circuit. Momentum therefore never increases or decreases. A part of every momentum circuit is under compressive stress and the other part is under tensile stress.

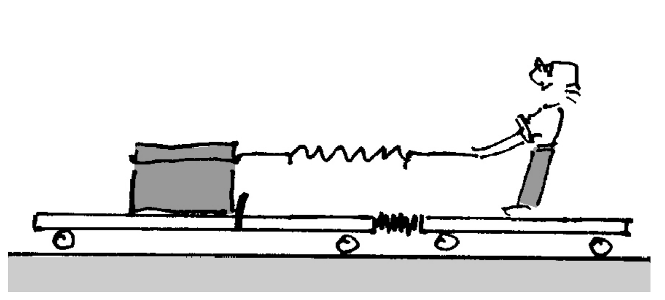


Fig. 3.39
The crate does not move. Even though, a momentum current is flowing.

We again change the experiment, this time in two steps. First, we block the crate, Fig. 3.39. The person pulls again, but the crate cannot move anymore. We conclude that we do not actually need the person: It is enough to just attach the tensed rope somewhere on the right, Fig. 3.40. As before, the rope is under tensile stress, and the board is under compressive stress. This means that the momentum current flows in a circuit as it did before, even though nothing is moving and even though we have no 'momentum pump' at all.

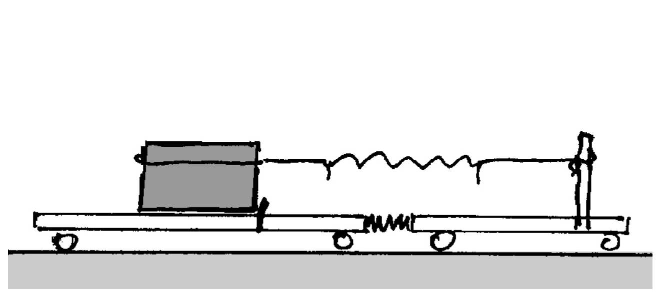


Fig. 3.40
Momentum current without a driving force.

It will surprise you that something can flow without a driving force. After all, earlier in Chapter 3 we learned that a driving force is necessary to make a current flow. We now see that this rule doesn't always hold. There are currents without driving forces. The fact that a driving force is not necessary just means that the current meets no resistance.

Later on you will see that electric currents usually need a driving force as well, but that there are electric conductors that have no resistance. These are the so-called *superconductors*. Electric currents can flow without any driving force in an electric circuit made of superconducting material.

Electric circuits without resistance are rare but non-resistant momentum current circuits are common. Figures 3.41 and 3.42 show two examples.

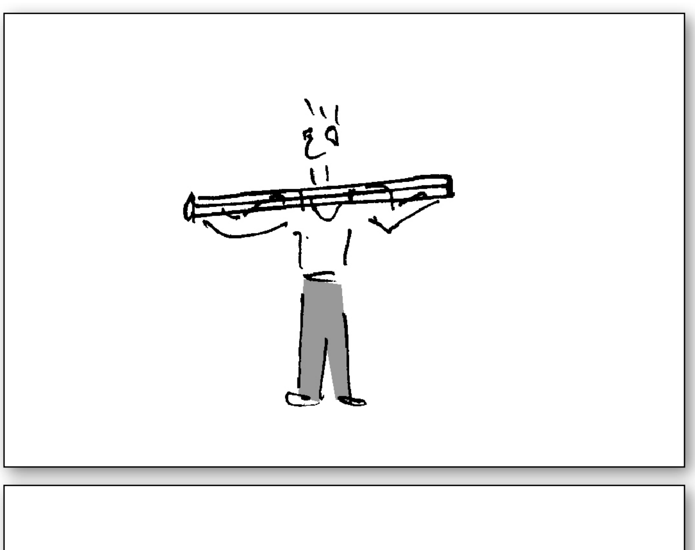


Fig. 3.41
A closed momentum current circuit.

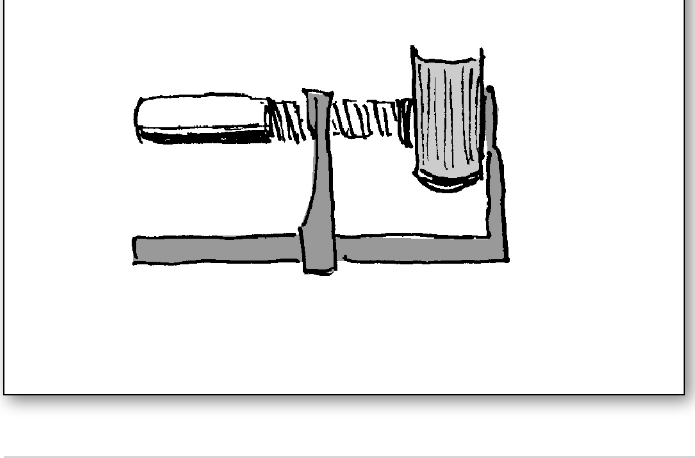


Fig. 3.42
A closed momentum current circuit.

Exercises

1. In Fig. 3.43a, a tractor tries to pull a tree out of the ground. Sketch the path of the momentum current.

2. Fig. 3.43b shows a taut clothesline. Sketch the path of the momentum current. Where do we find the tensile stress and where is the compressive stress?

3. How can a non-resistant current of matter be realized? Does such a thing exist in nature?

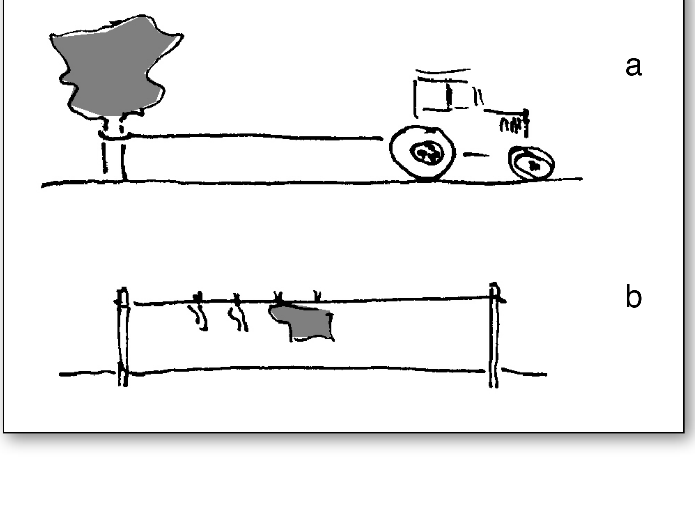


Fig. 3.43
For Exercises **1** and **2**

3.10 The strength of momentum currents

A momentum current flows at a constant rate from the tractor in Fig. 3.44 into the trailer. A certain number of Huygens per second flow through the coupling rod. The amount of momentum flowing through a conductor divided by the time span is called the *strength of the momentum current* (or simply the momentum current).

$$\text{momentum current} = \frac{\text{momentum}}{\text{time interval}}$$

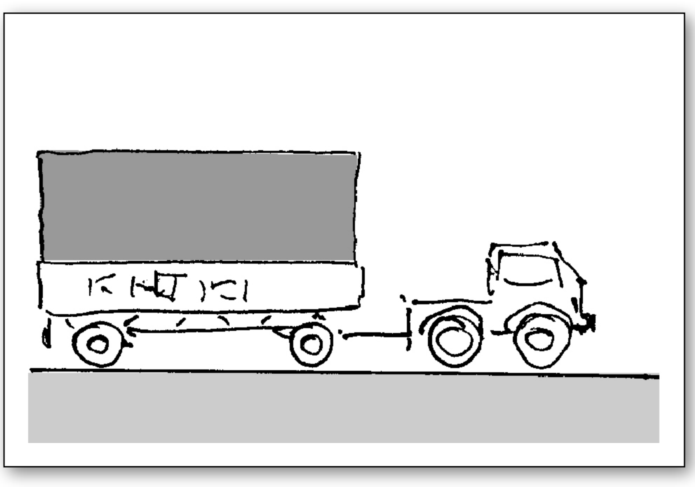


Fig. 3.44
A momentum current that is constant in time flows from the truck to the trailer.

This equation can be written much more briefly if the symbols for the quantities are used:

p = momentum

F = momentum current

t = time

Therefore we have

$$F = \frac{p}{t}$$

If, for example, 500 Hy per second flow through the trailer coupling as in Fig. 3.44, then

F = 500Hy/s.

The name *Newton* (N) is used for the unit Hy/s:

$$\text{N} = \frac{\text{Hy}}{\text{s}}$$

We can then write our momentum current so:

F = 500 N.

The unit of momentum currents is named for Isaac Newton (1643-1727). Newton gave mechanics the basic form we learn it in today. Among other equations, Newton gave us the equation $F = p/t$.

Momentum currents are easy to measure. A so-called force sensor is used for this, Fig. 3.45. A force sensor consists of a steel spring that elongates according to the strength of the momentum current flowing through it. The scale is calibrated in Newtons.

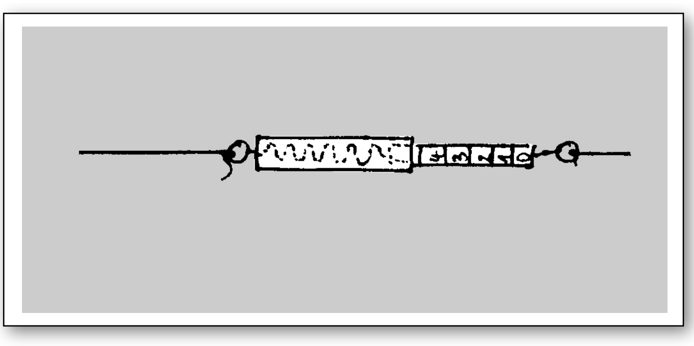


Fig. 3.45
A force sensor

Fig. 3.46 shows how to use a force sensor. The strength of the momentum current flowing through the rope in Fig. 3.46a is to be measured. The rope is cut at an arbitrarily chosen position and the newly created ends are connected to the two hooks of the force sensor, Fig. 3.46b.

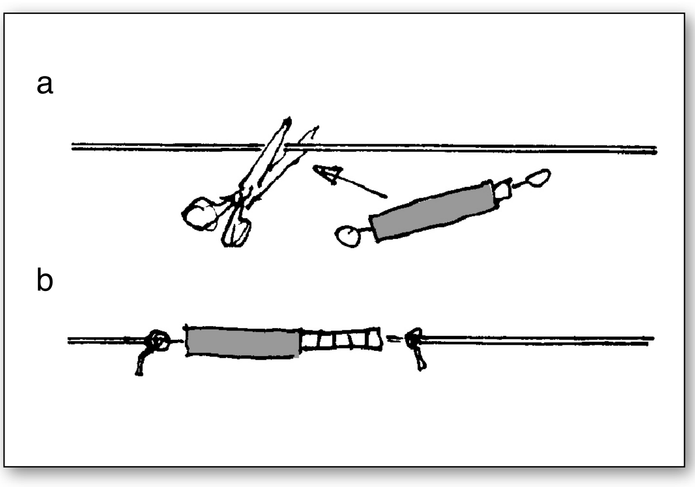


Fig. 3.46
(a) The strength of the momentum current in a rope is to be measured. (b) The rope is cut and the force sensor is attached to the newly created ends.

In Fig. 3.47a, the strength of the same momentum current is measured twice successively. Of course, both force sensors show the same reading. They also show exactly what just one force sensor would show.

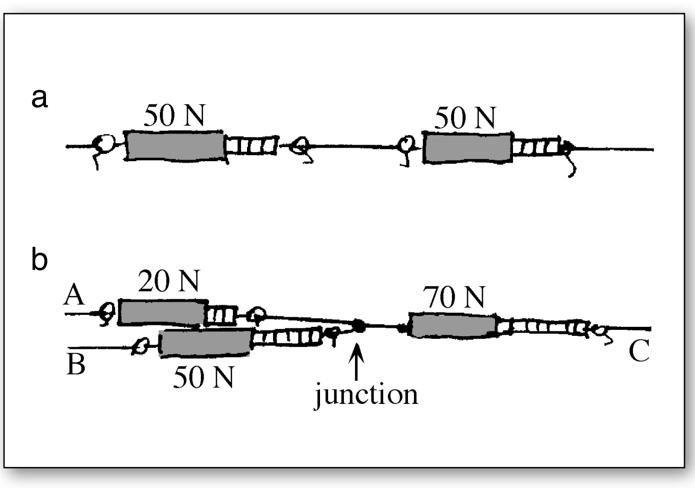


Fig. 3.47
(a) A current flows consecutively through two sensors. (b) A branched momentum current.

Momentum currents can branch just like water currents do. Fig. 3.47b gives an example. The sum of the currents in ropes A and B must equal that of C. Here we have used the junction rule that you already know from water currents (see section 2.5):

The sum of the currents flowing into a junction equals the sum of the currents flowing out of it.

Exercises

1. A constant momentum current flows into a wagon with perfect bearings. Within 10 seconds, 200 Huygens of momentum has collected. What was the current?
2. A truck is starting to drive off and a momentum current of 6,000 N flows through the coupling to the trailer. What is the momentum of the trailer after 5 s? (Friction loss by the trailer can be ignored).
3. In Fig. 3.48a, what do the force sensors C and D show?
4. The boxes in Fig. 3.48b are pulled at constant speed across the floor. What is the momentum current flowing from the box on the left into the ground? What is the current into the ground in the case of the one on the right?
5. A constant momentum current of 40 N flows into a vehicle (friction can be ignored). Represent the momentum as a function of time.

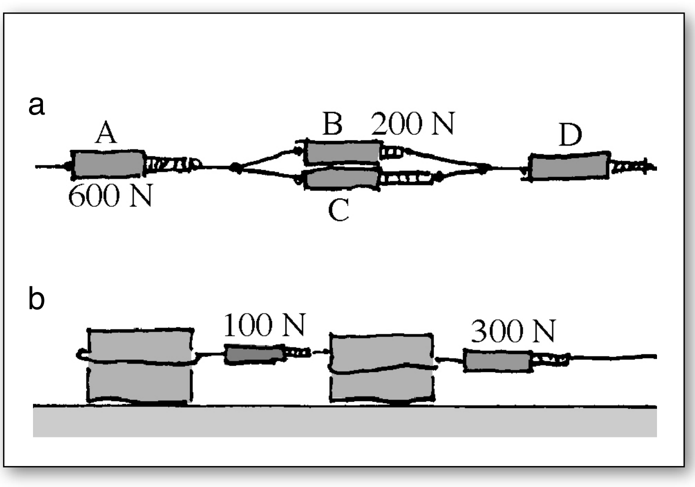


Fig. 3.48
(a) For exercise **3**;
(b) For exercise **4**

3.11 Force

In this section we will do nothing other than learn a new word for a well-known concept.

The name momentum current for the quantity *F* has only existed since the beginning of the 20th century. The quantity itself, though, has been around since Newton's time, or for about 300 years. At that time the quantity was given another name: it was called *force*. The *F* is the first letter in the word "force". The name force for the quantity *F* is still widely used. In fact it is more often used than the name momentum current. We must therefore get used to using it, although there is a problem in doing so: Even though "force" describes the same physical quantity as "momentum current", the two words are dealt with very differently. We will call a description using momentum currents the *momentum current model* and one with force, the *force model*.

You already understand why our measuring device for momentum currents is called a "force sensor".

We will illustrate handling a force model by using Figures 3.49 and 3.50. In Fig. 3.49, a person pulls a wagon with good bearings and sets it in motion to the right. Remember the description in the momentum current model: The person pumps momentum out of the Earth over the rope into the wagon. In doing so, the wagon's momentum increases. The force model describes the same process in this way: A force is exerted upon the wagon, thereby increasing its momentum.

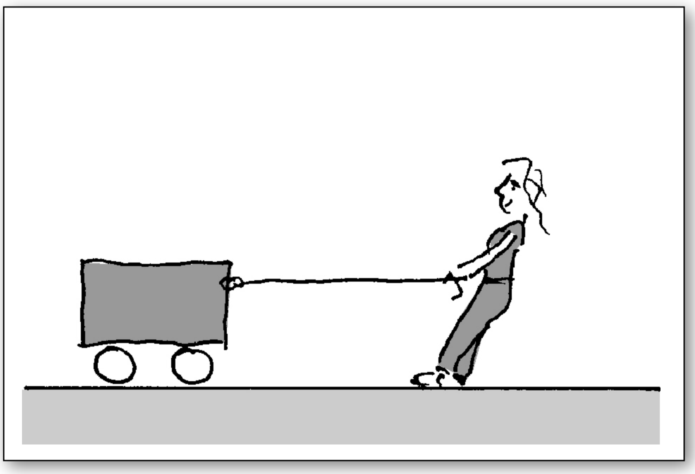


Fig. 3.49
The person exerts a force upon the wagon. The momentum of the wagon changes because of this.

It gets a bit more difficult in the description of Fig. 3.50. Here, there are two springs, A and B, pulling on a car. A pulls to the left and B pulls to the right. Naturally, both force sensors show the same thing. Let us assume it is 50 N. We will once again describe the situation in the momentum current model: A momentum current of 50 N flows out of the ground through spring B, and into the car from the right. From there it flows through A and back into the ground. A force model would describe it as follows: Spring A exerts a force of 50 N on the car pointing to the left, Spring B exerts a force of 50 N on the car pointing to the right. Because the forces have the same absolute value, but act in opposite directions, the momentum of the car does not change.

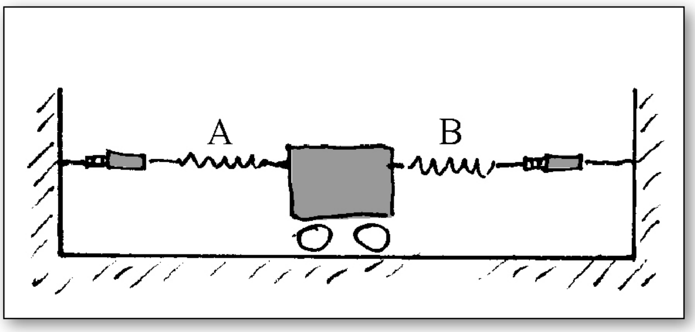


Fig. 3.50
Spring A exerts a force to the left and spring B a force to the right upon the wagon. Because the forces are of the same magnitude, the wagon's momentum doesn't change.

3.12 Measuring momentum currents

We wish to build a device for measuring currents (a force sensor) ourselves. We pretend that the spring force sensor has not been invented yet, and that the unit of measuring momentum currents has not yet been determined.

We will start by deciding what our unit of measurement will be. To do this, we need a large number of identical rubber bands. We hold one of them up to a ruler so that it is stretched out but not beyond it's normal length, Fig. 3.51, and measure its length. Let's assume we find a length of 10 cm = 0.1 m. Because the rubber band is slack, no momentum current is flowing through it yet. We stretch it until it is 0.15 m long. Naturally, a momentum current is flowing through it now. We declare the strength of this momentum current to be our unit of current. (Because the band is made up of two parallel rubber threads, only half of the unit of momentum current flows through each thread.)

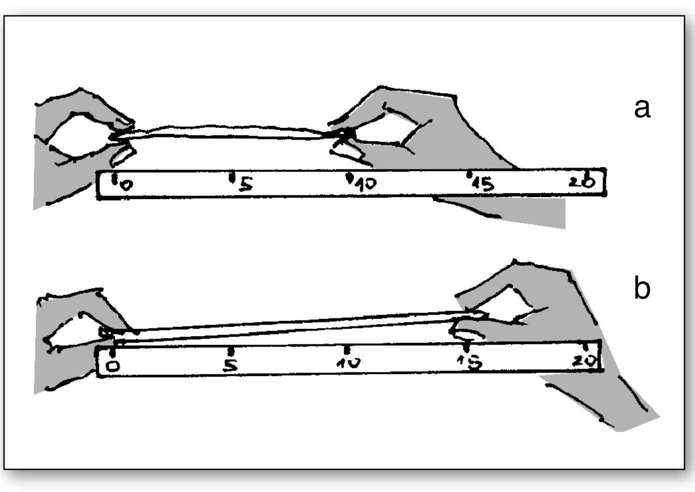


Fig. 3.51
How to determine a momentum current unit. (a) The rubber band is laid out but not stretched. (b) The rubber band is stretched by 5 cm.

Now we can create as many units of momentum current as we wish with other rubber bands. This means that we can create multiples of our unit of momentum current. For example, if we connect three uniform rubber rings together in parallel, three units of momentum current flow through all three of them.

With the help of our supply of rubber bands, we can now *calibrate* another elastic object such as an elastic cord, Fig. 3.52. To do this, we allow one or more units of momentum current to flow through the cord, all the time measuring the changes in its length as compared to its relaxed state.

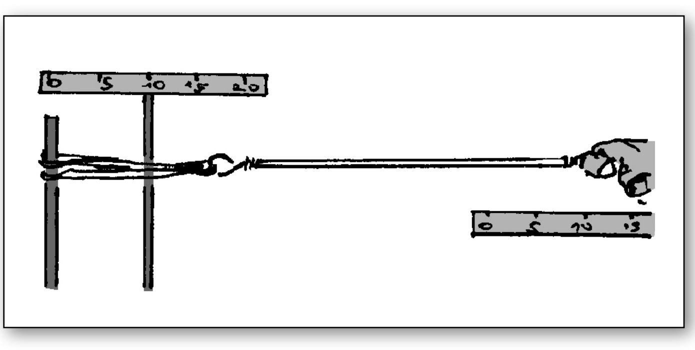


Fig. 3.52
The cord of an expander is calibrated using the rubber band unit.

In Fig. 3.53, the momentum current is shown as a function of the "stretching". This curve represents the *calibration curve* of the expandable cord. If we now wish to measure a momentum current, we can forego our rather cumbersome process with the rubber band units, and use the rubber cord instead.

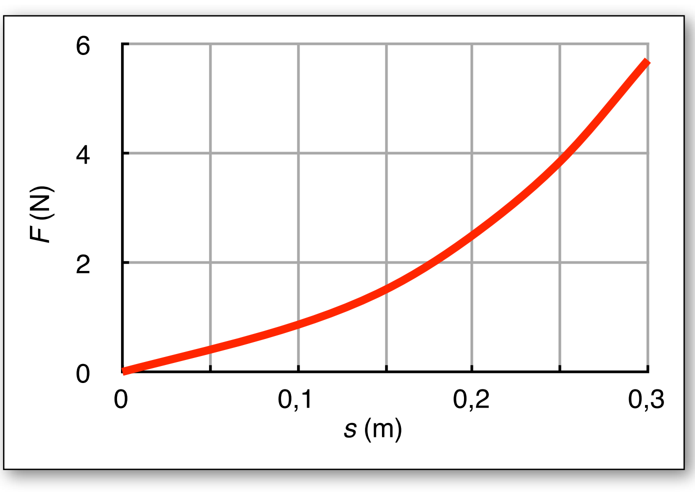


Fig. 3.53
The calibration curve of the expander: The momentum current is given as a function of the stretching of the cord.

For example, we wish to measure the strength of the current flowing into a wagon we are pulling. To do this, we pull the wagon by the elastic cord and measure by how much the cord lengthens. If the amount of stretching is 0.25 m, we can see from the calibration curve that the momentum current has a value of 4 units.

Now we will take another object to show the relationship between lengthening and momentum current: a steel spring. The result is shown in Fig. 3.54. The relation here is simpler than with the elastic cord: It is linear. The spring's lengthening s and momentum current F are proportional to each other. One says that the spring follows *Hooke's law*, which can be formulated as follows:

$$F = D \cdot s$$

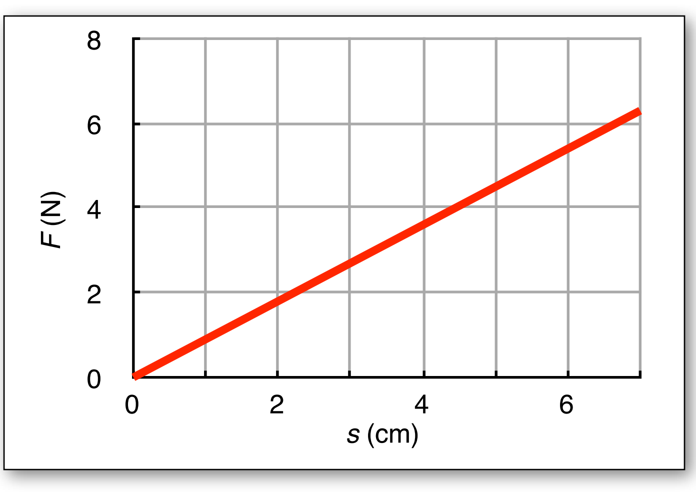


Fig. 3.54
In a steel spring, the relation between momentum current and stretching is linear.

D is a constant for a given spring. It is called the *spring constant*. Its unit is N/m. In general, the spring constant has different values for different springs. Fig. 3.55 shows the relation between F and s for two different springs. D has a greater value for spring A than for spring B. If spring A and spring B are both stretched by the same amount, the momentum current in A is greater than the one in B. The spring with the higher spring constant is the harder spring.

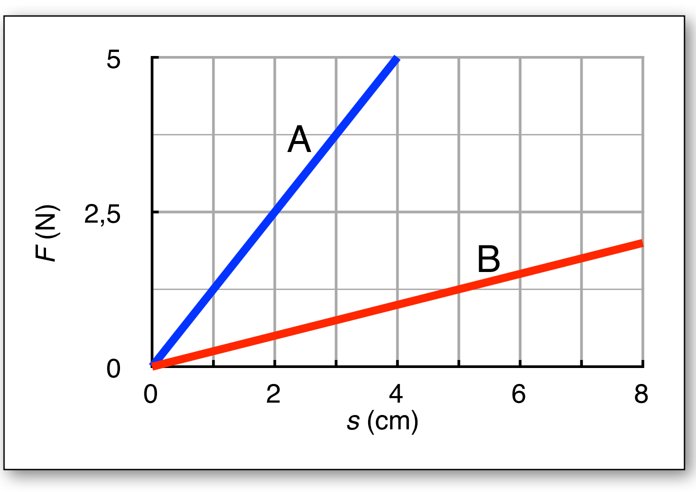


Fig. 3.55
The spring constant of spring A is greater than that of spring B. Spring A is harder than spring B.

Many springs can have tensile stress as well as compressive stress. Hooke's law holds for such springs, meaning the linear relation between change of length and momentum current, for lengthening (positive values of s) as well as for shortening (negative values of s).

Exercises

1. A spring's spring constant is D = 150 N/m. How much does it lengthen when a momentum current of

a) 12 N

b) 24 N

flows through it?

2. The F-s relation for the rope represented in Fig. 3.56 is measured.

a) By how much does the rope stretch when a momentum current of 15 N flows through it? How much will it stretch for a momentum current of 30 N?

b) If the rope lengthens by 20 cm, what is the momentum current?

c) What does one feel when one pulls the rope at each end with both hands? Compare it to the steel spring.

3. How would a device be made whose F-s relation looks like the one in Fig. 3.57?

4. Two springs are hooked up to each other and inserted into a rope through which a momentum current flows. One spring stretches four times as much as the other. What is the relation between the two spring constants?

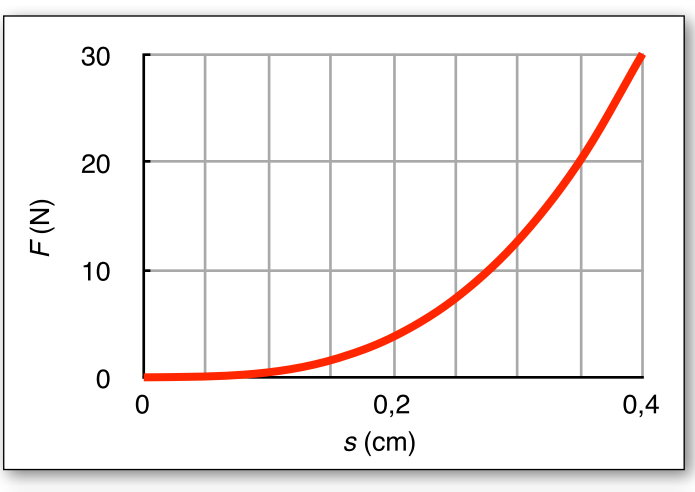


Fig. 3.56
For exercise **2**.

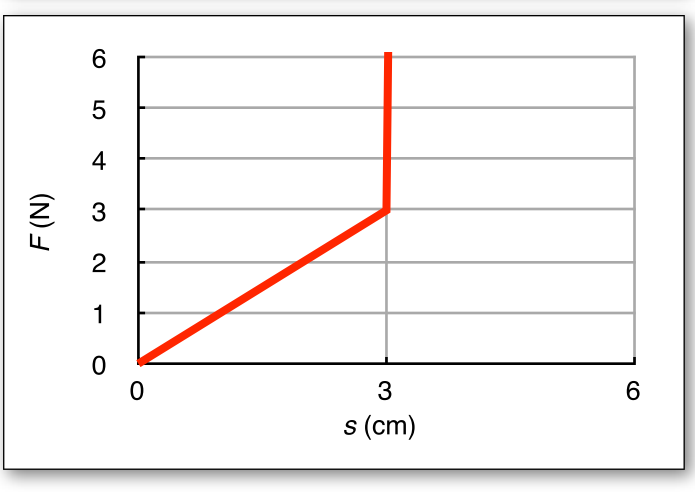


Fig. 3.57
For exercise **3**.

3.13 Momentum currents can be destructive

If a momentum current becomes to great, the conductor it flows through can break, Fig. 3.58. Mostly this is undesirable. However, sometimes one wishes to break, tear or crumble something. We will discuss examples of both cases.

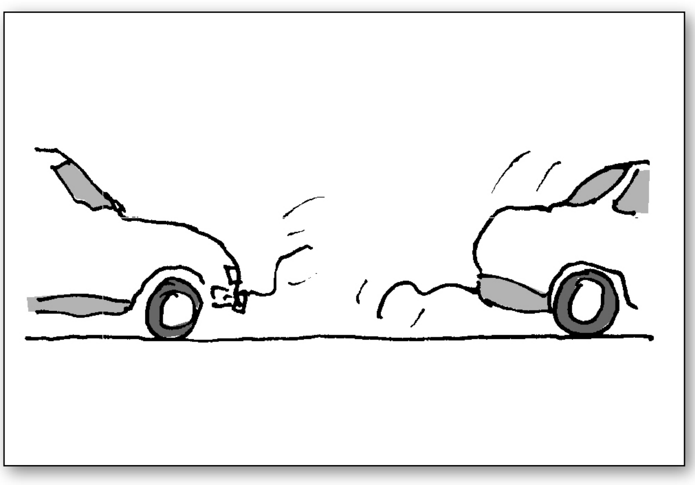


Fig. 3.58
If the momentum current is very large, the conductor can break.

Towing a car

The towing rope in Fig. 3.58 has torn. How could this have been avoided? Try to bring a heavy car up to a certain speed by pulling on it with a thin thread. If you pull too hard, i.e., if you let a momentum current flow that is too strong, the thread will snap. However, it is possible to charge the car with the desired amount of momentum. One must allow a momentum current that is weak enough but flows over a longer time span. In other words: You need to pull less strongly, but for longer. In the case of towing a car this means that one must start off slowly so that the momentum current in the rope is not too great.

Catching a stone

A stone hitting a window pane transfers its momentum to the window pane very quickly. The current is very strong, and the glass breaks. If you catch a stone with your hands, you follow the motion of the stone with your hands while stopping it. In doing so, the duration for the momentum to flow out of the stone is increased and the momentum current is decreased. No damage is done.

A hammer

Sometimes one wishes to break something, a brick, perhaps. A hammer could be used for this. At first, the hammer is charged with momentum relatively slowly when it is put into motion by a hand. When it hits the stone, its momentum flows in a short span of time. The momentum current is very strong and the stone breaks.

Whether or not a momentum current is destructive does not only depend upon its strength. It is clear that it is possible to avoid the tearing of the towrope in Fig. 3.58 by other means, say by using a thicker rope. One sees that for tearing, not only a strong momentum current but a strong momentum current through a small cross sectional area is decisive, and not just a strong momentum current. Rope A in Fig. 3.59 tears when a current of 50 N flows through it. Rope B, with double the cross sectional area, does not tear. This is easy to understand. If rope A has a cross sectional area of 1 cm^2 and rope B has one of 2 cm^2, one can imagine rope B as two parallel ropes of 1 cm^2 cross sectional area each where 25 N flows through each one. They are stressed less strongly. The stress of the material of a conductor can be reduced by making the conductor thicker. Here are some more examples of this.

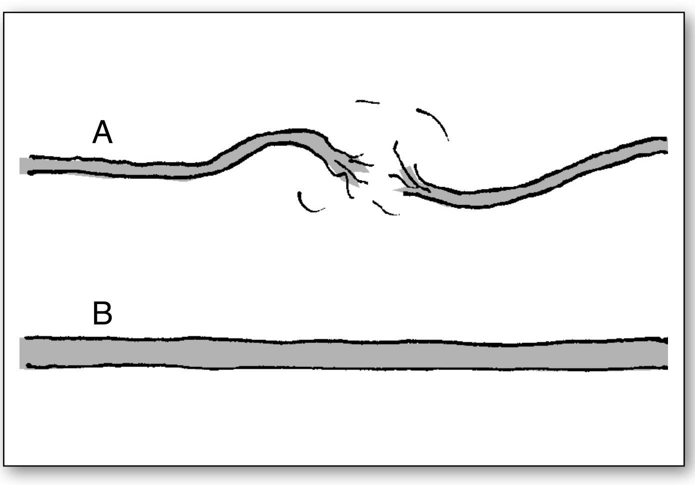


Fig. 3.59
The cross sectional area of rope B is double that of rope A.

Nails, thumb tacks, knives, chisels

These can be used to destroy things (making a hole in a wall is a kind of destruction). In each case, a momentum current is conducted through a tip or other narrow point into the material being worked upon. The stress of the material at this point is so great that it breaks.

Seat belt and air bag

A car comes to an abrupt stop during an accident. It transfers its momentum very quickly: to a tree, a guard rail, or another car. The people in the car also have momentum and must get rid of it during the accident. The strong momentum currents which flow during the process lead to the destruction of the car and injuries of the passengers involved. In vehicle construction, a so-called crush zone is included in order to reduce some of the momentum currents. At impact, the car folds up somewhat. As a result, the process of transfer of momentum is prolonged, and the momentum currents become weaker.

Seat belts have several functions.

First, they stretch out somewhat at impact. In the process, the transfer of momentum from passenger to the car is prolonged and the current is made smaller.

Second, because seat belts are wide, the momentum current flowing out of the person is distributed over a large surface. As we have seen, the destructive effects of the currents are reduced by this kind of momentum distribution. Without the belt, the passengers might be thrown against some pointed object inside the car.

Finally, the momentum currents are conducted through parts of the passenger's body that are less in danger of being injured. It would be much worse if the passenger transferred his momentum via his head.

The situation is even better with an air bag. The surface to which the passenger's momentum flows is still greater.

Exercise

A car needs to tow another one. A momentum current of 2000 N may be expected for accomplishing this. Unfortunately, the drivers do not have a towrope with them. They finally locate a big roll of string that can take a momentum current of only 100 N. What do you suggest they do?

3.14 Velocity

The physical quantity that indicates how fast a vehicle or anything else moves, is called *velocity*, abbreviated to v.

The driver of a car must always know how fast he is driving, he must know the velocity of his vehicle. For this purpose, every car has a measuring device for velocity: the *speedometer*. It shows the velocity in the unit of kilometers per hour, abbreviated to km/h.

Fig. 3.60 shows the record of a tachograph: The velocity of a truck is automatically recorded over time. We will try to interpret the diagram. The truck moves off at time t = 0 minutes. After 4 minutes, it must stop shortly and after 9 minutes, this happens again. Probably it came to traffic lights on red. From the 12th to the 16th minutes, it drove rather slowly, at 35 km/h. Maybe it was traveling uphill or there was a lot of traffic. From the 18th minute onwards, it drove at a constant high speed of 85 km/h. Apparently, it had left the city limits.

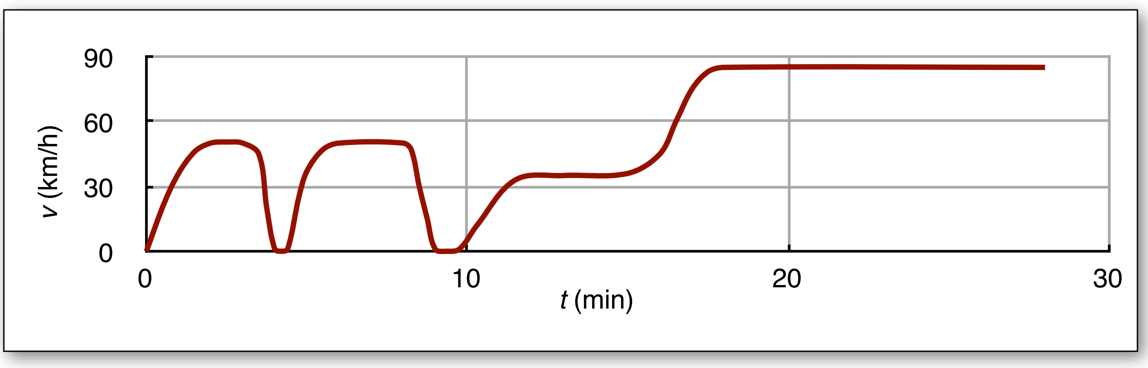


Fig. 3.60
Tachograph record of a truck. The speed is given as a function of time.

As long as a body moves at constant speed, there is a simple relation between the velocity, the distance to be covered and the time needed to cover it.

If a car moving at a constant velocity needs half an hour to cover 60 km, it needs 0.75 hours to cover 90 km, 1 hour to cover 120 km, 2 hours to cover 240 km, etc. (see table 3.5). The distance s is proportional to the time t:

$$s \sim t$$

s in km	t in h	s/t in km/h
60	0,50	120
90	0,75	120
120	1,00	120
180	1,50	120
240	2,00	120

Table 3.5
Covered distance, time necessary to cover that distance, and quotient of distance and time for a car that moves with constant velocity.

In Fig. 3.61, the relation is represented graphically. Another way of expressing the same facts would be: The quotient s/t is constant. It is

$$\frac{60\text{ km}}{0{,}5\text{ h}} = \frac{90\text{ km}}{0{,}75\text{ h}} = \frac{120\text{ km}}{1\text{ h}} = \frac{240\text{ km}}{2\text{ h}} = \ldots$$

and this quotient equals the velocity v = 120 km/h. Therefore, in the case of constant velocity, we can write:

$$v = \frac{s}{t}$$

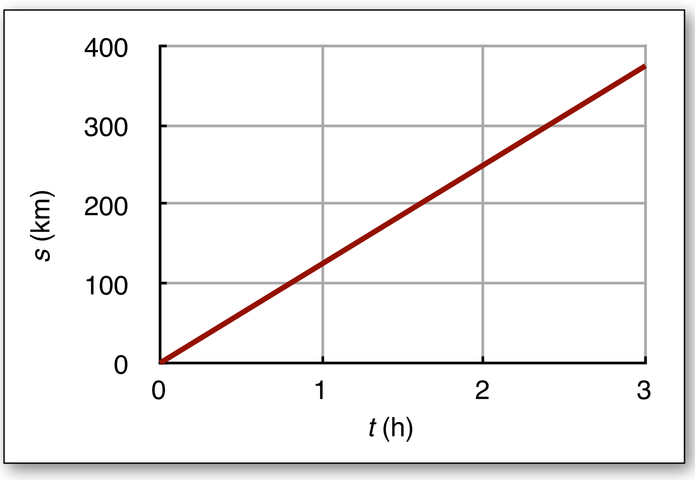


Fig. 3.61
Distance-time relation of a car

Similarly to various other quantities, velocity can have many units. For example, a car's velocity is given in km/h, a ship's in knots. The internationally agreed upon physical unit is meters per second, abbreviated to m/s.

We convert the unit km/h into m/s:

$$1\frac{\text{km}}{\text{h}} = \frac{1\text{ km}}{1\text{ h}} = \frac{1000\text{ m}}{3600\text{ s}} = 0{,}2778\text{ m/s}.$$

Exercises

1. A bicyclist needs 40 minutes to cover a distance of 10 km. What is his velocity (in km/h)?
2. A train travels at constant velocity for 1 h 32 min covering a distance of 185 km. What is its velocity? Give the result in km/h and m/s.
3. A car drives at 90 km/h for 10 minutes. How many km does it cover in this time?
4. An airplane traveling at 800 km/h flies 1600 km. How long is the flight?
5. The speed of light is 300,000 km/s. The distance from the Earth to the sun is 150,000,000 km. How long does light from the sun take to reach the Earth?

3.15 The relation between momentum, mass, and velocity

The heavier and the faster an object is, the more momentum it has. This sentence makes a statement about the relation between three physical quantities: momentum *p*, mass *m* and velocity *v*. We will now investigate this relationship by looking for a "quantitative" expression.

We ask about the dependence of momentum upon the two other quantities. Solving our problem will be much easier if we separate it into two parts. First we will investigate how the momentum relates to the mass of the object in question. After that, we will look into how it depends upon the velocity.

In order to obtain the influence of mass upon momentum we consider several bodies of varying masses, all moving at the same velocity. Our problem becomes clear if we choose bodies like those shown in Fig. 3.62. Body A is a glider on an air track and body B is composed of two coupled gliders each of which is exactly as heavy as body A. The mass of B is double that of A:

$m_B = 2 \cdot m_A$

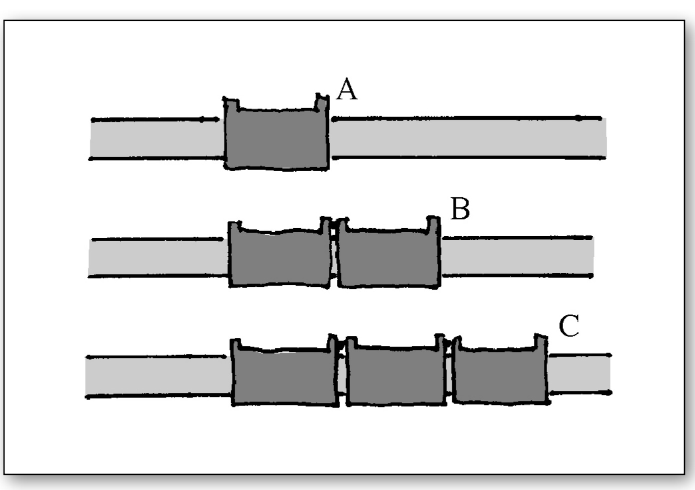


Fig. 3.62
Body B has twice, body C three times the mass of body A. B also has twice, and body C, three times the momentum of A.

Body C is composed of three such gliders, it therefore has three times the mass of A:

$m_C = 3 \cdot m_A$

We could continue in this way with more bodies of four times the mass of A, five times the mass of A, etc. Now A, B, C, etc., are all moving at the same velocity. How do their momenta relate to each other? Body B is nothing more than two coupled bodies A. If A has momentum p_A then B must have twice the momentum p_A:

$p_B = 2 \cdot p_A$

C is composed of three coupled A-type bodies. Each of these moves exactly as fast as body A, meaning that C must have three times the momentum of A:

$p_C = 3 \cdot p_A$

We recognize the following relationship: The momenta of two bodies differ by the same factor as their masses. In other words, momentum and mass are proportional:

$p \sim m \qquad \text{at } v = \text{const}$

This was the first of the relations we are looking for. We will need to do a bit more to find our second one, the relationship between momentum and velocity.

Our approach will be to lower the momentum of a body to half and then measure by how much the velocity changes. Next, we will lower the momentum down to a third and check again how *v* changes, etc. Fig. 3.63 shows how the experiment looks.

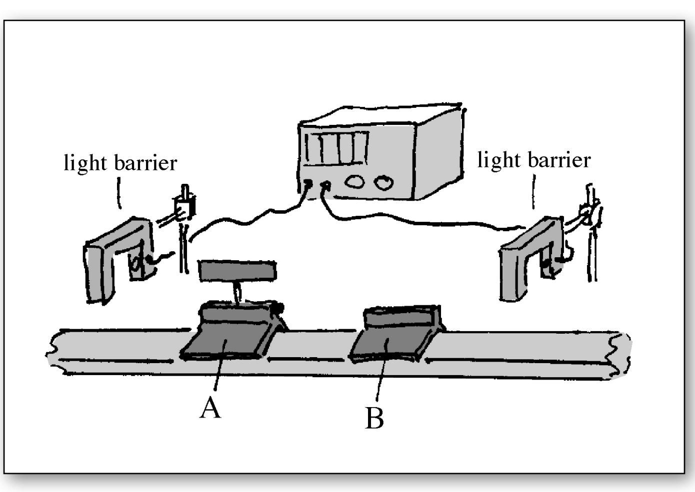


Fig. 3.63
The momentum of body A decreases by half at impact. Measurement shows that the velocity also reduces to half.

Body A moves to the right and towards body B, which is at rest. A hits B and couples to it so that A and B, together, move further to the right. We can measure the velocity of A before and after impact. (After impact, it is naturally exactly as great as that of B.) We now seek the momentum and velocity values of body A both before and after impact.

We call the momentum of A before impact p_i (*i* means initial), and its momentum afterwards p_f (*f* means final). Because at impact, p_i distributes evenly over both bodies A and B, body A has exactly half as much momentum after impact as it did before impact. Therefore:

$p_f = (1/2) \cdot p_i$

The velocities before and after the collision are found by experiment. It demonstrates that the velocity v_f after the impact is half that of the velocity v_i before it:

$v_f = (1/2) \cdot v_i$

If a body A is allowed to collide with two bodies B and C at rest, Fig. 3.64, the momentum distributes over all three bodies and we have

$p_f = (1/3) \cdot p_i$

In this case measuring velocities yields

$v_f = (1/3) \cdot v_i$

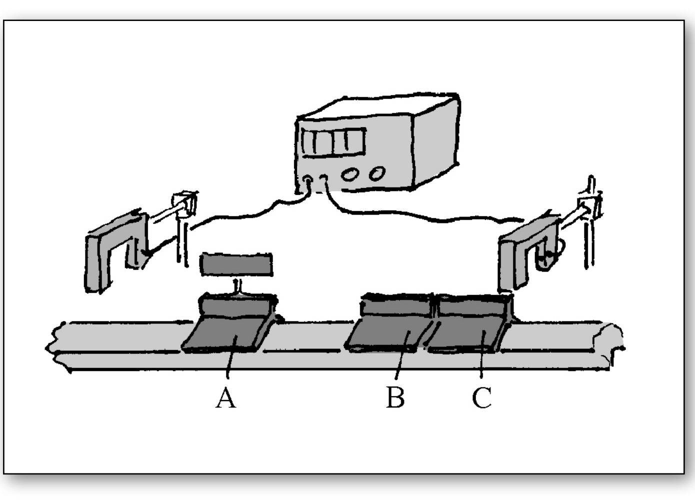


Fig. 3.64
At impact, momentum and velocity of body A both reduce to one third of their initial values.

We conclude from this that for a body, i.e., for constant mass, momentum and velocity are proportional:

$p \sim v \qquad \text{for } m = \text{const}$

Now we have the two relations individually: the one between *p* and *m,* as well as the one between *p* and *v*. We will write them once again:

$p \sim m \qquad \text{for } v = \text{const} \qquad (1)$

$p \sim v \qquad \text{for } m = \text{const} \qquad (2)$

Mathematics tells us that we can combine these two relations into one:

$p \sim m \cdot v \qquad (3)$

The correctness of this proportionality can be seen in that the two relations (1) and (2) result from it. If one leaves *v* constant and only changes *m*, then relation (1) results from (3). If, on the other hand, *m* is left constant and *v* is changed, then the result is (2).

We still cannot calculate the momentum of a body from its mass and its velocity by using (3). To do this, we still need a factor of proportionality in (3). We are lucky here: Actually we don't need such a factor because the unit of momentum (Huygens) sets this factor to one if we set the mass in kg, and the velocity in m/s. Therefore:

> $p = m \cdot v$

This is our desired result. Now we have a very useful formula. We can calculate the momentum of a body if we know its mass and velocity. Mass and velocity are easily measured quantities. Thus, we have learned a simple method of determining values of momentum. Remember that in this formula, momentum has the unit Hy only if the mass is given in kg and the velocity in m/s.

> The momentum of a body is proportional to its mass and its velocity.

We consider the equation $p = m \cdot v$ from a different viewpoint. Imagine two bodies A and B with strongly differing masses:

$m_A = 1$ kg

and

$m_B = 1000$ kg.

Each of these bodies is given 1 Hy of momentum. How do the bodies react? Of course, they are set in motion, but differently. From

$p = m \cdot v$

follows

$$v = \frac{p}{m}\ .$$

The resulting velocity for body A is

$$v_A = \frac{p}{m_A} = \frac{1\ \text{Hy}}{1\ \text{kg}} = 1\ \text{m/s}$$

and for body B it is

$$v_B = \frac{p}{m_B} = \frac{1\ \text{Hy}}{1000\ \text{kg}} = 0.001\ \text{m/s}\,.$$

A is 1000 times as fast as B. It is easier to set a body of small mass in motion than one with more mass. A general statement about this would be:

> It is easier to change the velocity of a body with small mass than to change the velocity of a body with greater mass.

We also say that a heavy body has more *inertia* than a light one.

> The mass of a body is responsible for its inertia.

Exercises

1. A truck weighing 12 t (12,000 kg) travels at a velocity of 90 km/h. What is its momentum?

2. A goalie catches a ball which comes at a velocity of 20 m/s. What is the momentum that flows over the goalie into the ground? (The ball weighs 420g.)

3. A tennis ball is hit at a right angle against a wall. Its velocity is 30 m/s. What is the momentum that flows into the wall? (The tennis ball weighs 50g.)

4. A person accelerates a car with good bearings by pulling on it. A force sensor shows the momentum current flowing into the car. The person pulls for 5 seconds. What is the resulting velocity? (The car weighs 150 kg, the force sensor shows 15 N.)

5. A locomotive pulls a train. A momentum current of 200 kN flows through the coupling between the locomotive and the train cars. What is the train's momentum (without the locomotive) after 30 seconds? The train now has a velocity of 54 km/h. What does the train weigh?

6. A wagon at rest and weighing 42 kg is accelerated. The momentum current through the rod pulling the wagon is 20 N. How much momentum flows into the wagon in 3 seconds? At this point, its velocity is 1.2 km/s. What is its momentum? Where did the missing momentum go?

7. Water flows at a speed of 0.5 m/s in a long straight pipe having a length of 2 km and a diameter of 10 cm. The water is blocked off at one end by a valve. Calculate the momentum given up by the water. Where does this momentum go? The blocking lasts 2 s. What is the force of the water upon the valve (the momentum current)? Hint: First calculate the water volume in liters. 1 l of water has a mass of 1 kg.

3.16 SI-units

In the part of physics that you have gotten to know, the statement we made about mechanics has proven true: Physical quantities are needed in order to describe the world physically. An important goal of physics, if not the most important goal, is to find relationships between these quantities.

In Table 3.6, we have put together some of the quantities we have encountered so far.

Name of the quantity (symbol)	SI-unit (symbol)
pressure (p)	Pascal (Pa)
energy (E)	Joule (J)
energy current (P)	Watt (W)
time (t)	seconds (s)
momentum (p)	Huygens (Hy)
momentum current (F)	Newton (N)
velocity (v)	meters per second (m/s)
distance (s)	meter (m)
mass (m)	kilogram (kg)

Table 3.6
Names and SI-units of some physical quantities and their abbreviations

You know that every quantity has a unit. Most quantities, though, have more than one unit, Table 3.7. There are various reasons for one and the same quantity having more than one unit. Often, different units of measuring were defined within the different fields of science, technology, or handicrafts: A tailor uses the cubit, the plumber, the inch, and the physicist, the meter. People agree on one unit in one country perhaps, but unfortunately, in other countries there will be other units. In most of Europe, mass is measured in kilograms, but in the USA the pound is used. The *Système International* was finally agreed upon as the binding system for units. According to this system, each quantity (with few exceptions) has only one unit. We call these units *SI-units*.

Name of the quantity	Units
pressure	Pascal, bar, atmosphere
energy	Joule, calorie
energy current	Watt, horsepower
time	second, minute, ... year
momentum current, force	Newton, dyne
velocity	meters per second, kilometers per hour, knots
distance	meter, inch, light year
mass	kilogram, pound

Table 3.7
Most physical quantities have beside the SI-unit other units.

The units in Table 3.6 behind the names of the quantities are SI-units.

Use of SI-units has not only the advantage of making international communication easier. It is also a unit system that makes physical formulas as simple as possible. If the values of the quantities on the right side of the formulas that you already know are entered in SI-units, the result (meaning the value of the quantity on the left) is also in SI-units. If, on the other hand, the values on the right were inserted in different units, the result would probably be in a unit not commonly used. We will look at two examples.

The equation $P = E/t$ can be used to calculate the energy current from energy and time. If the energy is given in Joules, and the time in seconds, the resulting energy current is Joules per second. Now, 1 J/s is equal to 1 Watt. We therefore obtain the SI-unit Watt. If we had given the energy in calories and the time in minutes, we would have gotten calories per minute. This would have been a totally inappropriate unit.

We have learned the following:

> If you want to solve a problem, and the initial values are not in SI-units, convert them to SI-units immediately.

4

The gravitational field

4.1 Vertical motion

In the following sections we will deal with the concepts of gravity, gravitational force, and objects that fall to the ground. Previously, we looked at horizontal motion, now we will consider vertical motion. Moreover, everything we learned about horizontal movement we can transfer to our description of vertical movement. We need only to turn our *x*-axis 90 degrees so that it is vertical and its positive side points downward. This means:

A body's momentum is positive when the body moves downward, and negative when it moves upward.

In section 3.8 we learned the following rule:

Momentum current to the right: compressive stress

Momentum current to the left: tensile stress

Because what was to the right is now down, and what was to the left, is now up, the new rule is:

Momentum current downward: compressive stress

Momentum current upward: tensile stress

We take the closed momentum current circuit of Fig. 4.1 as an example.

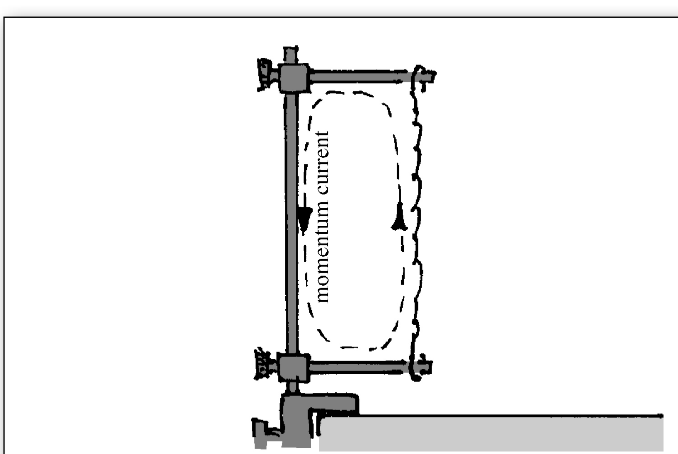


Fig. 4.1
Closed momentum current circuit with vertical *x*-axis.

4.2 Gravitational attraction – the gravitational field

Every object is attracted by the Earth. Two phenomena show this:

1. If you take an object into your hand and then let it go, it drops downward.

2. Every object has a weight.

Both of these phenomena show that the object receives momentum from the Earth. A falling body gets faster the longer it falls. Its momentum increases.

A body that is not falling also gets momentum. This can be seen, for example, if it is hung from a spring scale, Fig. 4.2. The spring shows that a continuous momentum current flows out of the body, over its mounting and into the Earth. The momentum must be supplied continuously. There is a constant momentum current flowing into the body. However, it flows through a connection between the body and the ground which is invisible.

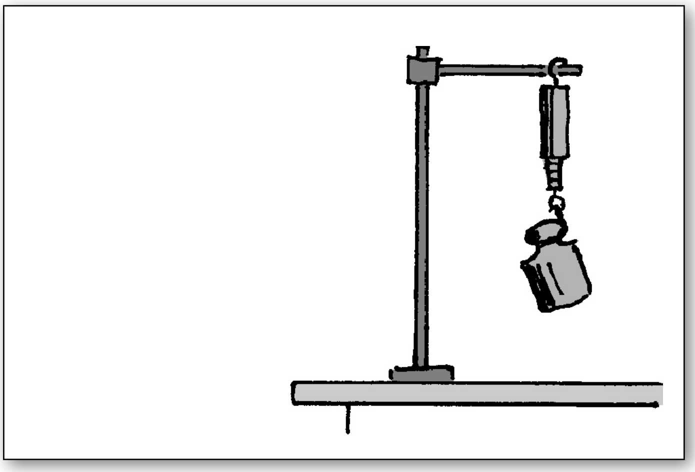


Fig. 4.2
The momentum which continuously flows through the spring scale and the mounting into the ground, reaches the body through an invisible connection.

We have already gotten to know a similar momentum conductor, a connection that cannot be seen: the magnetic field. In the case we want to consider at the moment though, there cannot be a magnetic field, because that would mean that only magnets or iron bodies would be attracted to the Earth. The connection is composed of an entity that is not a magnetic field but is similar to a magnetic field. It is called a *gravity field,* or *gravitational field*. All entities that have a mass, that means all bodies, are surrounded by a gravitational field in exactly the same way that a magnetic pole is surrounded by a magnetic field. The greater the mass of the body, the denser this field is.

Every body is surrounded by a gravitational field. The greater the mass of the body, the denser the field. Momentum flows from one body to another through the gravitational field. The Earth's gravitational pull results from the momentum current from the Earth to the object in question.

4.3 What the Earth's gravitational attraction depends on

Let us try an experiment. First we hang a piece of iron with a mass of 1 kg on a spring scale, i.e, a force sensor, and then we hang a 1 kg piece of wood on it. The force sensor shows the same thing both times. Is this a surprise? Of course not. How does one find out whether a piece of iron or a piece of wood has a mass of 1 kilogram, anyway? By putting it on a scale. Most scales function like our spring scale. Using the scale or the force sensor, we can define what we mean by two equal masses: If two bodies show the same reading on a force sensor, they have the same mass.

We can say this in another way as well: If the momentum currents flowing out of the Earth and into two bodies have the same strength, the bodies have the same mass.

We now take two bodies, each with a mass of 1 kg. If we put them together, we can consider them as one body with a mass of 2 kg. A momentum current flows into both of them which is twice as strong as the current that would flow into just one of them. This may seem obvious to you. You could probably imagine, though, that the momentum current into the first body is influenced by the addition of the second body.

How strong is the momentum current flowing into the 1 kg body? The force sensor shows that it has a strength of about 10 N. A more exact measurement gives us 9.81 N. Accordingly, 2 · 9.81 N = 19.62 N flows into a body of 2 kg mass, and 98.1 N flows into a body of 10 kg mass. Again, we are dealing with proportionality: the strength of the momentum current flowing from the Earth into a body is proportional to the mass of the body:

$F \sim m$

The factor of proportionality has a value of 9.81 N/kg:

$F = m \cdot 9.81$ N/kg

We are not quite finished with our considerations. A kilogram of iron weighs exactly as much as a kilogram of wood, but a kilogram of iron doesn't weigh as much on the Moon as it does on Earth. We will do the following thought experiment. We take an object with a mass of 1 kg and weigh it at different locations: here at home, at the north pole, at the equator, on the Moon, on Mars, on the surface of the Sun and finally, on a neutron star. The results are compiled in Table 4.1.

Location	*g* in N/kg
Central Europe	9.81
North and South Poles	9.83
Equator	9.78
Surface of the Moon	1.62
Surface of Mars	3.8
Surface of the Sun	274
Surface of a neutron star	1 000 000 000 000

Table 4.1
Values of the standard gravitation at various locations

At every location, the following proportionality is valid

$F \sim m$.

However, the factor of proportionality has a different value for each location. The values vary only slightly at different locations on the surface of the Earth. They deviate greatly, though, on the other celestial bodies from those on Earth. For this reason, we will write the relation between F and m in a general form

$$F = m \cdot g$$

The proportionality factor g is dependent upon where the body having the mass m is to be found. It is called the *gravitational field strength..*

The momentum current from the Earth into a body equals the product of the mass of the body, and the gravitational field strength. The strength of the gravitational field on the surface of the Earth is 9.81 N/kg ≈ 10 N/kg.

Here is a description of the Earth's gravitational pull in the force model: The quantity F is called *force of gravity* or *weight*. One can also say that the *gravitational force* acts upon a body.

What do we actually mean when we say that an object is very heavy? It probably means that it is difficult to lift it off the ground. Do we mean that it has a big mass? Probably not. On the Moon it wouldn't be difficult at all to lift this "heavy" object. The word "heavy" rather means that a strong momentum current is flowing into the body. In other words, the gravitational force acting upon it is great. The same object can be light or heavy depending upon where it happens to be.

Exercises

1. What is the momentum current that flows out of the Earth into your own body? (What is the gravitational force acting upon your body?) What would this momentum current be on the Moon? What would it be on a neutron star?

2. During an expedition to the Moon, astronauts using a force sensor determine the gravitational force acting upon a body. They find that F = 300 N. What is the body's mass?

4.4 Free fall

We take an object into our hands and let go of it. It falls to the ground. We can now explain this phenomenon: A momentum current of $m \cdot g$ flows into the object. Therefore, its momentum continuously increases. The longer it falls, the faster it falls.

Something here is odd, though. If two objects, one heavy and one light, are let go at the same time from the same height, they reach the ground at the same time. Shouldn't the heavier one hit the ground earlier since it receives more momentum from the Earth?

We will calculate how the momentum of the two bodies increases. We assume that the mass of the heavier body is 4 kg, and that of the lighter one is 1 kg. We insert

$F = m \cdot g$

into

$p = F \cdot t$

and obtain

$$p = m \cdot g \cdot t \quad (1)$$

Here we insert the mass and the gravitational field strength for the heavier body and get

$p = 4 \text{ kg} \cdot 10 \text{ N/kg} \cdot t = 40 \text{ N} \cdot t$

and for the light body

$p = 1 \text{ kg} \cdot 10 \text{ N/kg} \cdot t = 10 \text{ N} \cdot t$.

These *p-t*-relations are represented in Fig. 4.3. The figure shows that the momentum of both objects increases uniformly. However, the momentum of the heavier body increases more quickly than the momentum of the light one. At any given moment, the heavier one has four times the momentum of the lighter one.

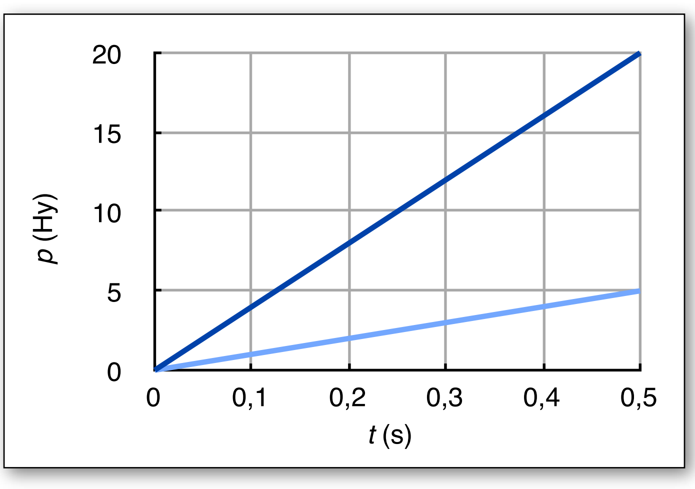


Fig. 4.3
Momentum as a function of time for two falling bodies of different weights.

Why do both the bodies fall at the same velocity, then? To answer this question, we need the formula

$$p = m \cdot v. \quad (2)$$

We conclude from it that four times the momentum is needed in order to bring the heavy body up to a certain velocity than is needed to bring the light one to the same velocity. The body with greater mass has greater inertia than the one with smaller mass.

A simple calculation yields the same result. We equate the right sides of (1) and (2) and obtain

$m \cdot g \cdot t = m \cdot v$

Dividing both sides of this equation by m yields

$$v = g \cdot t \quad (3)$$

This equation demonstrates that the velocity of a falling body increases uniformly. Because the mass plays no role anymore, it also tells us that the velocity with which a body falls, is not dependent upon the body's mass. In Fig. 4.4, the speed of an arbitrary freely falling object is represented as a function of time.

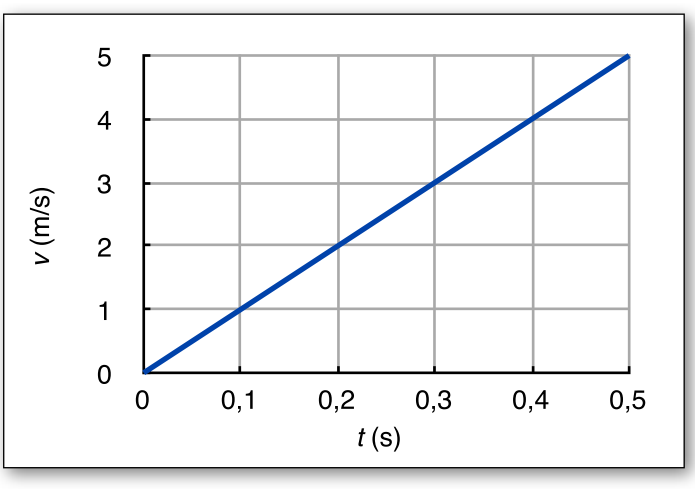


Fig. 4.4
The velocity of a freely falling body increases linearly with time.

The fact that the gravitational field strength appears in Equation (3) means that the velocity of free fall depends upon where the falling object is located. For example, objects on the Moon fall six times as slowly as on Earth.

We have so far assumed that a falling body gets momentum only from the Earth and that it does not lose momentum while falling. We have rather simplified the situation by making this assumption. A falling body actually does lose momentum through friction with the air. If a body is not too light and falls only a short distance, then our simplification is justified. This situation is called *free fall*. However, if a body is very light and has a large surface as well, our considerations become invalid.

> Three rules for falling bodies:
>
> If a body A has twice the mass of a body B, it receives double the amount of momentum per second from the Earth. It also needs twice the momentum to reach the same velocity as body B.
>
> The velocity of falling bodies increases uniformly.
>
> All bodies fall equally fast.

We will now consider a variant of free fall. We don't let an object fall from a state of rest, but we throw it vertically up into the air. At the beginning, it has negative momentum. Like before, it constantly receives new positive momentum from the Earth, leading to a gradual reduction of its negative momentum. It becomes slower and slower, comes to rest and finally begins to move in the positive direction (downward).

Upward motion is the mirror image of downward motion. In the process of falling down, the momentum of an object increases uniformly. In the process of moving upward, the negative momentum of an object decreases uniformly. The equivalent is true for the velocity: The negative velocity decreases linearly with time during upward movement, and when an object falls downward, its (positive) velocity increases linearly with time.

Fig. 4.5 shows the velocity as a function of time. We have set the point of reversal as the zero point of the time axis. The toss up into the air takes place at the time "minus 0.4 seconds". One sees in the figure that the object needs the same amount of time to fly upward as it does to fall back down.

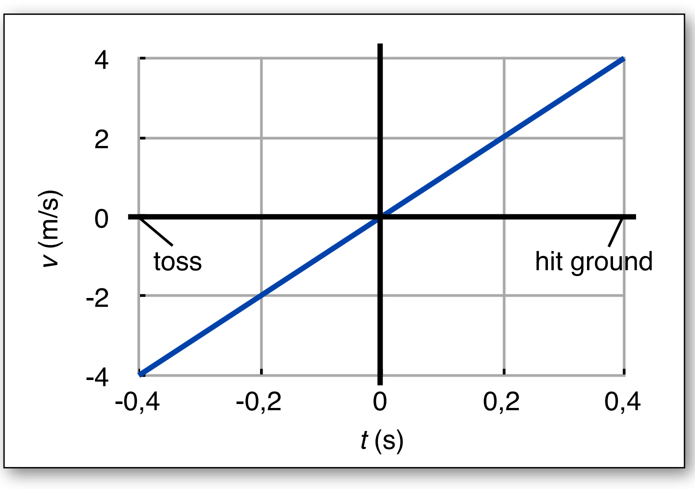


Fig. 4.5
The velocity of a body thrown upward. During upward motion the velocity is negative, for downward motion, it is positive.

Exercises

1. You jump from a 3-meter diving board into water. The free fall lasts 0.77 s. What is your momentum when you hit the surface of the water? What is your speed?

2. What is the velocity of a freely falling body after 1/2 seconds on Earth, on the Moon and on the Sun?

3. A stone is thrown upward. Its initial speed is 15 m/s. How long does it take to hit the ground?

4. A stone is shot upward by a slingshot. After 5 seconds it hits the ground. What was its initial speed?

4.5 Falling with friction

We cannot always ignore friction. How great it is depends upon

1. the form of the body
2. the velocity of the body.

You already know from cars that:

1. One tries to keep air friction to a minimum by designing the car body accordingly.
2. If you drive fast, friction and gas usage (per kilometer) are much higher than if you drive slowly.

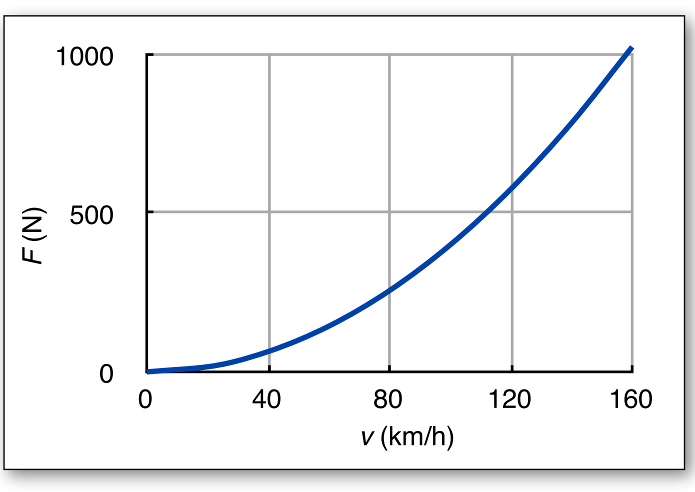


Fig. 4.6
Air resistance for a typical automobile: The momentum current flowing into the air as a function of the velocity.

Figs 4.6 and 4.7 show that friction, meaning the momentum current that flows into the air, grows very fast with increasing speed. Both figures show the momentum loss due to friction as a function of speed. In Fig. 4.6, this is for a typical automobile, and in Fig. 4.7, for a much smaller object, a ball with a diameter of 30 cm.

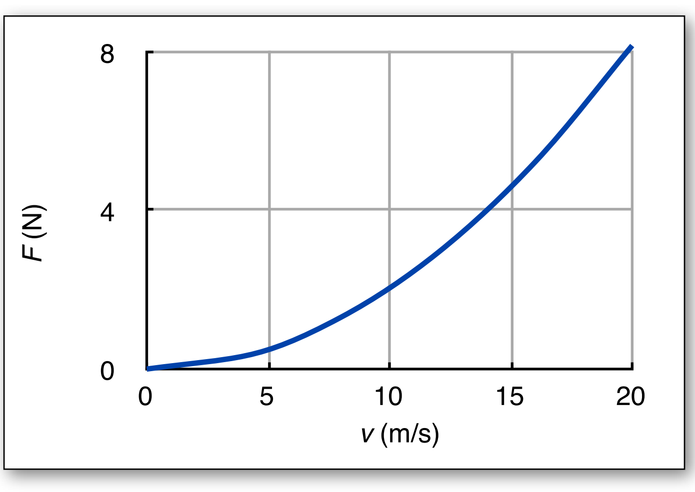
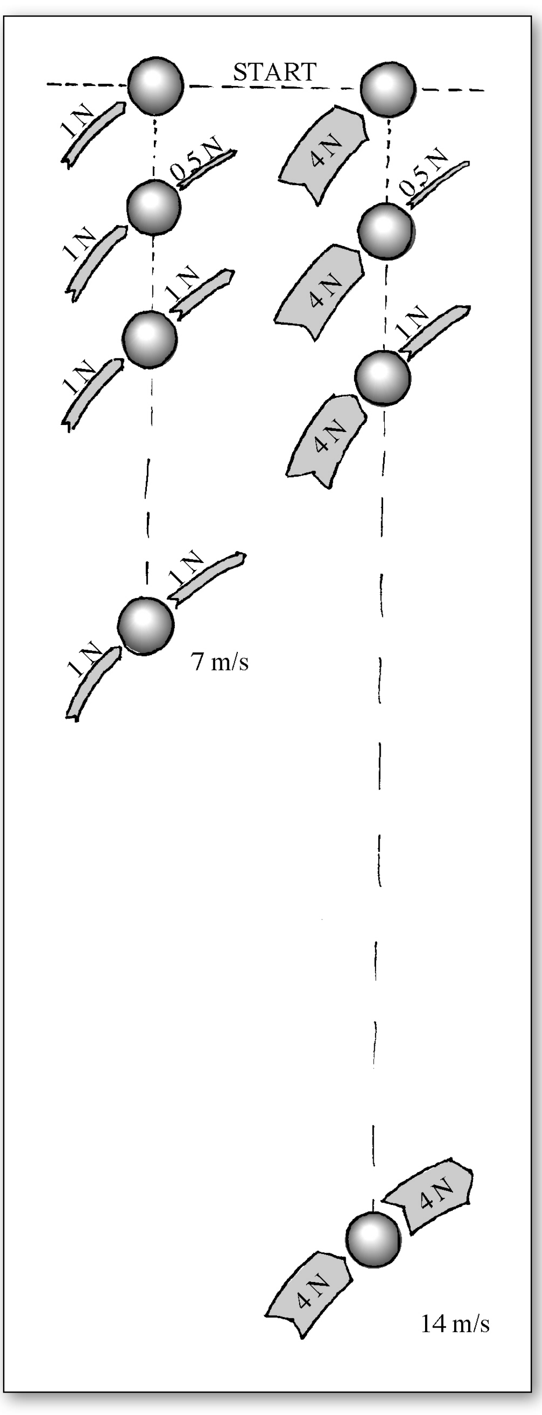


Fig. 4.7
The momentum current which flows off into the air as a function of velocity, in the case of a sphere with a diameter of 30 cm.

We have seen that if we didn't have this friction loss, or as long as it could be ignored, all bodies fall at the same velocity. How does the velocity of a falling body behave when friction cannot be ignored any longer?



We let a large and very light ball fall (Fig. 4.8, on the left). Its mass is m = 100 g = 0.1 kg, and its diameter is 30 cm = 0.3 m. A momentum current flows continuously from the Earth into the ball. It is

$F = m \cdot g$ = 0,1 kg · 10 N/kg = 1 N.

When it starts to fall, its speed is small as is its momentum loss to the air. At a speed of 2 m/s, the momentum current flowing into the air is less than 0.1 N (see Fig. 4.7). The loss is still small compared to the momentum current of 1 N coming out of the Earth. However, the loss quickly becomes greater until finally the ball loses exactly as much momentum per second to the air as it receives from the Earth. From this point on, its momentum does not increase anymore. Fig. 4.7 shows that the ball then has a velocity of about 7 m/s.

Fig. 4.8
A light sphere (left) and a heavy sphere (right) fall to the ground. The light one reaches its terminal velocity earlier than the heavy one.

Fig. 4.9 shows the velocity of our ball as a function of time: At the beginning, its velocity increases linearly with time. It behaves like a freely falling ball. Gradually the loss becomes greater though. Finally, when the currents flowing into and out of it become equal, its momentum and therefore its velocity, do not increase any longer. It has reached its final or *terminal velocity.* The ball is now in a *steady state.*

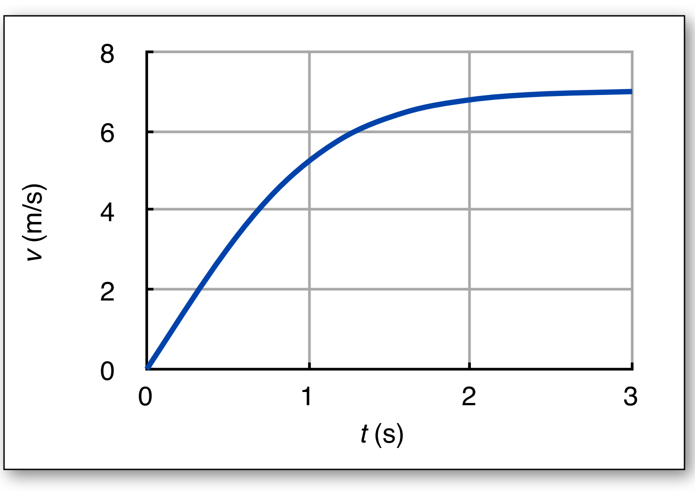


Fig. 4.9
If there is air friction, the velocity of a falling body increases until it reaches a terminal velocity.

We let another ball fall. It has the same diameter (30 cm), but it is four times as heavy as the first ball, Fig. 4.8, on the right:

m = 0.4 kg.

A momentum current of

$F = m \cdot g$ = 0.4 kg · 10 N/kg = 4 N

flows from the Earth, through the gravitational field, and into the ball.

At which speed does this ball stop becoming faster? Let's take another look at Fig. 4.7. The momentum current loss is exactly the same as the momentum current flowing in from the Earth when the velocity is 14 m/s. The heavy ball reaches steady state at a higher speed than the lighter one.

> At high velocities, friction cannot be ignored.
>
> The speed of a falling body increases only to a terminal velocity. The terminal velocity of an object depends upon its form, and it is greater for heavy bodies than for light ones.

An interesting application of our considerations would be parachute jumping. The parachutist jumps out of an airplane and reaches her terminal velocity of 50 m/s within a few seconds. She then "falls" at this speed for quite a while. The momentum current flowing through the gravitational field and into the person has the same strength as the one flowing out due to friction.

The parachute opens up at about 400 m above ground. The opening of the parachute means that air friction increases abruptly and strongly. The momentum current flowing off is suddenly much greater than the one flowing in. In the process, the momentum decreases. Along with the momentum, the velocity decreases, and with it, the loss by friction. Finally the momentum current due to friction reaches the same value as the gravitational momentum current, although at a velocity of only about 4 m/s. The parachute, with its passenger, now floats at a constant low velocity to the ground. Fig. 4.10 shows the velocity of the parachutist as a function of time.

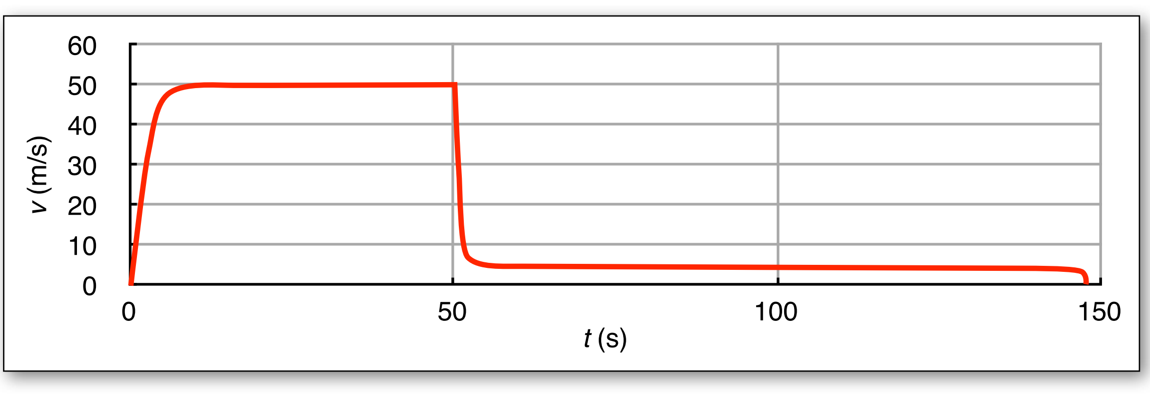


Fig. 4.10
The velocity of a parachutist as a function of time

Our considerations of the terminal velocity become invalid if there is no air or any other resistive medium involved. The Moon has no atmosphere. For this reason, all objects there fall at the same velocity: a sheet of paper falls just as quickly to the ground as a large stone. This can also be observed on Earth. The experiments must be carried out in a container from which the air has been removed. We let some small objects having different mass fall in an evacuated glass pipe. As expected, they all fall equally fast.

> ***Exercise***
>
> What terminal velocity does a falling sphere with a diameter of 30 cm and a mass of 0.8 kg reach?

4.6 Weightlessness

The man in Fig. 4.11a feels heavy. His body must carry the weight of his heavy head, and his feet have it even worse. They must carry the weight of his whole body. The man has an idea, see Fig. 4.11b. His legs are relieved of their burden, but now his arms must support his entire weight. In Fig. 4.11c, the man makes a third try at ridding himself of his weight, but he fails again.

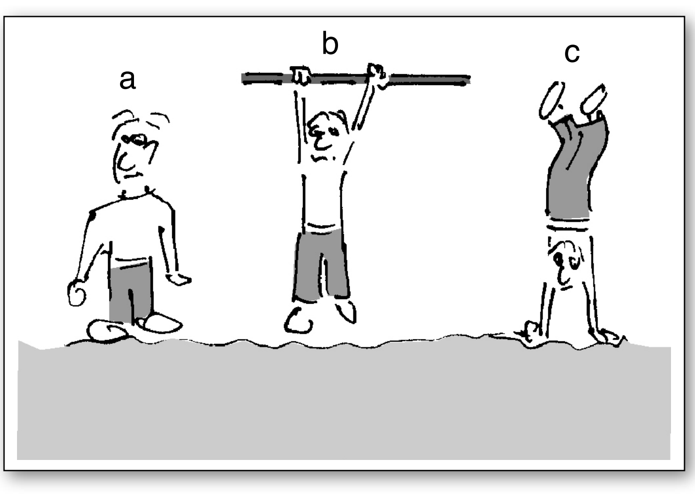


Fig. 4.11
No matter what he does, the man cannot get rid of his feeling of weight.

The man in Fig. 4.11 is bothered by the "feeling of gravity". We will try to define this feeling physically. In each of the three cases, what the man is feeling are momentum currents flowing through his body. Momentum flows through the gravitational field into every part of his body and must be conducted off: it must flow back into the ground. In Fig. 4.12, these currents have been sketched for a standing person. Momentum flows into the head, into the arms, into the upper body, etc. All of it must flow downward through the legs and feet and into the ground. The momentum current in the feet is therefore the strongest.

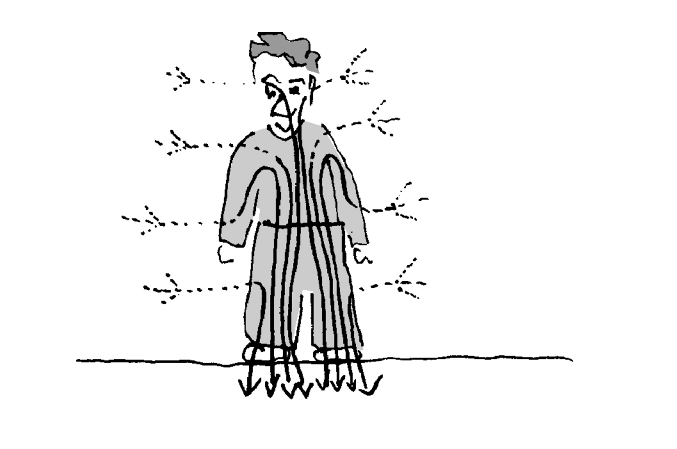


Fig. 4.12
The momentum currents flowing through the gravitational field into a person must also flow out again.

In the following we will consider a model of a person. It is composed of two blocks, one on top of the other (upper body and lower body), Fig. 4.13. One sees that the momentum current on the bottom side of the lower block is twice that on the bottom of the one above.

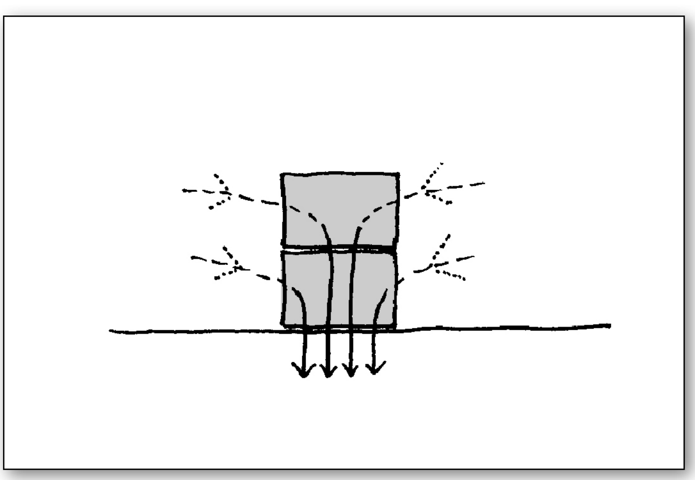


Fig. 4.13
A model person. It is composed of an upper and lower body.

We will now put our model into a state of weightlessness: a state where no momentum flows through it. In other words, a state in which no part of it is under compressive or tensile stress.

You probably think that it would be necessary to bring the model person very far away from the Earth to a place where the Earth's gravitational field is very weak. There would not be any momentum flowing into our model there. There wouldn't be any momentum flowing through it either. This would be a possibility indeed. There is another, simpler method though. We let momentum flow into the model, but not back out again. When there isn't any momentum flowing through it, it feels weightless.

How can this be done? Very simply. It is enough to interrupt the connection to the ground so that the momentum cannot flow out of the model person and back into the Earth. We must just let our person fall freely, Fig. 4.14. Now momentum is flowing from the gravitational field into each block (into every part of the person), and into every part of each block. It doesn't flow within the blocks though. In particular, no momentum flows from one of the blocks into the other. The result: There is no more compressive or tensile stress. The block below does not feel the weight of the one above.

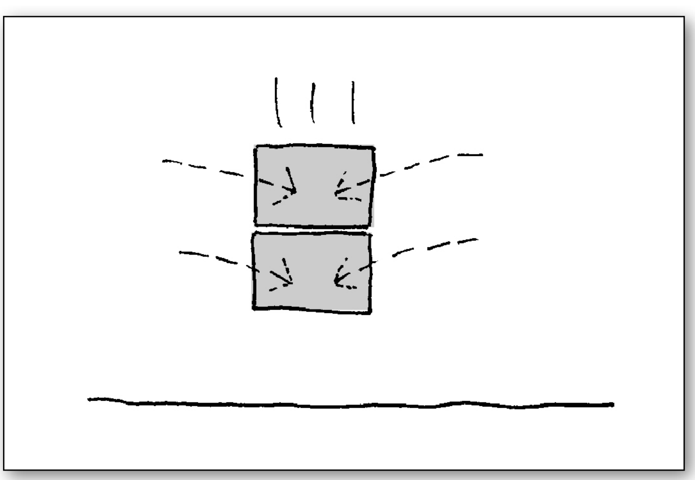


Fig. 4.14
A freely falling body is weightless. There are no momentum currents flowing within it.

For you, meaning a real person, the same holds: If you jump down from somewhere, you are weightless as long as you are falling. Even when you jump upward, you are weightless as soon as you lose contact with the ground. You remain so until you touch down again.

Now the time spent falling through the air is too short for one to really notice the feeling of weightlessness. We will therefore do an experiment with our model person, Fig. 4.15. The two blocks stand upon a platform. This platform hangs from strings similarly to a scale. There is a thin board between the two blocks attached to a taut rubber band connected to the wall. The rubber band would pull the board out if it wasn't held in place between the two blocks because of the weight of the upper block.

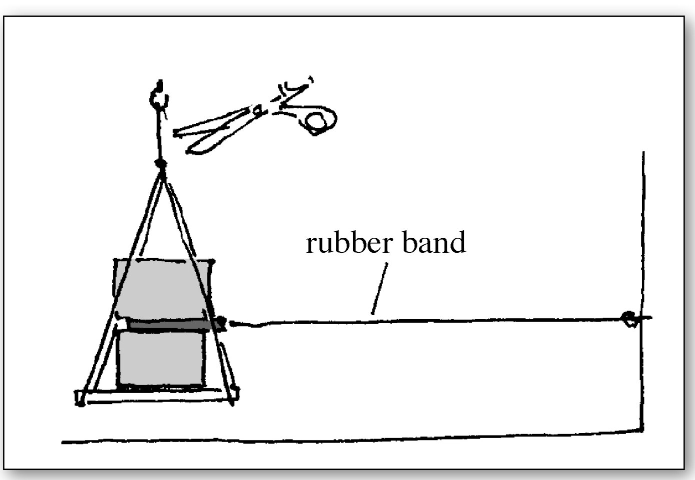


Fig. 4.15
During free fall, the blocks are weightless. The board between the blocks is released.

Now the experiment: We cut the string holding up the whole arrangement. At the same time, pulled by the rubber band, the board shoots out from between the blocks. Why? The stack of blocks was in free fall for a short moment. During this short period, it was weightless. The upper block was not pressing down upon the lower one. It let go off the board.

You know that astronauts feel weightless in their space ship. What is the explanation for this? Is it because they are so far away from the Earth? Not at all. The ISS (the *International Space Station*) flies at an altitude of about 250 km. Compared to the Earth's radius, this is not very high. Actually, it flies very close to the Earth's surface, Fig. 4.16. The gravitational field up there is almost exactly as dense as down here where we are: The strength of the field at 400 km altitude is g = 8.68 N/kg. This is hardly smaller than on the Earth's surface.

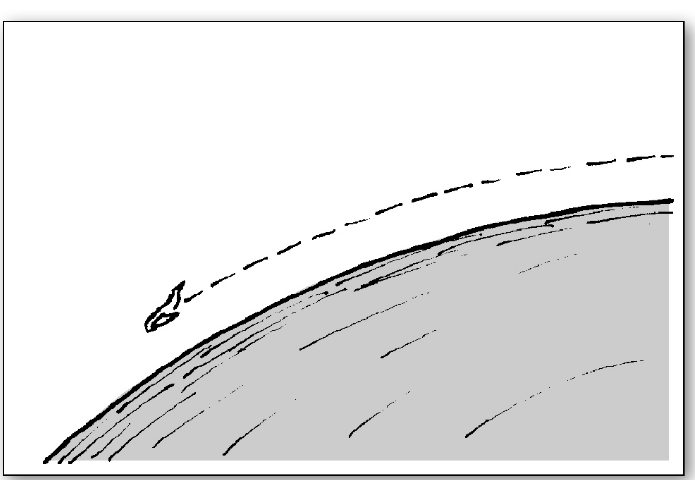


Fig. 4.16
The ISS flies at an altitude of only 400 km. The strength of the gravitational field is hardly smaller here than at the surface of the earth.

There must be another reason for weightlessness then. It is exactly the same as the one we have for falling objects: As soon as the propulsion rockets have burned their fuel, the space ship becomes a freely falling body. Why doesn't the ISS or a satellite fall to Earth? Now this is exactly what it does. However, it has a lot of horizontal momentum. It falls like a stone thrown sideways. It falls so far away that it always "falls behind the Earth". It "falls" in a continuous circle and never touches the Earth's surface.

Free falling bodies are weightless.

Exercises

1. An astronaut has two identical looking objects of different mass before him. Can he find out which of them has the greater mass? If yes, how?

2. A space ship is so far away from Earth that there is almost no gravitational field. The astronauts would like to feel their weight. How can they do this without returning to the Earth or flying to another celestial body?

4.7 Density of materials

"Which is heavier: 1 kg of iron or 1 kg of wood?" We have all heard this question. It is asked to trick someone. The right answer is naturally: "Both are equally heavy". However, someone not listening carefully, and not noticing the word "kg", would probably say that the iron is heavier.

We see that the words "heavy" and "light" can be understood in two somewhat different ways:

First, to represent a weight or mass: 1.5 kg of sugar is heavier than 0.8 kg of flour.

Second, to express a characteristic of the substance: One says that iron is heavier than wood because a piece of iron has greater mass than a piece of wood of the same volume.

This second definition of "heavier" and "lighter" is expressed quantitatively by the *density* of the material. We understand the density ρ of a material to be the ratio of mass m and volume V. In short, the mass per volume:

$$\rho = \frac{m}{V}$$

The resulting SI-unit is kg/m³. The densities of various materials are given in Table 4.2.

Table 4.2
Density of some substances at p = 1 bar and ϑ = 20 °C

	ρ (kg/m³)
Beech wood	600 – 900
Granite	2600
Aluminum	2700
Iron	7800
Copper	8960
Gold	19300
Gasoline	720
Ethyl alcohol (standard alcohol)	790
Water	998
Trichloroethylene	1460
Mercury	13550
Hydrogen	90
Nitrogen	1.25
Air	1.29
Oxygen	1.43
Carbon dioxide	1.9

Here is another point to consider: Some substances, namely gases, can be easily compressed. For this reason, their densities can be changed by just altering the pressure or temperature. If the density is given, the corresponding pressure and temperature must also be given. This effect is very small for solid and liquid substances, though. The values in the Table are based upon *standard conditions*: They are valid for p = 1 bar and ϑ = 20°C. The densities of gases in the table are noticeably much smaller than those of liquids and solids. We will keep in mind a basic rule-of-thumb:

Under standard conditions, the density of liquids and solids is about 1000 times that of gases.

In order to measure the density of a substance, one takes an arbitrary amount of it and determines its mass m and its volume V, and then divides m by V.

Sometimes it is easy to measure m and V, sometimes it is not. For example, to determine the density of gasoline, it is enough to weigh 1 l = 0.001 m³. One finds m = 0.72 kg. This leads to the following density:

$$\rho_{gasoline} = \frac{0.72\text{ kg}}{0.001\text{ m}^3} = 720\text{ kg/m}^3$$

It is more difficult to determine the volume of a solid substance if it has an irregular form. Fig. 4.17 shows how one might approach this problem. The object is submerged in water and the amount of water it displaces is measured.

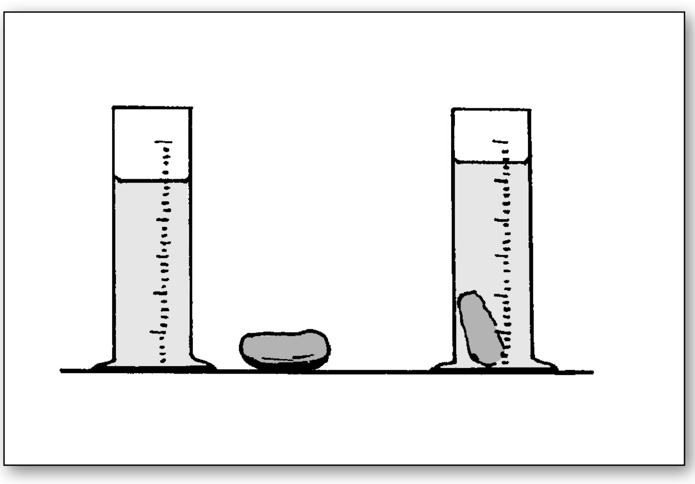


Fig. 4.17
In order to determine the volume of a solid body, the volume of the displaced water is measured.

Finding the mass of gases is the more difficult part of determining their density. Let us find the density of air. We take a container that can be sealed, having a volume of 1 l, which we weigh. We then pump the air out of it with a vacuum pump and weigh it again. The difference of the two results is the mass of the air that was in the container at the beginning.

Exercises

1. The weight of 1.6 liters of a liquid is determined. One finds m = 1.3 kg. What is the density of the liquid?

2. A granite paving stone weighs 2.2 kg. What is its volume?

3. The capacity of a car's gas tank is 40 l. How much does the gasoline of a full tank weigh?

4. A copper sheet is 120 cm long and 80 cm wide. It weighs 8.2 kg. How thick is it?

5. What is the mass of the air in your living room?

4.8 When a body floats and when it sinks

A piece of wood, some gasoline, or a drop of oil float on water. Iron, copper or aluminum sink. What about a drop of water in water? Does it float or sink? A silly question, you may think. You cannot distinguish one drop of water from the rest of it! However, it is really not difficult to do this. Just dye it a color. The result: It doesn't float and it doesn't sink, it is suspended.

Whether or not a body floats upon a liquid depends upon how heavy the body is. What do "heavy" and "light" mean here though? Surely not the mass. A piece of wood floats upon water no matter how great its mass. Density is important in this case. A body floats upon a liquid if its density is less than the density of the liquid. If it has greater density, it sinks. If the densities of the body and the liquid are the same, the body is suspended.

(We have created an alternative use of the word body. It can also mean a portion of a liquid.)

We will test this once more with water and gasoline. If we put a drop of water into a container of gasoline, it sinks. Gasoline dripped onto a container filled with water spreads out over its surface.

Both cases deal with the same phenomenon. This can be clearly seen in the following experiment: Several liquids of differing densities are poured into a glass. These might be trichloroethylene, water, and gasoline. The three liquids arrange themselves in layers with the one with greatest density at the bottom. Just on top of it is the one with the next greatest density, etc. (Fig. 4.18). Now a few solid bodies can be put into the glass. A metal body sinks to the bottom; a body of hard rubber (ρ = 1200 kg/m^3) floats upon the trichloroethylene but not upon the water. A body of light plastic (ρ = 900 kg/m^3) floats upon the water, but not upon the gasoline. Finally at the top, a piece of wood floats upon the gasoline. The seven different substances have arranged themselves according to their densities.

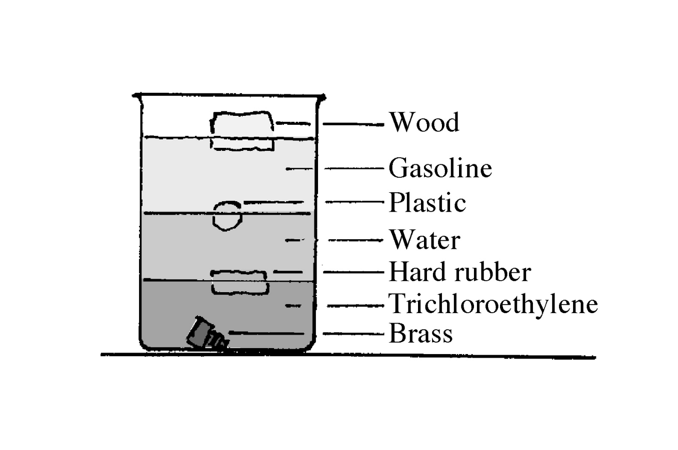


Fig. 4.18
The 7 bodies (liquid and solid) arrange themselves according to their densities. (Wood, Gasoline, Plastic, Water, Hard rubber, Trichloroethylene, Brass.)

Because gases have lower densities than liquids, all gases "float" upon all liquids. This is why an air bubble in water or a carbon dioxide bubble in cola rise to the top.

So far we have asked what types of bodies float upon *liquids*. The same question could be asked for a *gas*. Of course all liquids and solids sink in gases. Water drops or solid particles fall downward through the air. In a glass, though, one gas can "float" upon another. This phenomenon is put to use by balloons. If a balloon is filled with a gas with a lower density than that of air, say hydrogen, the balloon will rise upward (assuming that the balloon's shell is not too heavy because the hydrogen must lift it as well). The dirigibles popular at the beginning of the 20th century functioned by this principle.

We sum up:

> A body with a density lower than that of its environment, rises upward. If its density is higher than that of its environment, it sinks.

Exercises

1. Is there a liquid upon which iron would float? Give reasons!

2. A balloon is filled with carbon dioxide. Does it rise or fall? Give reasons!

4.9 The relation between pressure and altitude in liquids and gases

One feels “pressure in the ears” when diving into a swimming pool or when an elevator rises or moves downward quickly in a tall building.

In both cases, the pressure changes. Ears are our most sensitive sense organs for detecting changes of pressure.

We fill a tall container with water. The container has holes on its side at three different levels, Fig. 4.19. The water sprays out of all three holes. The pressure forcing the water out is called *hydrostatic pressure*. It is created by the weight of the water. The water jet at the bottom sprays the farthest, so the pressure there must be the greatest. The jet at the top is the weakest, so the pressure there must be the lowest. Hydrostatic pressure of water increases with depth.

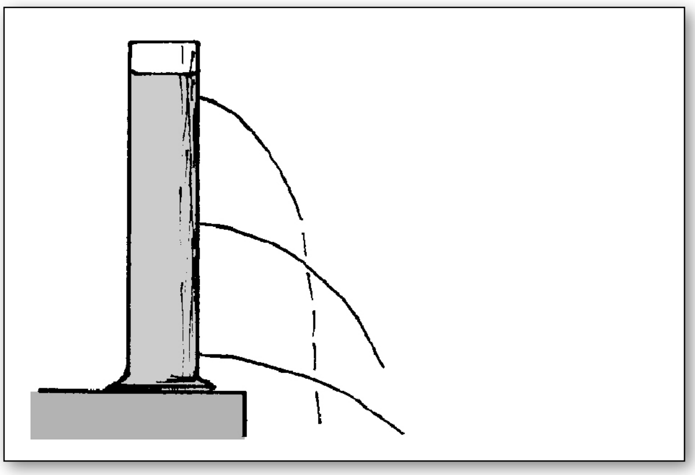


Fig. 4.19
Water pressure increases with depth.

This increase of pressure can be measured. One finds that hydrostatic pressure increases 1 bar per 10 m water depth. At the deepest part of the ocean (about 10,000 m below sea level) pressure is at 1000 bar. You can now see why diving capsules that go to these depths need to have such thick walls.

Hydroelectric power plants are often constructed as shown in the schematic in Fig. 4.20. At a high altitude in the mountains there is a water reservoir: a storage lake for water coming from various rivers and streams. Several thick pipes lead from this reservoir down into the valley below to the actual power plant with its turbines and generators. If, for example, the reservoir lies 500 m above the turbines, the pressure at the entrance to the turbines is 50 bar.

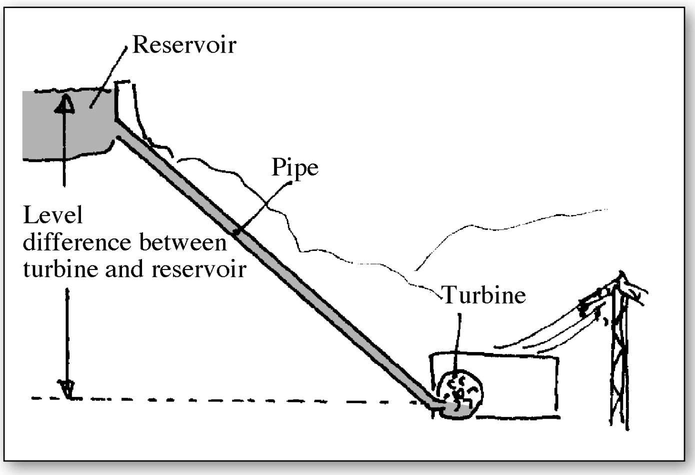


Fig. 4.20
The pressure at the entrance to the turbine depends upon the difference in height between the turbine and the water reservoir.

This increase of pressure with depth can also be observed in the Earth’s atmosphere, or in other words, in the “sea of air” surrounding the Earth. On the floor of this sea, meaning at the Earth’s surface, the hydrostatic pressure is about 1 bar, as you know. It decreases in the upward direction. Close to the Earth’s surface it decreases at about 1 mbar per 10 m. Not only does the pressure grow smaller in an upward direction, but the pressure change per altitude difference does as well (also see Section 2.2).

The hydrostatic pressure of liquids and gases increases with depth.

5

Momentum and energy

5.1 Momentum as an energy carrier

Physical effort uses up energy. What is meant here by "uses up"? For example, one needs to eat a lot in order to keep up his or her exertions. One receives energy through food, and gets rid of this energy through physical activity. "You use a lot of energy" actually means "A lot of energy is flowing through you", you take in a lot of energy and you get rid of a lot of energy.

The person in Fig. 5.1 is pulling a box across a floor. (Of course there are less exhausting ways to transport it, but then we wouldn't be able to discuss our problem so well.) She exerts herself, she is getting rid of energy. Where does this energy go? It goes to the bottom surface of the box, creating heat there. Then it distributes into the environment together with the heat.

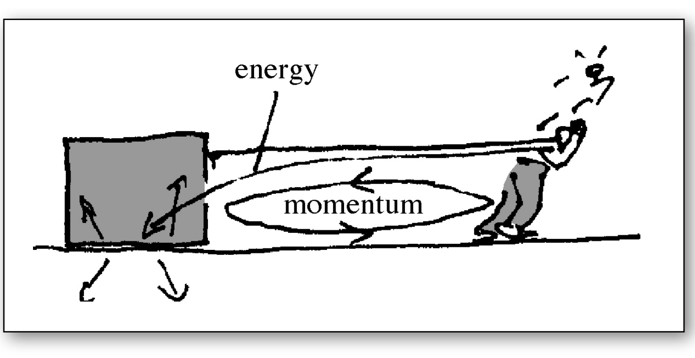


Fig. 5.1
The momentum flows in a closed circuit. The energy flows from the person's muscles to the underside of the crate.

We now wish to investigate the transport of energy between the person and the box. The first point to be dealt with is: Which one is the energy carrier? In the rope between the person and the box, a momentum current flows simultaneously with the energy current. We suspect that the energy carrier we are looking for is momentum.

> Momentum is an energy carrier.

However, we immediately see that not every momentum current is accompanied by an energy current. The momentum current in Fig. 5.1 flows, as we know, from the box through the ground and back to the person. The energy takes its own course from the bottom of the box. Therefore, the momentum flowing back does not carry any energy.

What does the strength of the energy current depend upon? Put more generally: What do we need to do in order to transport as much energy as possible through a rope or a rod?

If we attach a taut rope to a wall, Fig. 5.2, a momentum current flows, but no energy current flows because nothing is being heated and nothing is moving. What is the difference between the ropes in Fig. 5.1 and Fig. 5.2? The first rope is moving, the second one is not. We see that in energy transport, the velocity with which the momentum conductor moves is important.

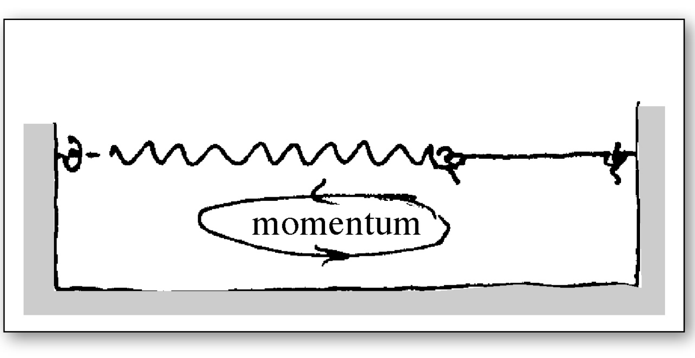


Fig. 5.2
No energy current flows although a momentum current is flowing.

In addition, the strength of the energy current is dependent upon the strength of the momentum current. If a rope is not under mechanical tension, no energy can be transported by it.

We draw a conclusion:

> The strength of the energy current P through a rope depends upon
> – the strength F of the momentum current in the rope and
> – the velocity v of the rope.

We want to see how the relation looks quantitatively. What kind of equation relates the three quantities P, F, and v?

The dependence of the energy current P upon the momentum current F is easy to find. Fig. 5.3 shows from above how two identical boxes are pulled across a floor. We compare both pieces of rope A and B. Both move at the same velocity. The momentum current as well as the energy current split evenly at the node P. The momentum current in B is half that of A. This is also true of the energy current. Therefore we see that at constant speed the energy current is proportional to the momentum current:

$P \sim F$

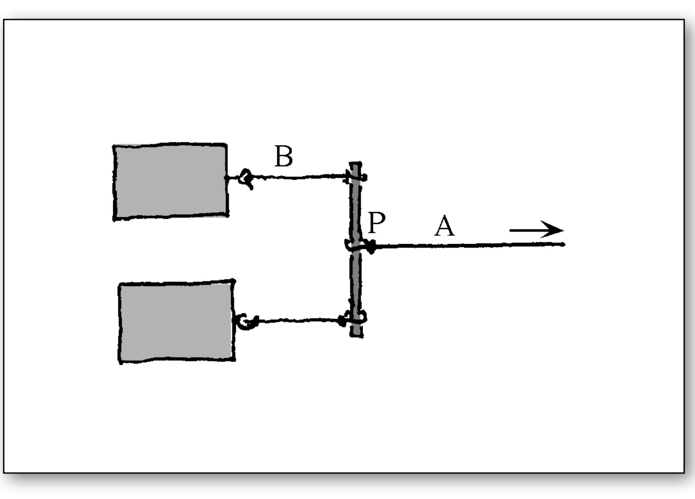


Fig. 5.3
Two crates are pulled across the floor. View from above.

We will do an experiment in order to find the relation between P and v. A box is pulled with the help of a "pulley", Fig. 5.4. We compare ropes A and B. A first comment about the energy current: All of the energy that flows into rope B from the right goes through the pulley and through rope A. No energy can flow in rope C because C does not move. We therefore have

$P_A = P_B$

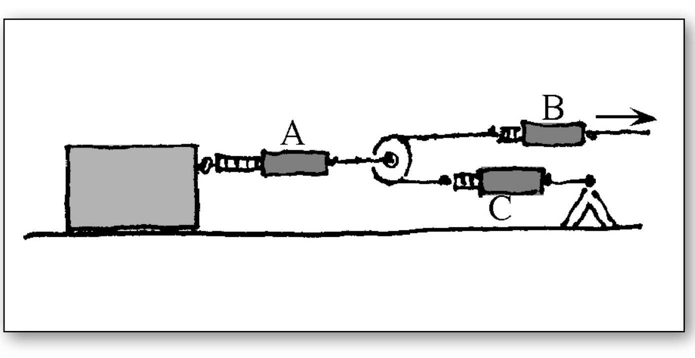


Fig. 5.4
The momentum current in rope A is twice that in rope B. The velocity of rope A is half that of rope B.

Next we compare the velocities of A and B. If the box, and rope A along with it, moves a certain distance to the right, the right end of B moves twice this distance to the right. Let us assume that the box moves 10 cm to the right. The pulley also moves 10 cm to the right. If rope B was not rolled over the pulley, but attached to the right end of A, then B would also move 10 cm to the right. However, rope C becomes 10 cm shorter because of the pulley, and rope B takes these 10 cm coming from rope C. B becomes 20 cm longer and this means that the velocity of B is always twice that of A. Therefore:

$v_B = 2v_A$

Finally, we will compare the momentum currents in A and B. We can only do this by measuring. Our measurements show that the momentum current in B is half that in A. (In C it is exactly the same as in B, so the junction rule is satisfied). We can then write:

$F_A = 2F_B$

All of these results together can be written as follows:

$P \sim v \cdot F$

This proportionality tells us that P is proportional to F if the velocity is kept constant. It also tells us that if v is doubled and F is halved, P stays constant. This is what we have found in our experiment with the pulley.

> If energy is transported by the energy carrier momentum, the strength of the energy current is proportional to the strength of the momentum current and to the velocity with which the conductor moves.

In order to form an equation out of this proportionality, we should introduce a factor of proportionality. Fortunately, the SI-units of the three quantities have been so chosen that the following is valid:

> $P = v \cdot F$

This is the desired result. We can use it to calculate the energy current in our rope if we know the momentum current through it and its velocity.

An example: We pull on a rope with a force sensor mounted into it. The force sensor shows 120 N and the rope moves at 0.5 m/s. The resulting energy current is:

$P = v \cdot F = 0{,}5\ \text{m/s} \cdot 120\ \text{N} = 60\ \text{W}.$

Remember that velocity must be given in m/s and momentum current in N so that the energy current is given in the SI-unit Watt.

The formula

$P = v \cdot F$

can be transformed into an equation that is handier for some problems. We replace P by E/t and v by s/t:

$\frac{E}{t} = \frac{s}{t} \cdot F$

and multiply both on the right and the left by t. The result is:

> $E = s \cdot F$

The equation tells us, for example, that if one pushes against a rod and the rod moves a distance s, the amount of energy flowing through the rod is $s \cdot F$. Here, F is the momentum current flowing through the rod being pushed.

An example: We pull on a rope so that we have a momentum current of 120 N and the rope moves 2 m. How much energy is transported through the rope? We apply our new formula. With $F = 120$ N and $s = 2$ m, we have

$E = s \cdot F = 2\ \text{m} \cdot 120\ \text{N} = 240\ \text{Nm} = 240\ \text{J}$

Exercises

1. A tractor pulls a trailer on a horizontal street at a speed of 20 km/h. A momentum current of 900 N flows through the trailer coupling. What is the energy used by the trailer? (What is the energy current from the tractor to the trailer?) Where does the momentum that flows through the trailer go? Where does the energy go?

2. A truck is pulling a trailer along a horizontal street from one city to another. The distance between the two cities is 35 km. A momentum current of 900 N flows through the coupling. How much energy in all has flown from the truck to the trailer?

3. The drive belt of a machine runs at a speed of 10 m/s. The energy current transported by the drive belt has a strength of 800 W. What is the force of the belt upon the belt pulley? (What is the momentum current in the belt?)

4. A crane lifts a weight of 50 kg at a velocity of 0.8 m/s. What is the energy current through the crane cable? The weight is lifted 5 m high. How long does this take? How much energy flows through the cabel during this process?

5.2 Mechanical energy storage

a) Elastically deformed bodies as energy stores

We stretch a long, strong spring, Fig. 5.5. This is tiring because energy is used for it. We consider the right end of the spring (point A in Fig. 5.5). This end of the spring is under mechanical tension, meaning a momentum current *F* is flowing through it, and it is moving at a velocity *v*. According to our formula, $P = v \cdot F$, an energy current is flowing through it as well. Now we look at its left end (point C). The momentum current here is the same as at A, but because C does not move, no energy current flows. The energy flowing in at A does not flow out again at C but is stored in the spring.

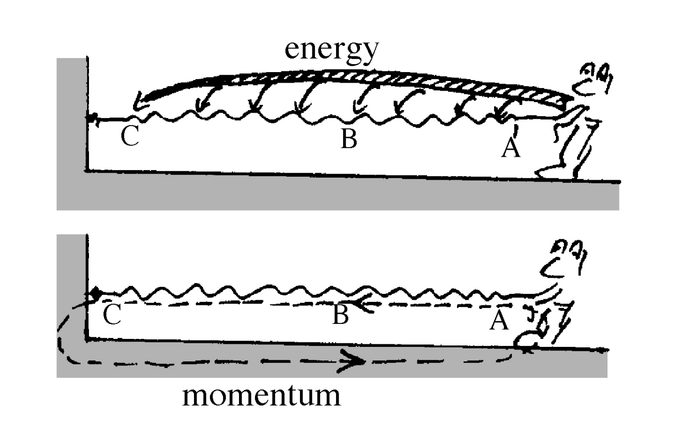


Fig. 5.5
Energy flows into the spring from the right when it is stretched.

We can check the currents at other points of the spring, for example, in the middle. There the momentum current is exactly the same as at A and at C. The velocity at the middle of the spring is exactly half that at point A. Therefore the energy current flowing at point B is only half that flowing into the spring at point A. This is understandable: Half the energy is stored in the right half of the spring and the rest flows further to the left half. This idea can be taken further: A third of the energy is stored in each third of the spring. In every quarter of the spring, a quarter of the energy is stored, etc. In short: The energy distributes evenly over the entire length of the spring.

If a spring can be compressed so that it doesn't expand sideways, this method can be used to store energy as well.

> A spring is an energy storage unit. The more a spring is expanded or shortened, the more energy it contains.

Of course, these considerations are not only valid for springs but for any other elastically deformable object as well. An extended expander contains energy in the same way a slingshot does, a bent diving board or a dented football, Fig. 5.6.

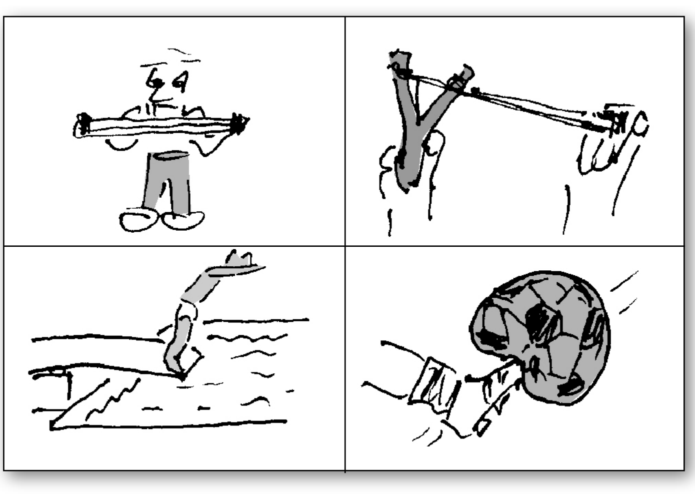


Fig. 5.6
Energy is stored in a stretched expander, in a stretched slingshot, in a bent diving board and in a dented football.

b) Moving bodies as energy stores

We charge a car which is unimpeded by friction, with momentum. We have done this often in the past, Fig. 5.7, but this time we know that not only momentum but also energy flows in the rope. The energy cannot leave the car any more than the momentum can. In the process of pulling, the car accumulates momentum and energy.

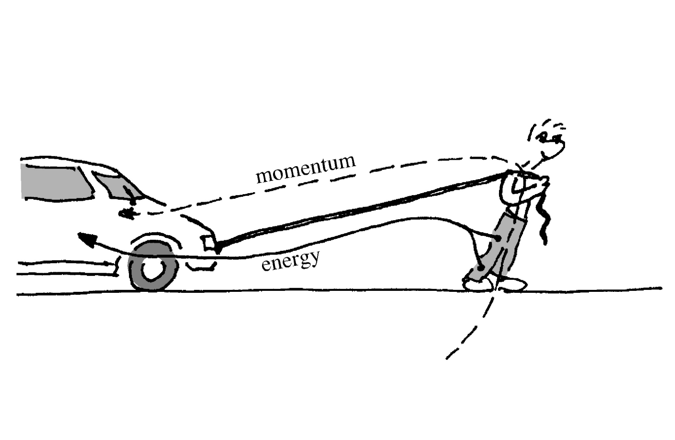


Fig. 5.7
Energy as well as momentum flows into a car when it accelerates.

> A moving body contains energy. The greater its speed, the more energy it contains.

When a moving car rolls to a stop, its momentum flows into the ground. The energy takes another path. It is used to create heat. Wherever we have friction, heat is produced. In the process, the energy is distributed into the environment: a part goes into the ground, and a part goes into the car and into the air.

The energy contained in a moving wagon can also be put into a spring and stored there. To do this, a wagon pulls a spring, Fig. 5.8. The wagon comes to a stop. There are two ways to pull the spring. Either its left end is attached to the wall, and its right end is pulled to the right by the wagon, or the right end is attached to the wall, and the left end is pulled to the left. In both cases the result is identical: a taut spring charged with energy. In the first case, the energy came from a wagon with positive momentum and in the second case, from a wagon with negative momentum. However, both wagons have positive energy.

> No matter in what direction an object is moving, its energy is positive.

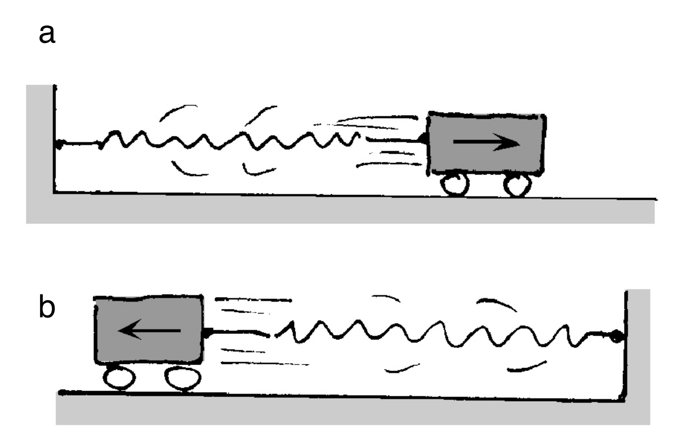


Fig. 5.8
A moving wagon gives its energy to a spring. (a) The wagon moves to the right. (b) The wagon moves to the left.

c) The gravitational field as energy store

In Fig. 5.9, a heavy body is pulled upward. Again, energy is flowing in the rope along with momentum. The momentum comes out of the Earth, over the gravitational field into the body.

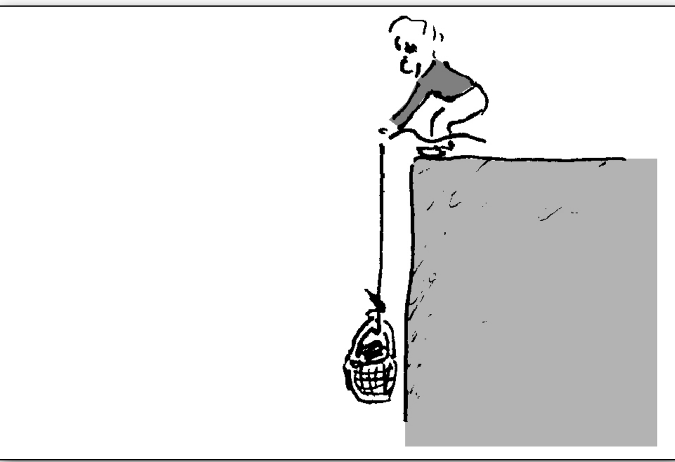


Fig. 5.9
Energy is stored in the gravitational field while the object is being hoisted.

The gravitational field can be pictured as an invisible spring pulling on the body. When the object is lifted, energy is stored in the gravitational field exactly as energy is stored in a spring by expanding it. When the object is lowered again, the energy is given back from the gravitational field.

It takes more energy to lift a heavy object than to lift a light one. The heavier the object being lifted, the more energy is stored in the field.

> The gravitational field is an energy storage unit. The higher an object is lifted, and the heavier it is, the more energy is put into the gravitational field.

The energy of the gravitational field is put to use in hydroelectric plants, Fig. 5.10.

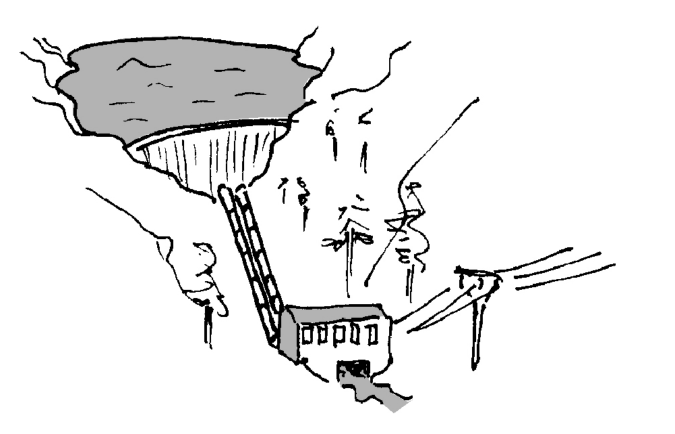


Fig. 5.10
Hydroelectric power plant. While flowing downward, the water takes energy out of the gravitational field. It gives it up again in the turbine.

Water from streams and rivers is collected at high elevations and piped downward. In the process of flowing down, water takes energy out of the gravitational field. It then flows through the turbines of the power plant and gives its energy up there. Energy flows into the turbines with the energy carrier "water". From the turbines, the energy continues to the generator carried by the energy carrier angular momentum.

5.3 The complex paths of energy and momentum

In the following, we will investigate two processes of motion: The motion of a stone thrown upward, Fig. 5.11, and the motion of a body oscillating between two springs, Fig. 5.12. In both cases we ask the same questions:

– What path does the energy take?
– What path does the momentum take?

a) The stone thrown upward

Energy

When a person throws a stone up into the air, Fig. 11, energy goes from his muscles and into the stone. While the stone is moving upward, the energy flows into the gravitational field. At the turnaround point, it has completely left the stone. During the descent it flows out of the field and back into the stone. When the stone hits the ground, heat is created. The energy is distributed into the environment, i.e., into the stone, the ground and the air, along with the heat.

Momentum

In the act of throwing the person "pumps" negative momentum from the ground into the stone. While the stone is traveling upward, positive momentum flows out of the Earth, through the gravitational field, and into the stone. In the process, the negative momentum of the stone decreases. At the turnaround point, the entire negative momentum of the stone is compensated. The inflow of positive momentum does not stop here though. The stone now takes the positive direction (downward) and its positive momentum increases with falling. At impact with the ground it gives the momentum back to the ground.

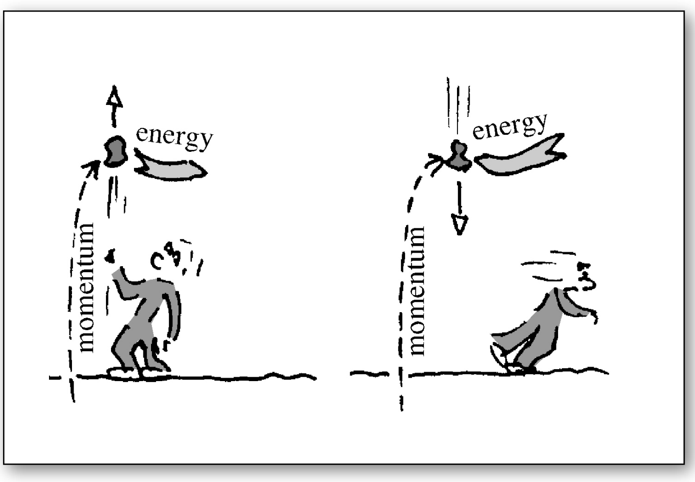


Fig. 5.11
The path of energy and momentum for an object that is thrown upward.

b) The oscillating object

The air cushion glider in Fig. 5.12 moves back and forth, performing a so-called oscillation. You have certainly seen a lot of other cases of oscillatory motion. In many of these processes, the paths of energy and momentum are very similar to those in Fig. 5.12. We will therefore look a little closer at the glider in Fig. 5.12. We push the glider to the left a bit out of the equilibrium position and then let go of it.

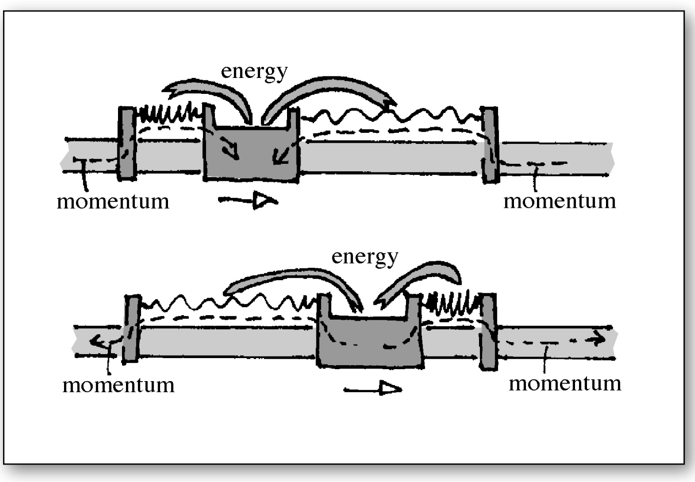


Fig. 5.12
The path of energy and momentum for an oscillating object.

Energy

At the moment of release, both springs are charged with energy: the left one because it is compressed, and the one on the right because it is stretched. The glider now starts moving to the right receiving energy from both springs because both of them relax. If the glider reaches the middle, both springs have given up their energy. All the energy is in the glider. The glider continues moving to the right gradually becoming slower. It now gives its energy to both springs. At the reversal point on the right, all the energy is in both springs again, and the whole process starts all over in the opposite direction.

Momentum

At the moment the glider is released, the left hand spring is under compressive stress and the right hand one under tensile stress. In the one on the left, a momentum current is flowing to the right. In the right hand one, it is flowing to the left. In total, two momentum currents are flowing from the ground into the body. The body's momentum increases until it has reached the middle. Now the springs are relaxed and momentum has ceased to flow. However, as soon as the glider moves beyond the middle point, the springs begin to be stressed again, but now the right one is under compressive stress and the left one is under tensile stress. The momentum is flowing in the opposite direction than before: out of the body, in both directions, and into the ground.

Exercises

1. A train car rolls against an elastic spring buffer. What path do energy and momentum take?

2. A ball falls to the ground and bounces back up. What path do energy and momentum take?

3. An object hangs from a ceiling on a rubber band so that it can oscillate up and down. Describe the path of energy and momentum.

6

Momentum as a vector

6.1 Vectors

There is thick fog in an area with a lot of ship traffic. Captain Amundsen gets the positions and speeds of the ships in the surroundings from his radio operator: “At a distance of 5.6 miles north-east of the Gigantic, a tanker is traveling at a speed of 35 knots (65 km/h).” Is this information enough to let Captain Amundsen avoid a collision? Of course not, Fig. 6.1. “What direction is it traveling in?” he asks. He knows that if the tanker is traveling westward, it will become dangerous. He would have to carry out an evasion maneuver. If the tanker is traveling eastward, there is no danger.

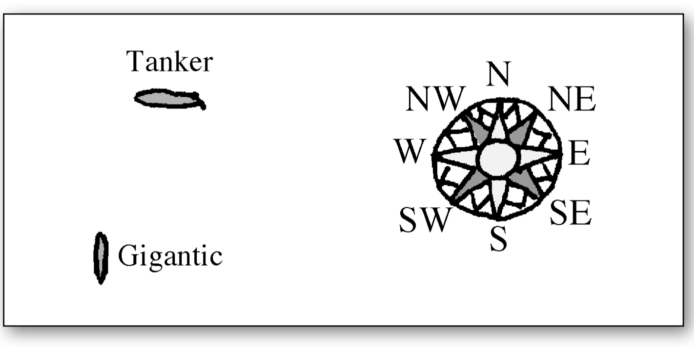


Fig. 6.1
The positions of the Gigantic and a tanker. The tanker sails eastward. There is no danger.

In order to unambiguously describe the motion of a body (in this case, the tanker), one needs the following information:

– how fast the body is traveling, for example, 65 km/h;

– in which direction the body is moving, for example, east.

Both statements are about velocity. “65 km/h” alone does not determine the velocity. Determining the direction of the motion is also a part of this.

Velocity has a *magnitude* and a *direction*. In the case of our tanker we have:

– magnitude of the velocity: 65 km/h;

– direction of the velocity: eastward.

There are other quantities that can only be defined through their magnitudes and directions. Momentum is one of these quantities.

Both the cars in Fig. 6.2 have a momentum of 2000 Hy. In spite of this, their momenta are not the same. The cars are not moving in the same direction. Car A is traveling in direction x and car B is traveling transversely to it.

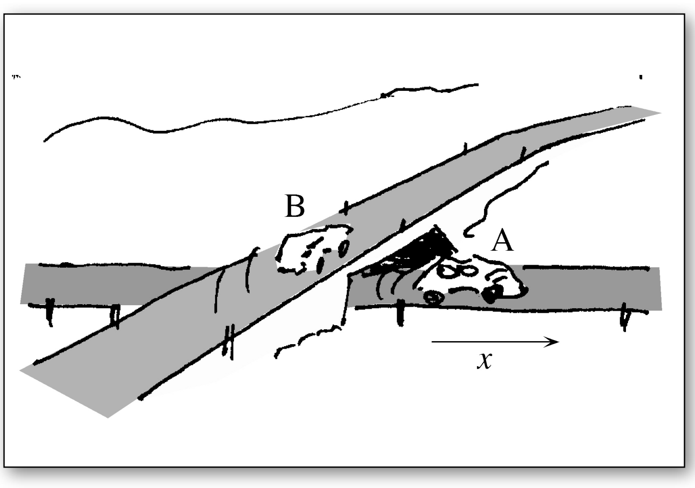


Fig. 6.2
The momenta of both cars are different.

In order to identify the momentum of a body, the magnitude and the direction must be given, as for velocity. In our example this means:

Momentum of car A
  Magnitude: 2000 Hy
  Direction: x

Momentum of car B
  Magnitude: 2000 Hy
  Direction: transverse to x

Two momenta are identical only when they have the same magnitudes and the same directions.

Physical quantities defined in this way are called *vectors.*

> A vector is determined by its magnitude and its direction
> Velocity and momentum are vectors.

“Normal” physical quantities, meaning those that can be defined by just a number, are called *scalars.*

To say that the mass of a body is

$m = 5$ kg

is unambiguous. To give a direction in this case would not make sense. The mass is therefore a scalar. Other examples of scalars are energy, electric current, and temperature.

How do you communicate the value of a vector quantity, say the momentum of a body? For example, like this:

– magnitude of momentum: 200 Hy;
– direction of momentum: 35 degrees to the x axis.

There is an even easier way to describe the momentum (or any other vector quantity): by a sketch. The scale must be set first, say:

1 cm in the sketch corresponds to 50 Hy.

The momentum can now be represented by an arrow. The length of the arrow indicates the magnitude of momentum, the direction of the arrow is the momentum’s direction.

In Fig. 6.3, the momenta of three bodies are represented. Note that 1 cm corresponds to 50 Hy.

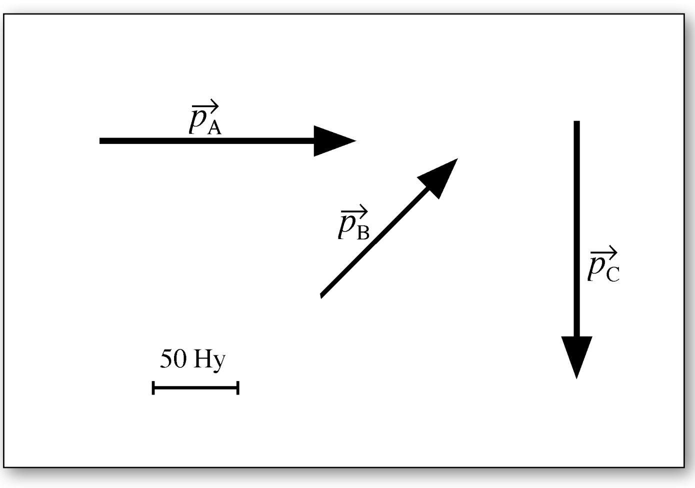


Fig. 6.3
Fig. 6.3. Momenta of three bodies A, B, and C, represented by arrows.

To make things a little easier for us, we will give the three types of momentum in Fig. 6.3 different names. The one in body A we will call 0-degree momentum because the momentum arrow makes a 0 degree angle to the x-axis. Body B’s momentum we call 45-degree momentum because its arrow forms a 45-degree angle to the x-axis. We accordingly name the momentum of C 270-degree momentum.

The fact that a quantity is a vector quantity is also expressed by its symbol: An arrow is written over the letter symbol. The symbol for the velocity vector is $\vec{v}$ and the symbol for the momentum vector is $\vec{p}$. These symbols were used in Fig. 6.3.

Exercises

1. Show the following momentum values graphically.

Body P: magnitude of momentum: 20 Hy
direction of momentum: 270 degrees to the x-direction

Body Q: magnitude of momentum: 1200 Hy
direction of momentum: 10 degrees to the x-direction

2. In the former chapters, we dealt only with motion parallel to one axis. In those cases positive and negative momentum values appeared. Show the momentum values p_1 = 3,5 Hy and p_2 = – 4,5 Hy as arrows.

3. What are the magnitudes and directions of the momenta represented as arrows in Fig. 6.4?

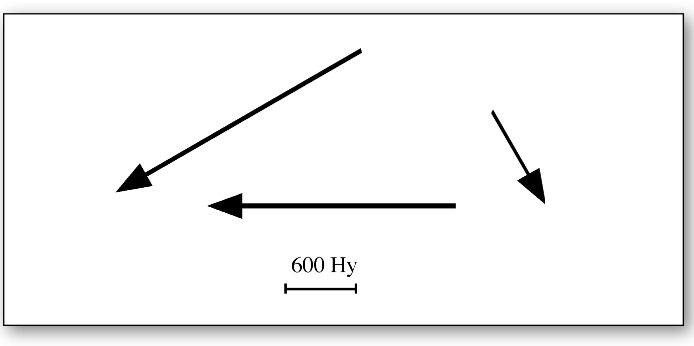


Fig. 6.4
For exercise **3**

6.2 Direction of flow and direction of that which flows

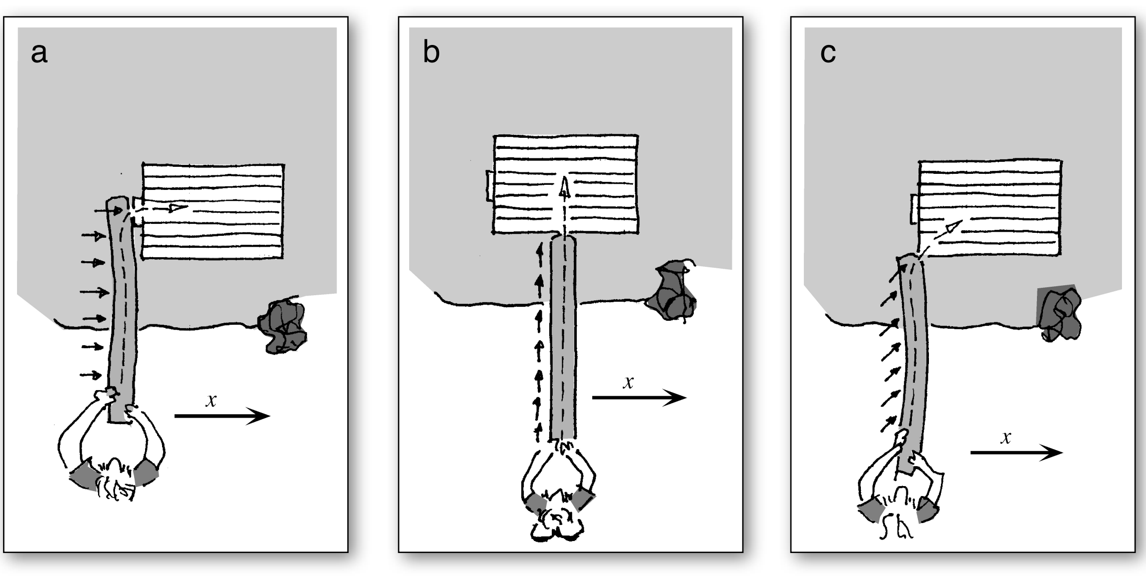


Fig. 6.5
(a) 0° momentum, (b) 90° momentum and (c) 45° momentum flows toward the raft, meaning in the rod from bottom to top.

Someone is standing at the edge of a pond trying to move a raft with the help of a pole. The Figs. 6.5a to 6.5c show three different scenes from above. We will describe what is happening.

In order to make things a bit easier, we have drawn in the x-axis. We always give the direction of momentum in relation to this direction.

In Fig. 6.5a, the person pushes the raft to the right, in the direction of the positive x-axis. We assume that he is pushing so that 150 Huygens per second flows into the raft. The momentum current is 150 Hy/s = 150 N. In Fig. 6.5b, he pushes the raft away from himself so that it moves toward the center of the pond. Again, he is pushing so that 150 Hy per second flows into the raft. In Fig. 6.5c, he pushes diagonally forward and to the right with 150 N.

Although the same number of Huygens per second is flowing in all three cases, the momentum currents are different in each case, because what is flowing is not the same.

In the first case, the raft gets 0-degree momentum. 0-degree momentum flowed through the pole. In the second, the raft received 90-degree momentum, so 90-degree momentum must have been flowing through the rod. In the third case, 45-degree momentum flowed through the rod.

The momentum flowing from the person to the raft is represented by the arrows on the left of the pole. You can imagine each arrow as a portion of the momentum flowing from the person to the raft.

The long dashed line shows the path the momentum takes.

You can tell that momentum currents present us with the same situation as momentum does. We need more than just a number to specify a momentum current. A momentum current is specified only if we give the direction of the flowing momentum (0 degrees, 90 degrees, or 45 degrees) in addition to its magnitude (here: 150 N).

> The momentum current strength is a vector.

We can represent this vector by an arrow, just as we did with other vectors. Length and direction have the following meanings:

– Length of the arrow: Magnitude of momentum current strength;

– Direction of the arrow: The direction of the momentum flowing through the conductor.

We use $\vec{F}$ as the symbol for the momentum current vector.

Fig. 6.6 shows the vectors of the currents belonging to Figs. 6.5a-c.

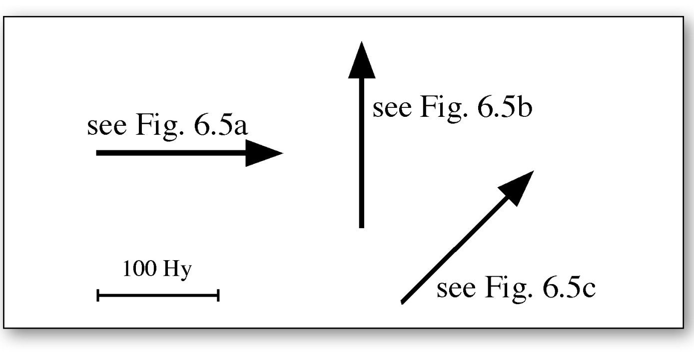


Fig. 6.6
Current strength vectors for momentum flowing toward the raft in Figs. 6.5a-c.

We now consider Fig. 6.7 and compare it to Fig. 6.5c. In Fig. 6.7, 45-degree momentum is flowing to the raft. In contrast to Fig. 6.5c though, it is flowing not through a straight pole but through one that is bent. The momentum must flow through an s-shaped path. In both Figures, the same momentum is flowing, and in both figures 150 N is flowing. It is flowing once through a straight conductor and once through a curved one. The vector arrows representing the situations in Figs. 6.5c and 6.7, are therefore the same.

The path the momentum takes is represented by the dashed lines in Figs. 6.5a-c and Fig. 6.7.

> Don't mistake the direction of the path with the direction of the momentum being transported.

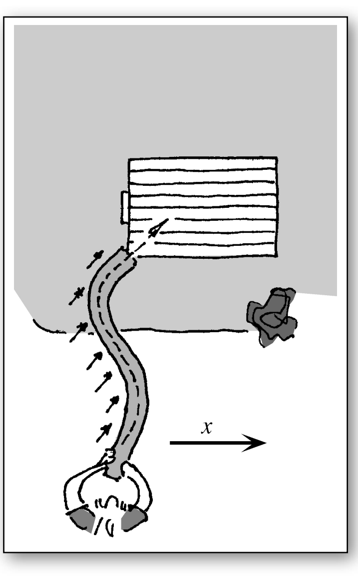


Fig. 6.7
45-degree momentum flows through a bent rod to the raft.

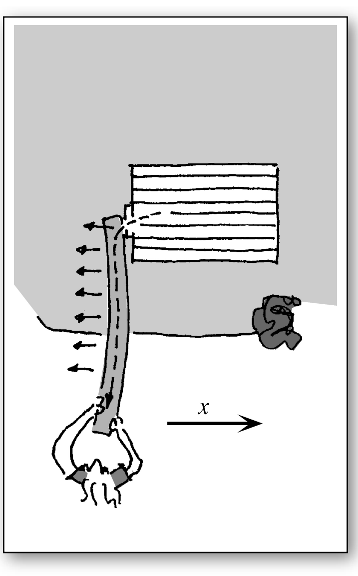


Fig. 6.8
The same process as in Fig. 6.5a, but described differently: 180-degree momentum flows into the ground.

In our descriptions of the three processes in Figs. 6.5a to 6.5c, we have said what kind of momentum and how much of it flows from the ground *to the raft*. We can also describe these processes by saying what kind of momentum and how much of it flows away from the raft, meaning *to the Earth*. Both descriptions are equivalent.

"x Huygens of momentum of a given direction flow per second from the ground to the raft" means the same as "x Huygens of momentum of the opposite direction flow per second to the ground".

Fig. 6.8 shows the same process as in Fig. 6.5a, but here it is described differently. Fig. 6.8 does not show what kind of momentum is flowing into the raft, but the kind of momentum that is flowing to the Earth. This is 180-degree momentum.

Notice that the arrows representing the current strength vectors in the two descriptions must point in opposite directions.

The following statements are equivalent:

– 150 Hy/s of 0-degree momentum flow from the Earth to the raft;

– 150 Hy/s of 180-degree momentum flow from the raft to the Earth.

Exercises

1. 300 N flows in a rod between a tractor and its trailer, Fig. 6.9. What kind of momentum is flowing *into the trailer?* Represent the momentum current by an arrow. Describe the same process by giving what kind of momentum and how much of it flows through the rod *away from the trailer*.

2. Someone pushes a wagon with a spiral rod in the positive x-direction, Fig. 6.10. A momentum current of 25 N flows.

a) What kind of momentum is flowing into the wagon?

b) Sketch the path of the momentum in Fig. 6.10.

c) Draw the arrow representing the current vector.

3. In Fig. 6.11, an apple falls to the ground. Momentum flows into the apple. (It comes from the Earth and travels through the gravitational field.) The apple weighs 300 g.

a) What is the magnitude of the momentum current?

b) What kind of momentum flows into the apple? (Give the angle in a vertical plane).

c) Draw the arrow representing the current vector.

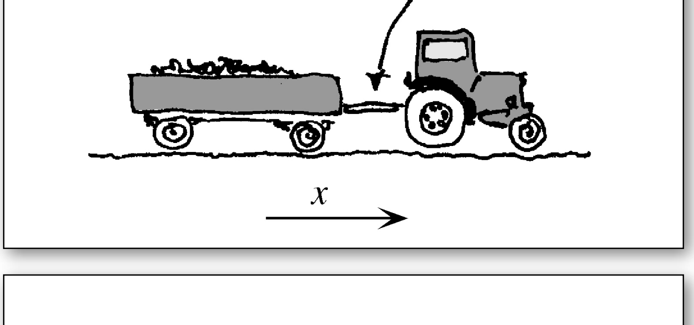


Fig. 6.9
For Exercise **1**

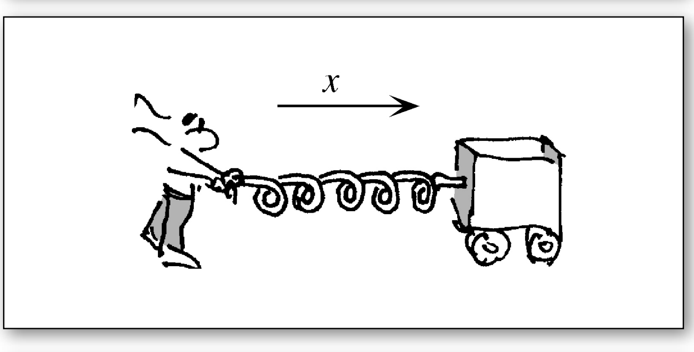


Fig. 6.10
For Exercise **2**

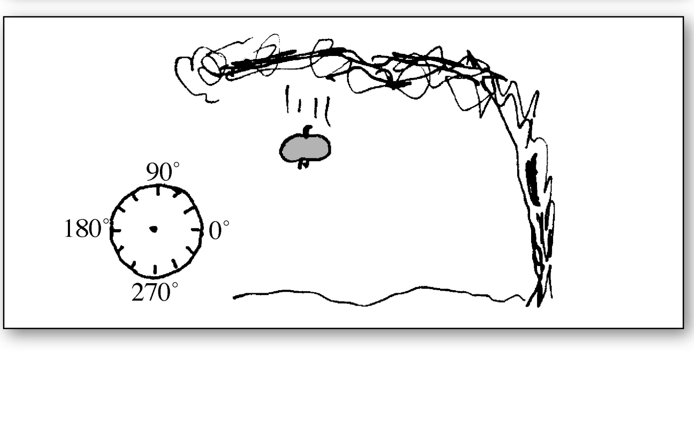


Fig. 6.11
For Exercise **3**

6.3 Adding vectors

Again we take the example of the pond with the raft, Fig. 6.12. The raft has momentum to the right. Physically speaking: It has 0-degree momentum, say 500 Hy. The person pushes the pole against the raft from below (in the figure). He guesses so that 50 Huygens per second of 90-degree momentum go into the raft. He pushes for three seconds. How much momentum does the raft have at the end?

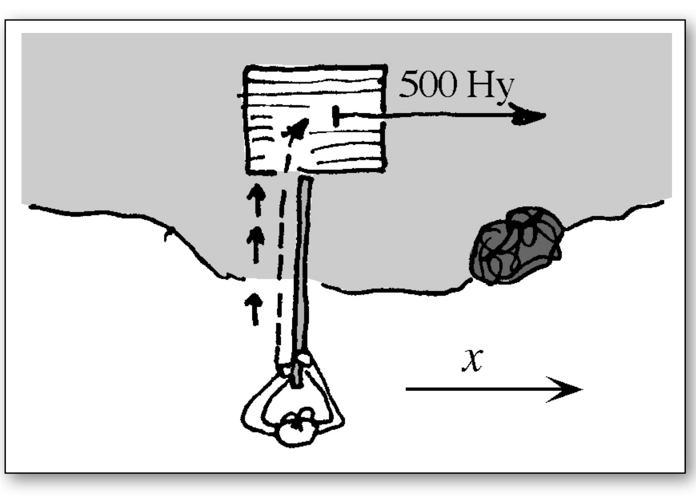


Fig. 6.12
The raft has 500 Hy of 0-degree momentum and receives 150 Hy of 90-degree momentum.

It now has 500 Hy of 0-degree momentum and 3 · 50 Hy of 90-degree momentum. How much is this in all? What kind of momentum is it?

You can imagine that the raft moves neither parallel nor transverse to the *x*-direction, but somehow diagonally (upward and to the right in our figure). The total momentum is neither 0-degree nor 90-degree.

The question of how much total momentum there is can be reformulated as follows: How do you add up vectors? How do you add up 500 Hy of 0-degree momentum und 150 Hy of 90-degree momentum?

It is easy to find the answer if the arrow representation of the momentum vectors are used.

We represent each momentum by an arrow, Fig. 6.13a. We call the arrows $\vec{p}_1$ and $\vec{p}_2$. We now connect them in such a way that the starting point of $\vec{p}_2$ coincides with the tip of $\vec{p}_1$, Fig 6.13b. Then we draw a third arrow $\vec{p}_3$ whose starting point coincides with the initial point of $\vec{p}_1$ and whose tip coincides with the tip of $\vec{p}_2$. Arrow $\vec{p}_3$ shows the total momentum we are looking for.

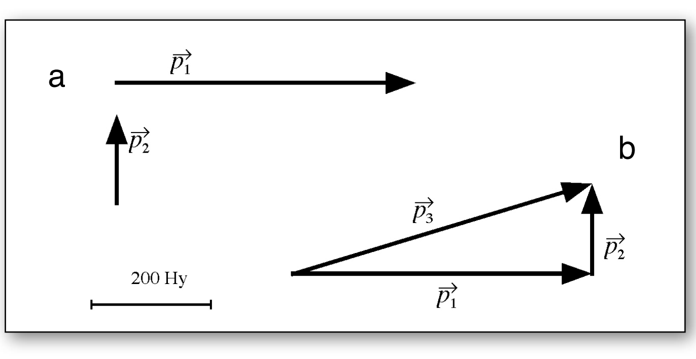


Fig. 6.13
Vector addition

We call what we just did *vector addition*. Symbolically it is expressed by

$\vec{p}_1 + \vec{p}_2 = \vec{p}_3$.

One obtains the same result if $\vec{p}_2$ is attached to $\vec{p}_1$ as one does by attaching $\vec{p}_1$ to $\vec{p}_2$, Fig. 6.14. Like usual addition, vector addition is commutative.

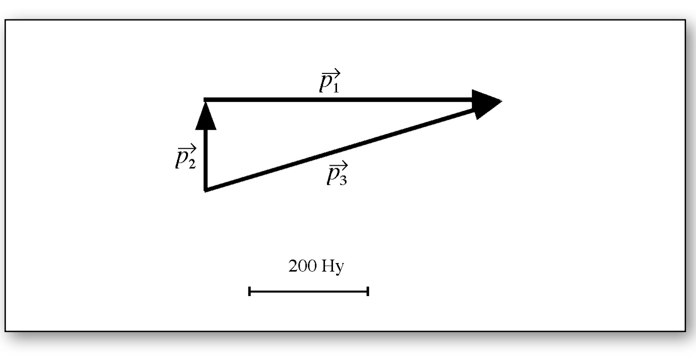


Fig. 6.14
Vector addition is commutative.

Example

A 0.5 kg stone is thrown horizontally, Fig. 6.15. Just after being thrown it has 3 Hy of 0-degree momentum. Because of its gravity, it constantly receives momentum from the Earth. This is 270-degree momentum. How much and what kind of momentum does it have after 2 seconds?

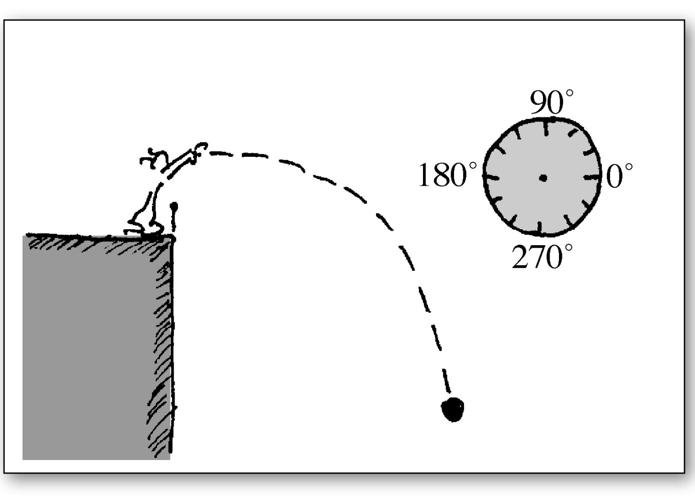


Fig. 6.15
At the beginning, the stone has 0-degree momentum. It constantly receives 270-degree momentum through the gravitational field.

We calculate the momentum current from the Earth:

$F = m \cdot g = 0.5 \text{ kg} \cdot 10 \text{ N/kg} = 5 \text{ N}$

The stone receives a momentum current of 5 Hy per second from the Earth. The momentum coming from the Earth in 2 seconds is

$p = F \cdot t = 5 \text{ Hy/s} \cdot 2 \text{ s} = 10 \text{ Hy}$

Now we add

$\vec{p}_1 + \vec{p}_2 = \vec{p}_3$.

where

$\vec{p}_1$: 3 Hy of 0-degree momentum

$\vec{p}_2$: 10 Hy of 270-degree momentum

Fig. 6.16 shows the solution. We can find the magnitude of the total momentum from Pythagoras' rule:

$$\text{Magnitude of } \vec{p}_3 = \sqrt{(3 \text{ Hy})^2 + (10 \text{ Hy})^2}$$
$$= \sqrt{9 + 100} \text{ Hy}$$
$$= 10.44 \text{ Hy}$$

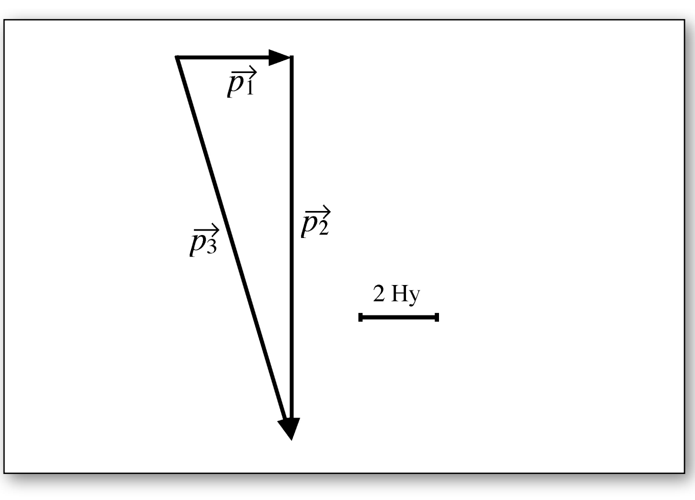


Fig. 6.16
Vector arrows of the toss in Fig. 6.15.

Example

Two people pull a boat through a canal. One person is on one bank and the other person is on the opposite bank, Fig. 6.17. The ropes are at 30-degree angles to the canal. (Or better: the upper rope in the figure creates a 30-degree angle and the lower one a 330-degree angle). A momentum current of 90 N flows in each rope. How much momentum per second does the boat receive? What kind of momentum is it?

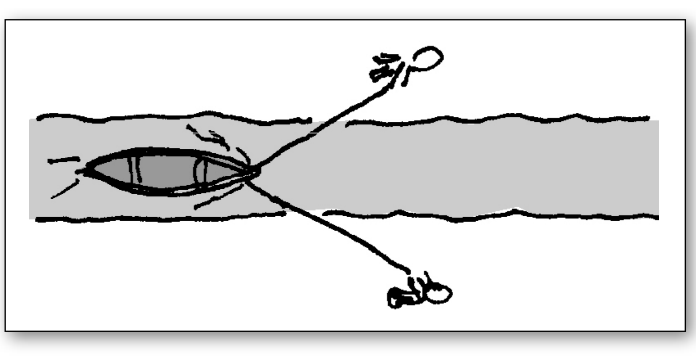


Fig. 6.17
The boat receives 30-degree momentum from one person, and 330-degree momentum from the other person.

In the upper rope, 90 N of 30-degree momentum flows, and in the lower one, 90 N of 330-degree momentum flows. In Fig. 6.18, the two current vectors have been combined. The total current into the boat is the sum of the vectors. From the drawing we see that:

Total current: 156 N of 0-degree momentum.

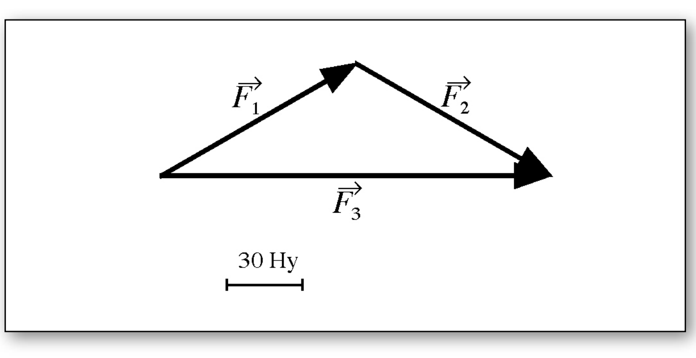


Fig. 6.18
The current vectors for Fig. 6.17.

Example

A car drives around a 90-degree curve, Fig. 6.19a. The magnitude of its momentum is the same before and after, namely, 30,000 Hy. While going through the curve, the car receives momentum from the ground. We have the following rule: initial momentum of the car + momentum from the ground = final momentum of the car. The plus sign symbolizes addition of vectors.

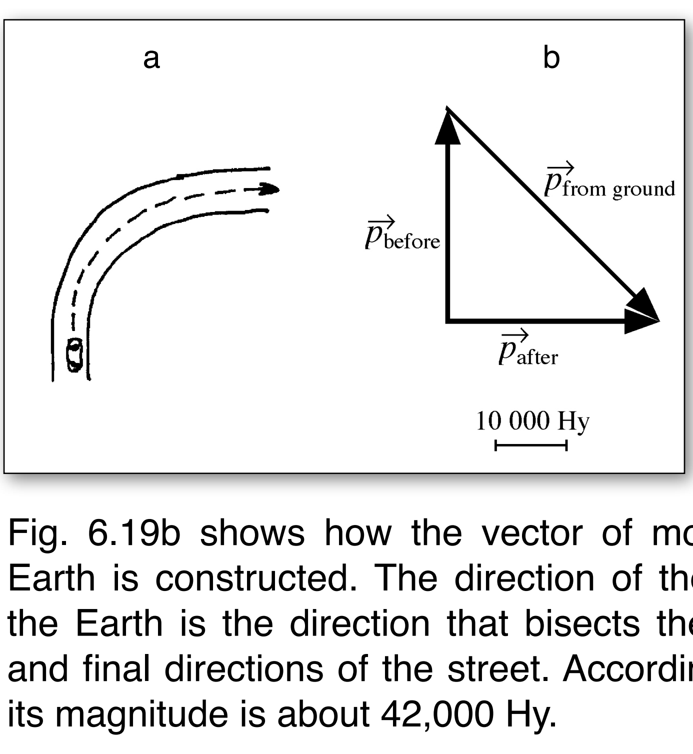


Fig. 6.19
(a) An automobile drives through a 90° curve. (b) Vector construction of the momentum received by the auto from the ground.

Fig. 6.19b shows how the vector of momentum coming from the Earth is constructed. The direction of the momentum coming from the Earth is the direction that bisects the angle between the initial and final directions of the street. According to the Pythagorean law, its magnitude is about 42,000 Hy.

Exercises

1. A stone weighing 100 g is thrown horizontally off a tower. Its initial momentum is 0.5 Hy.

a) How much and what kind of momentum does it receive from the Earth within one second?

b) Construct the vector of the momentum of the stone one second after being thrown.

c) What is the magnitude of the momentum in the stone one second after being thrown?

2. A 0.3 kg stone is thrown horizontally from a tower. Its initial velocity is 5 m/s.

a) What is the magnitude of its initial momentum?

b) At a certain point, the angle at which the stone falls is 45° to the vertical. How much momentum has the stone received from the Earth until this point? Draw the vector diagram. What is the magnitude of the momentum at this point?

3. A sphere having a mass of 3 kg is pushed diagonally upward at a 45° angle. At the beginning, it has 12 Hy of momentum. After what amount of time does it move diagonally downwards at an angle of 45°?

4. A train with a mass of 1,200 t and a velocity of 70 km/h, goes around a 30° curve. Construct the vector of the momentum received by the train from the Earth.

5. A car goes around a 90° curve. Its velocity before going around the curve was 30 km/h, afterwards it is 50 km/h. The car's mass is 1,400 kg. Draw the vector of the momentum going into the car while it is traveling around the curve. What is the magnitude of this momentum?

6. The goalie throws the ball out onto the field. A player kicks it right back to the goal. Use words to describe where the ball gets its momentum from or to where it loses it on its way. Take air friction into account.

6.4 Satellites, moons, and planets

We have seen that an object near the Earth's surface receives 270-degree momentum. If it is let go from a state of rest, the 270-degree momentum increases and the object moves towards the Earth.

If the object is not only let go, but thrown horizontally, it also falls to the Earth, Fig. 6.15.

Imagine that we throw the object off a high mountain in such a way that it receives a lot of 0-degree momentum. Fig. 6.20 shows the flight path for three different values of the initial 0-degree momentum.

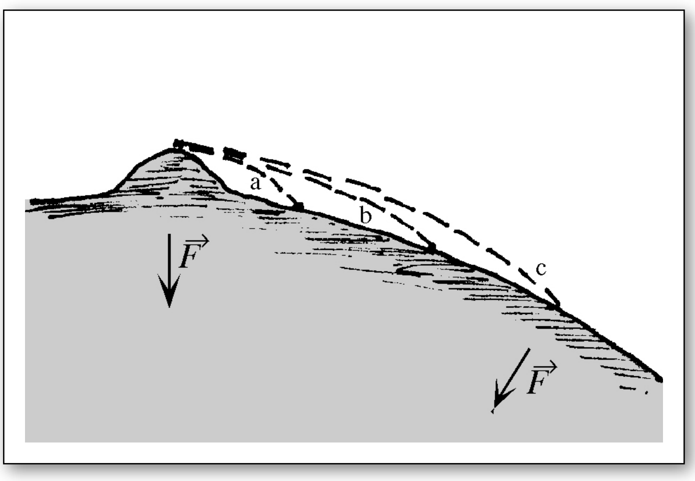


Fig. 6.20
An object is thrown from the top of a high mountain. The three trajectories correspond to three different values of the initial momentum.

The body travels so far that the curvature of the Earth becomes noticeable. Now something remarkable happens: Near the starting point, the body receives 270-degree momentum. During the body's flight, the direction of the new momentum flowing into it, changes. At the end of the trajectory c, the body in Fig. 6.20 receives 240-degree momentum instead of 270-degree momentum.

If we manage to give the initial momentum a very large value, the situation as seen in Fig. 6.21 will occur. The body falls and falls but in spite of this, never approaches the Earth.

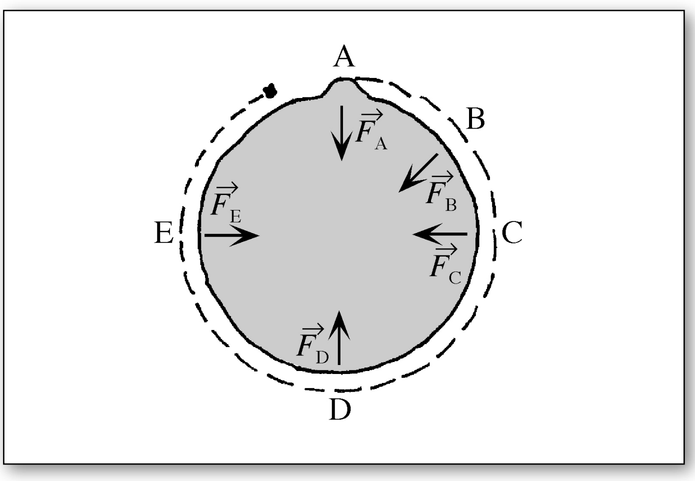


Fig. 6.21
The direction of the momentum coming from the Earth is transverse to the direction of the momentum already in the flying object.

At point A it receives 270-degree momentum, at B it receives 225-degree momentum, at C 180-degree momentum, at D 90-degree momentum, at E 0-degree momentum, etc. The momentum flowing in causes the path of the body to be bent toward the Earth at every point. If the initial momentum is chosen appropriately, the body will describe a circular orbit.

The direction of the momentum flowing into the body at each moment is at a right angle to the direction of the momentum that it has at that moment. The body at location B contains 315-degree momentum and it receives 225-degree momentum. At C it has 270-degree momentum and it receives 180-degree momentum, etc.

Hopefully you have noticed that we have not discussed a silly and totally unrealistic thought experiment.

When it is said that a satellite has been brought into its "orbit", this means

– that it has been brought to a certain altitude, and

– that it has been given the right amount of horizontal momentum so that it travels around a circular orbit.

If the satellite receives too little momentum, it falls to the Earth like the one in Fig. 6.20. If the body receives too much momentum for the circular orbit, it flies out further: It then takes an elliptical path, Fig. 6.22.

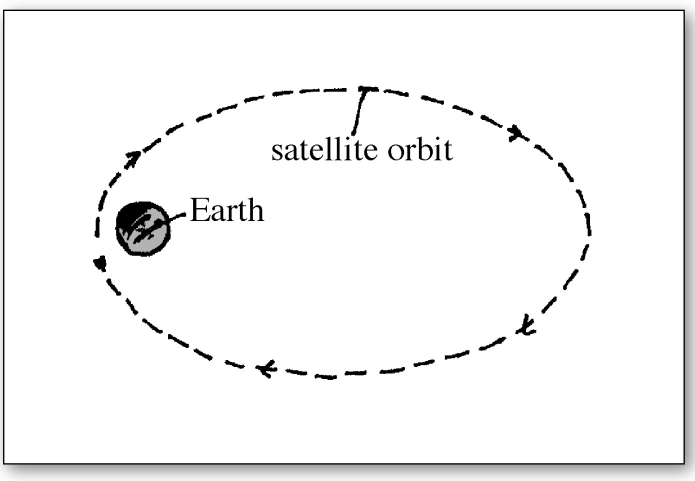


Fig. 6.22
If the initial momentum is greater than needed for a circular orbit, the satellite moves in an elliptic orbit.

Only at a much greater initial momentum does it actually fly away from the Earth. If this happens, it is no longer a satellite, but a *space probe*. (A very successful space probe was Voyager 2. After about ten years in flight, it left the solar system.)

Satellite movement is not something that people invented. It existed in nature long before people ever did. The motion of our natural satellite, i.e., that of the Moon, around the Earth is the same as the motion of artificial satellites. However, the Moon is much further away. Satellites move at an altitude of between 200 km and 40,000 km from the Earth's surface, while the Moon is at a distance of almost 400,000 km.

You probably know that not only the Earth, but other planets as well, have moons.

The movement of the Earth and of the other planets around the sun is of the type you just learned about. The Earth and the other planets constantly receive momentum from the Sun. The direction of the momentum that the Earth receives at any moment is transverse to the momentum it already has at that moment.

6.5 Wheels

We were dealing with momentum conductors and insulators before we knew that momentum is a vector. At that point, there was only one sort of momentum and we observed motion in only one direction. We discovered the rule that tells us that wheels serve the purpose of momentum insulation.

This rule becomes more complicated when momentum of different directions flows.

Fig. 6.23 shows once again how wheels hinder the flow of momentum into the ground. The person pulls and momentum flows through the rope and into the wagon. It builds up in the wagon because it cannot flow out through the wheels into the Earth. The wagon then picks up speed.

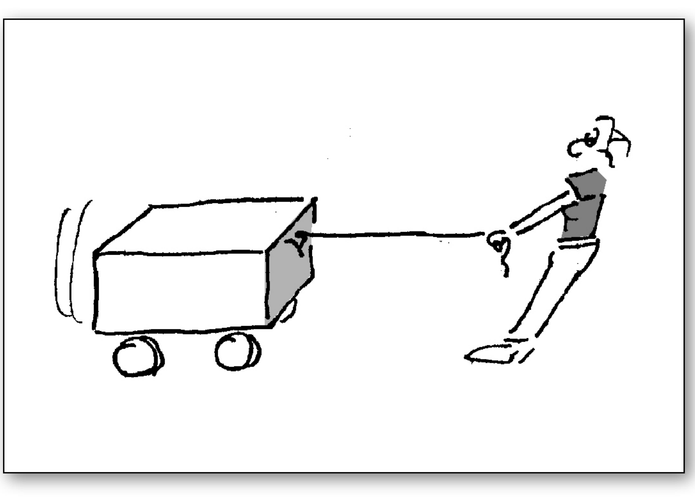


Fig. 6.23
The wagon is insulated from the ground by the wheels so the momentum coming from the person cannot flow off. It builds up in the wagon.

In Fig. 6.24, a person is also pulling on a wagon. In spite of the wheels, the momentum does not stay in the wagon. It flows off and the wagon remains still. The difference to what is happening in Fig. 6.23, is that the momentum flowing into the wagon is transverse to the wheels. So:

> Wheels let transverse momentum flow into the Earth. Lengthwise momentum is not allowed to pass.

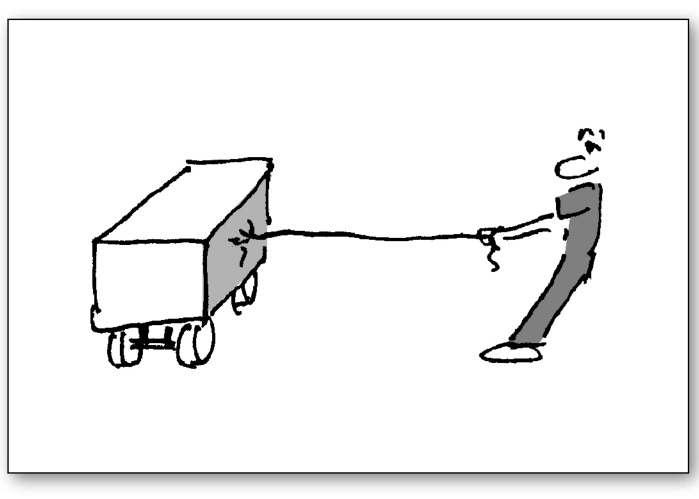


Fig. 6.24
The momentum coming from the person flows into the ground. Transverse momentum is not retained by the wheels.

Wheels let transverse momentum flow into the Earth. Lengthwise momentum is not allowed to pass.

We have perhaps been a bit too black-and-white in our description here. Actually, a little lengthwise momentum flows into the Earth due to friction. It is also possible, by pulling very strongly (producing a very strong momentum current), to make the conducting connection for the transverse momentum in Fig. 6.24 break down, Fig. 6.25.

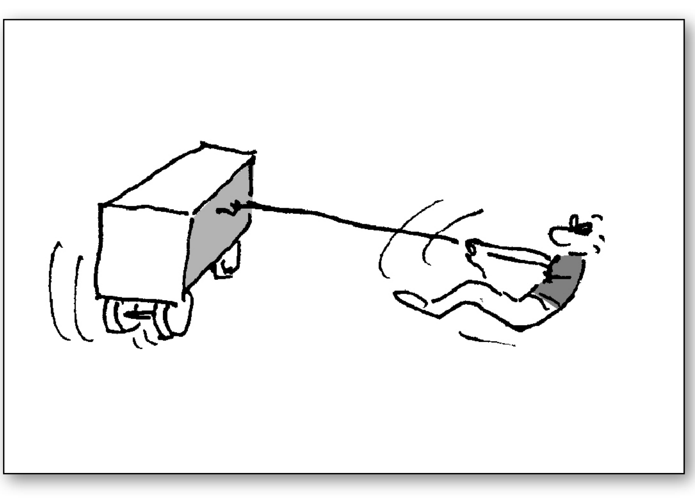


Fig. 6.25
If the momentum current is too strong, the conductive connection breaks down.

The car in Fig. 6.26 that is going around a curve, needs to get rid of its 0-degree momentum. This works because the tires let the transverse momentum flow into the ground, except in the case of an icy road when tires are insulators for momentum of every direction. Vehicles on tracks are safer in that situation. Transverse momentum can always be conducted off by them very well.

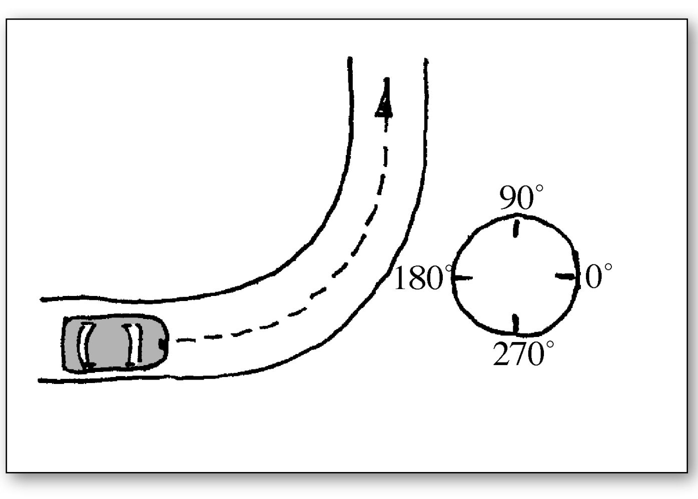


Fig. 6.26
Going around the curve, the car must give its 0-degree momentum to the ground and receive 90-degree momentum from the ground.

The conditions are much less clear with a ship. Lengthwise momentum does not flow off into water as well as transverse momentum does, but the difference is not as great as with vehicles on land.

Sometimes a vehicle is not supposed to lose lengthwise momentum or transverse momentum to the ground. A method to achieve this is to attach the wheels so that they can change their direction. You probably have tables for experiments in your physics classroom that move on these kinds of wheels, Fig. 6.27.

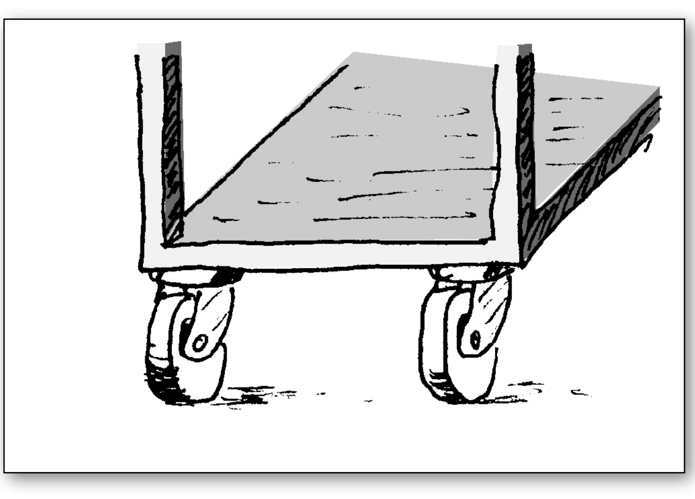


Fig. 6.27
The wheels pivot. They allow neither their longitudinal momentum nor transverse momentum to pass through.

Are we finished with wheels yet? Not quite. We have observed motion on the surface upon which the wagon rolls. The third direction is still missing.

Take a little car and press it from above against a tabletop. Of course it does not move. Lift it vertically upward and it moves upward. Do the same thing again but this time against a wall instead of the tabletop. If the wagon is pressed against the wall it doesn't move, momentum flows off, Fig. 6.28. If you pull it away from the wall, it starts to move, momentum is not flowing out. By the way, the wheels were totally unnecessary in this experiment.

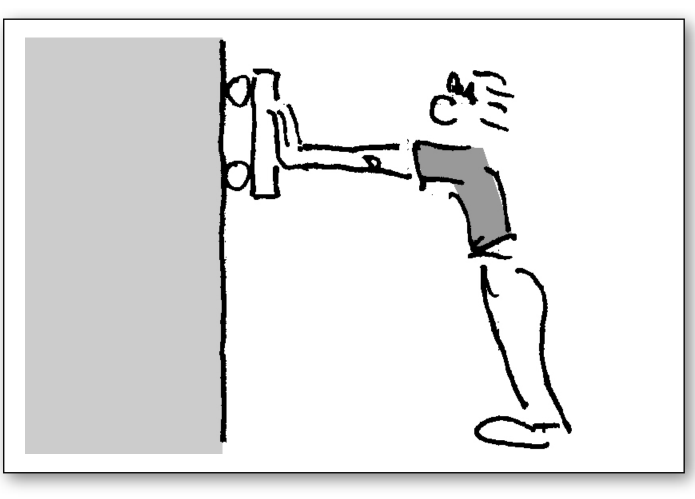


Fig. 6.28
Momentum that is transverse to the platform of the car, is conducted into the wall.

Example

A 20 kg wagon stands upon a sloping street and the brakes are then released, Fig. 6.29. What does the wagon do? Through the gravitational field, 270-degree momentum continuously flows into the wagon. The magnitude of the momentum current is

$F = m \cdot g = 20\ \text{kg} \cdot 10\ \text{N/kg} = 200\ \text{N}$

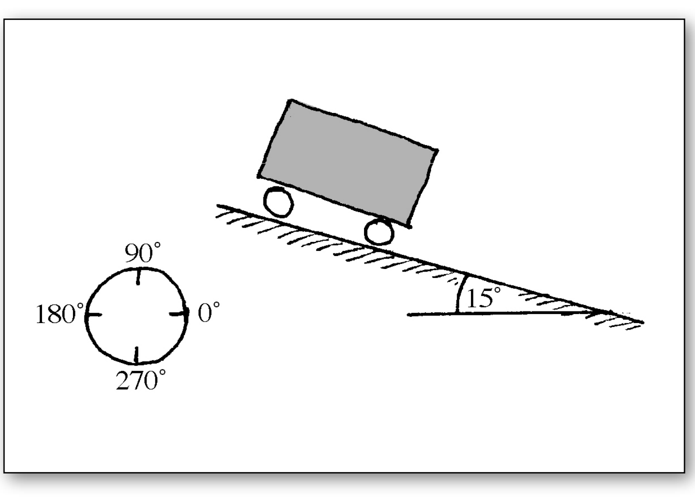


Fig. 6.29
270-degree momentum flows into the wagon. This is divided into a portion flowing into the Earth and a portion that builds up in the wagon.

What happens with this momentum? Does it flow into the ground? Does it build up?

We do not know what happens with the 270-degree momentum, but we do know what happens with the momentum that is parallel to the bottom of the car and the momentum that is perpendicular to its bottom.

The lengthwise momentum of the wagon is 345-degree momentum. It cannot flow off, so it has to build up in the wagon.

The momentum that is transverse to the car's bottom is 255-degree momentum. It flows entirely into the ground.

All we have to do is decompose the 270-degree momentum current $\vec{F}$ coming in, into a 345-degree current $\vec{F}_{long}$, and one of 255-degree current $\vec{F}_{trans}$, Fig. 6.30. From the figure, we can extract that

$\vec{F}_{long} = 50\ \text{N}$
$\vec{F}_{trans} = 190\ \text{N}$

Therefore: 190 Hy per second flow into the Earth, and the momentum of the wagon increases by 50 Hy.

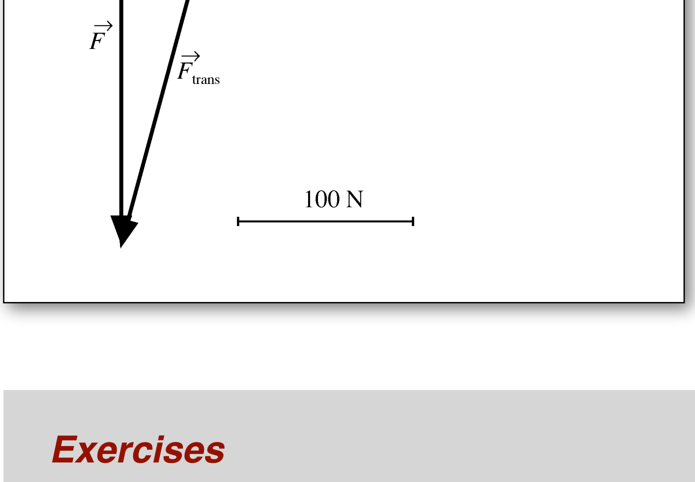


Fig. 6.30
The current vector of 270-degree momentum decomposed into components of 255-degree and 345-degree momentum.

Exercises

1. A cylindrically formed handle Z can slide without friction along a rod S, Fig. 6.31a. What kind of momentum can pass through the connection between handle and rod? Which momentum cannot pass?

2. Cylinder Z1 can glide along the rod S and cylinders Z2 and Z3 can glide back and forth on the frame R, Fig. 6.31b. Which kind of momentum can pass through the connection between Z1 and the frame? Which one cannot pass?

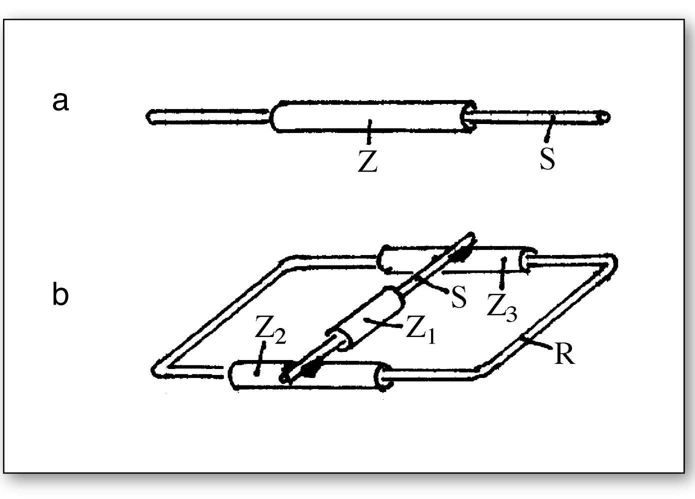


Fig. 6.31
(a) For Exercise **1**
(b) For Exercise **2**

6.6 Ropes

Here is an old rule we still have to complete: Ropes conduct momentum in only one direction.

It isn't easy to see what is happening in Fig. 6.32: The person is trying to set the wagon in motion sideways with the help of the rope. Of course he doesn't succeed. Using momentum for a description: He tries to send 90-degree momentum through a rope lying in 0-degree direction. This doesn't work because ropes are choosy.

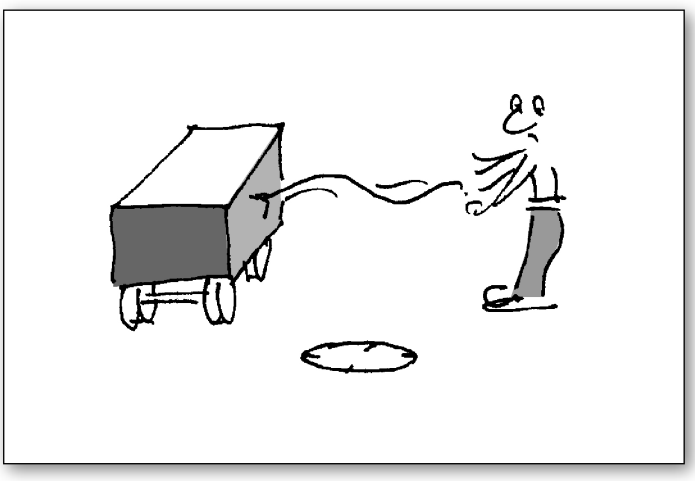


Fig. 6.32
Someone is trying to move a wagon transversely to the direction of the rope.

> Only momentum lying parallel to a rope can be sent through it, and in only one direction.

We want to apply this newly formulated rule. Fig. 6.33 shows a wagon from above that is being pulled by a rope. It is not being pulled forward, but a bit to the side. A momentum current of 40 N flows through the rope. What is the change of momentum per second of the wagon? How much momentum flows into the ground?

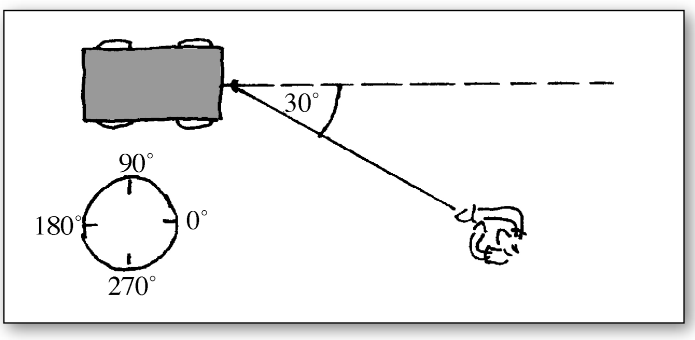


Fig. 6.33
Wagon with rope, seen from above. The rope is pulled. The 0-degree momentum of the wagon increases.

The momentum flowing through the rope must have the same direction as the rope. We call the corresponding current vector $\vec{F}$, Fig. 6.34. It is decomposed into two components:

– a component $\vec{F}_{trans}$, which lies transverse to the wagon and flows off over the wheels;

– a component $\vec{F}_{long}$, which lies in the direction of the wagon and causes the increase of the wagon's momentum.

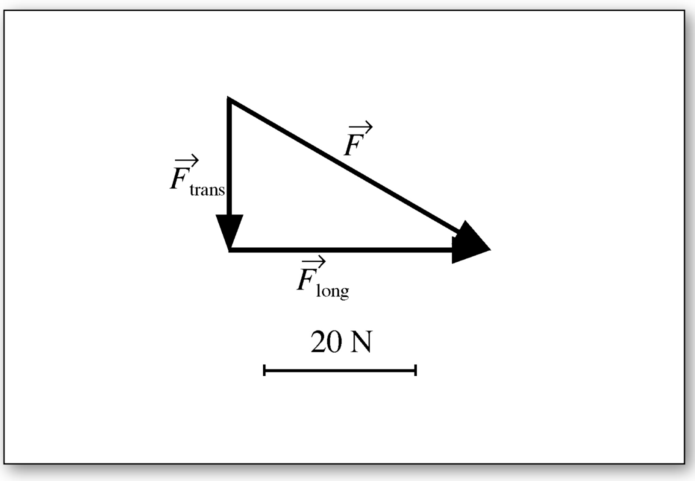


Fig. 6.34
The momentum current vector of the rope is decomposed into longitudinal and transversal components.

We extract from the Figure that:

F_{trans} = 20 N

F_{long} = 34 N.

As a result, 20 Hy of transverse momentum flows into the Earth, and the longitudinal momentum of the wagon increases by 34 Hy.

Exercises

1. A toy car is pulled by a rope across a horizontal floor. The rope runs diagonally upward between the person and the car, Fig. 6.35. A momentum current 20 N flows through the rope. How large is the part that contributes to the forward motion of the car?

2. One car tows another. The cars travel in the same direction, but shifted sideways by 1 m, Fig. 6.36. The tow rope is 3 m long. A momentum current of 500 N flows through the rope. Which momentum current contributes to the motion of the car?

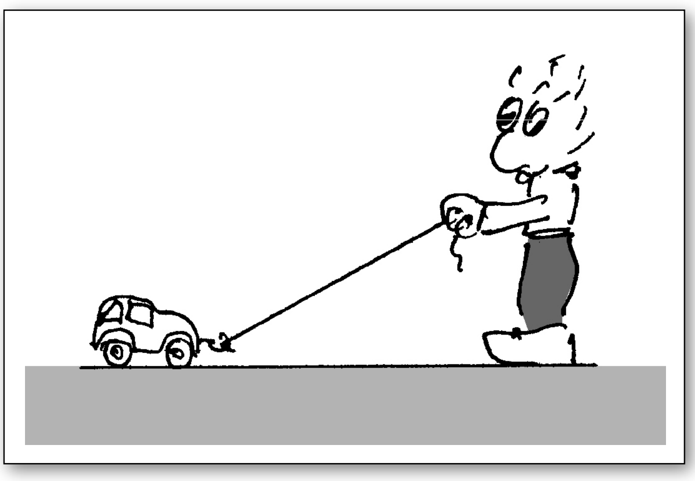


Fig. 6.35
For Exercise **1**

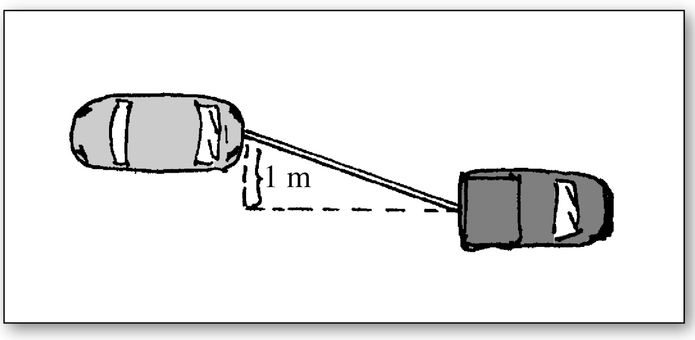


Fig. 6.36
For Exercise **2**

6.7 The junction rule for momentum currents

A lamp is hung by ropes from the walls of two neighboring houses, Fig. 6.37. It has a mass of 3.5 kg. How are the hooks on the walls stressed?

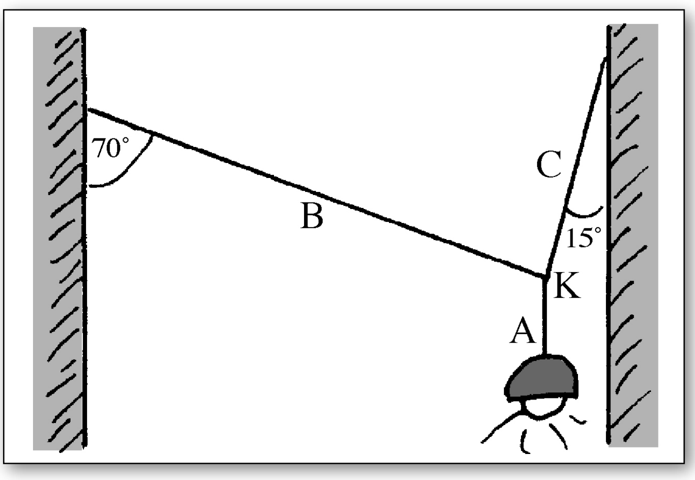


Fig. 6.37
How strongly are the two walls of the buildings stressed by the lamp?

Hopefully you have noticed that the momentum current vectors in ropes B and C are asked for here. We can easily find the momentum current in rope A. 270-degree momentum flows through the gravitational field into the lamp. The magnitude of the corresponding current is

$F = m \cdot g = 3.5\ \text{kg} \cdot 10\ \text{N/kg} = 35\ \text{N}$

This momentum current flows through rope A to *junction* K. How does it travel from K?

Because B is a rope, the only momentum that can flow here is the momentum lying parallel to B. The corresponding holds for C. Therefore, we must *decompose* vector $\vec{F}_A$ into two vectors. One, $\vec{F}_B$, lying parallel to B and the other, $\vec{F}_C$, lying parallel to C. The total current in B and C must be equal to that in A. This means that

$\vec{F}_A = \vec{F}_B + \vec{F}_C$.

Fig. 6.38 shows the decomposition. $\vec{F}_A$ is parallel to rope A, $\vec{F}_B$ is parallel to rope B, and $\vec{F}_C$ is parallel to rope C. By measuring the lengths of the vectors $\vec{F}_B$ and $\vec{F}_C$ we find

$\vec{F}_B = 9\ \text{N}$

and

$\vec{F}_C = 33\ \text{N}$.

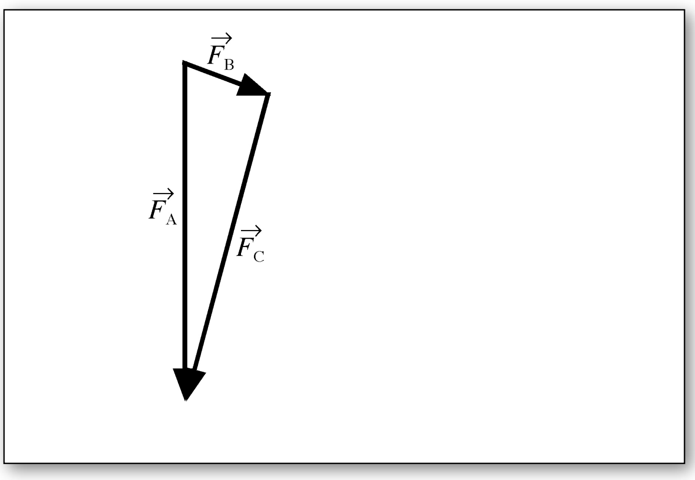


Fig. 6.38
The momentum current flowing in rope A to node K is equal to the momentum currents flowing away through ropes B and C.

Have you noticed that we are dealing with an old friend here? We have used a junction rule. The junction rule for momentum currents is:

> The total of momentum currents flowing into a junction are equal to the ones flowing out of it.

A junction is a location where three or more momentum currents meet. The currents are combined according to the rules of vector addition.

Exercises

1. Two tugboats tow a ship, Fig. 6.39. Each tugboat pulls with 15,000 N. What is the momentum current through the piece of rope attached to the ship?

2. A 10 kg object is to be hung from the hooks in Fig. 6.40. The rope can take a momentum current of up to 200 N. Beyond this, it will tear. What happens? Can it take the weight or not?

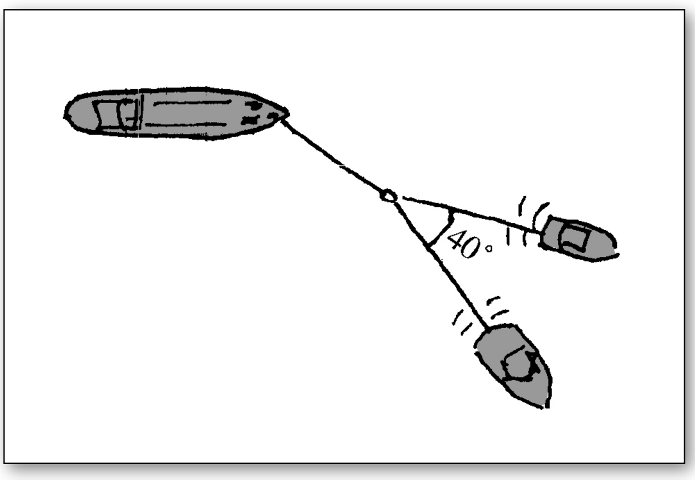


Fig. 6.39
For Exercise **1**

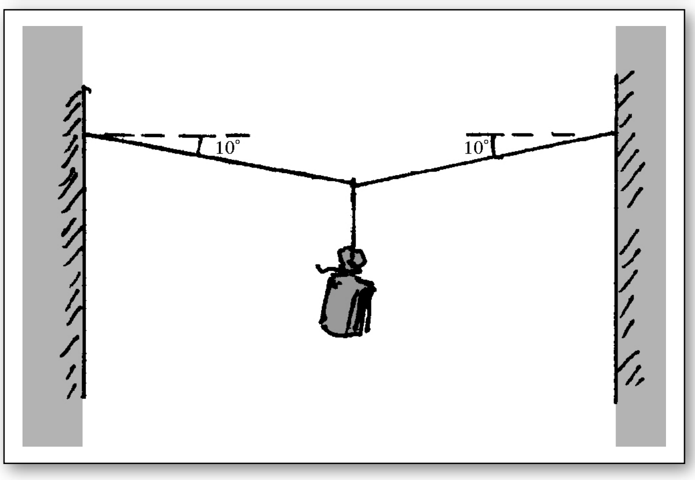


Fig. 6.40
For Exercise **2**

7

Torque and center of mass

7.1 Pulley wheels and pulley blocks

Wheels that are turned by chains or drive belts are very important in technology. We see wheels with ropes as deflection sheaves in cranes or pulleys (pulley blocks). You know chain wheels from bicycles and motorcycles. Often there are wheels with v belts hidden inside machines. Large drills would be an example of this. Flat belts were very often used for driving machines in the factories of earlier times.

In the following, we will deal with the flows of momentum and energy through such wheels. The first wheels we will consider are freely rotating ones: so-called *rollers, pulley wheels,* or simply *wheels*. They are not fixed to an axle driving something but are mounted in such a way that they spin easily.

Fig. 7.1 shows a wheel with a rope around it. There is a force sensor inserted into each of the ropes A, B, and C. They measure the strength of the momentum current through the corresponding rope. The loop at the right end of rope A is pulled so that the force sensor shows 12 N. What do the sensors on B and C show?

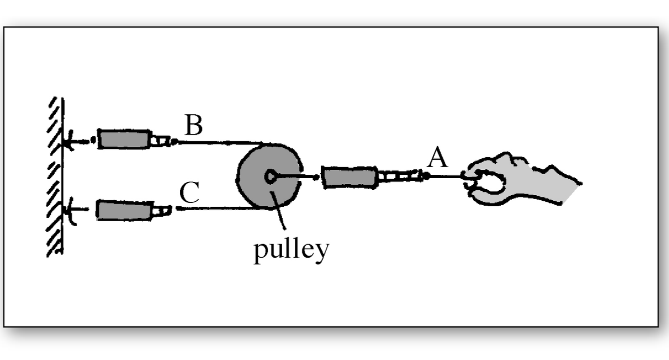


Fig. 7.1
Sensor A shows twice as much as B or C do.

We can predict the answer. We know that the momentum current flowing through rope A flows further through B and C. Therefore

$F_A = F_B + F_C$.

Because the whole arrangement is symmetrical, the following is also valid:

$F_B = F_C$.

As a result we have

$F_B = F_A/2$

and

$F_C = F_A/2$.

If F_A = 12 N, then F_B = 6 N und F_C = 6 N.

The dashed lines in Fig. 7.2 show the paths of the momentum in the ropes. The arrows next to the lines show the orientation of the momentum. These arrows are parallel to the ropes.

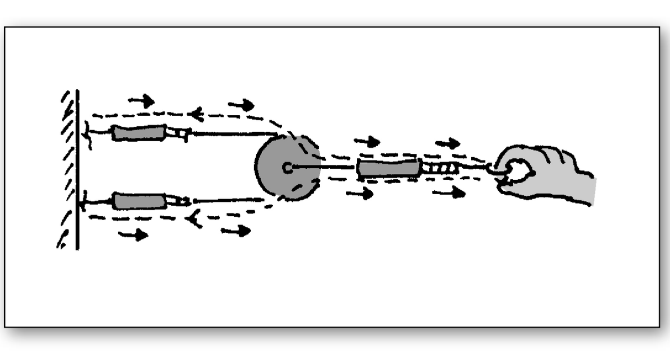


Fig. 7.2
The momentum current coming from the right branches at the wheel into two equal parts.

The loop can be pulled more strongly or less strongly. The force sensor in rope A will then show greater or smaller values. However, the magnitudes of the momentum currents in ropes B and C also change so that

$F_B = F_C = F_A/2$

is always valid.

Of course, nothing changes if ropes B or C are pulled instead of A, Fig. 7.3.

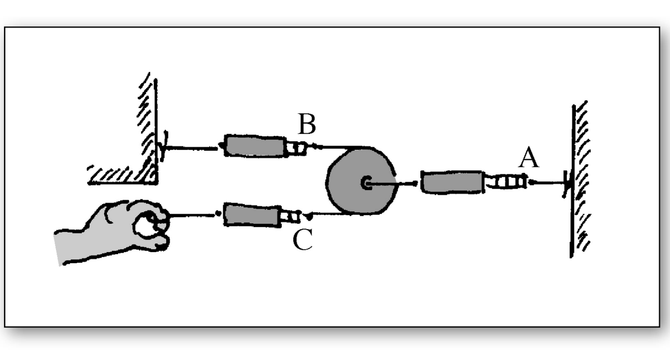


Fig. 7.3
It doesn't matter if you pull A, B or C.

We will now deal with a more complicated case, Fig. 7.4.

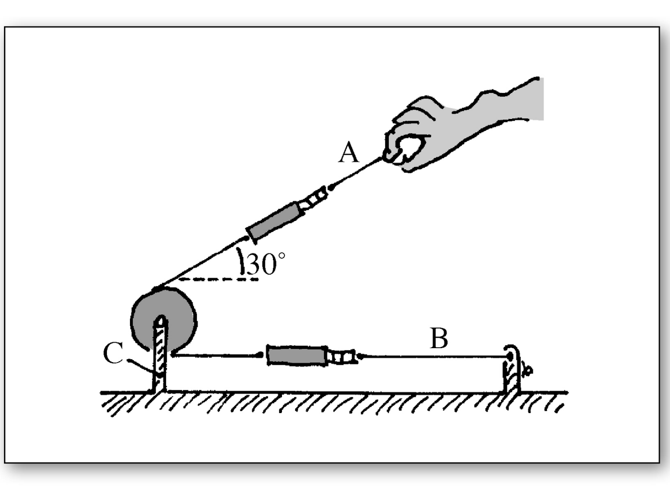


Fig. 7.4
Here, as well, the sensors A and B show the same value. However, the currents are not equal.

We pull on rope A and find out that no matter how hard we pull, the sensor in A shows the same as the one in B. In spite of this, the momentum currents in A and B are not the same. They have the same magnitude but different directions. Momentum always has the same direction as the rope it flows in. In rope A, 30-degree momentum flows toward the pulley wheel, and in rope B, 0-degree momentum flows toward the pulley wheel.

The sum of both types of momentum flows through the mounting of the wheel, and into the ground. By "sum" we mean the sum of the vectors, Fig. 7.5.

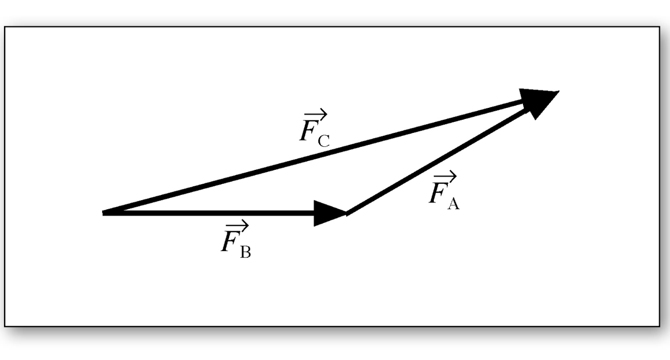


Fig. 7.5
Vector addition of momentum currents in the ropes of Fig. 7.4.

Because the magnitudes of vectors $\vec{F}_A$ and $\vec{F}_B$ are the same, $\vec{F}_C$ has the direction of the bisectrix of the angle between the two ropes.

Fig. 7.6 shows a variation of the situation in Fig. 7.4. Because the pulley wheel is attached to the wall by a rope (and not rigidly fixed), Conductor C must have the same direction as the momentum flowing there. This is noticed when A is pulled. Rope C adjusts itself so that the direction is the bisectrix of the angle between A and B.

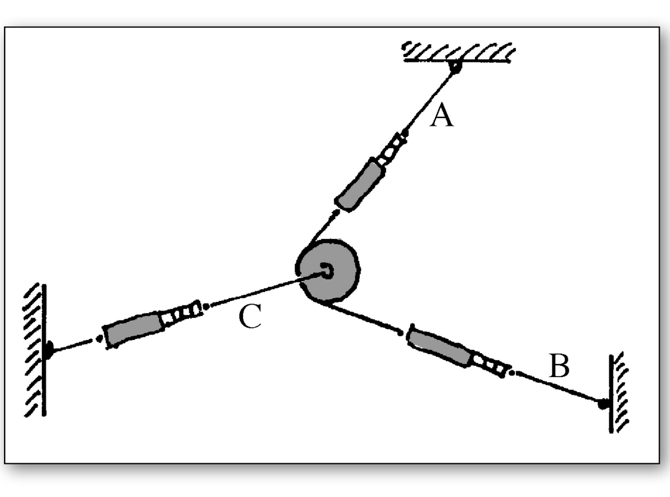


Fig. 7.6
Rope C adjusts itself so it is the bisectrix of the angle between A and B.

The three spring force sensors show that

$\vec{F}_C = \vec{F}_A + \vec{F}_B$

and

$\vec{F}_A = \vec{F}_B$.

We summarize:

If a rope runs over a freely rotating wheel (a pulley wheel), the momentum currents in both parts of the rope are of equal magnitudes.

The rule emphasizes the fact that the wheel must spin freely. Do you know why? Imagine that the wheel in Fig. 7.4 is blocked and the rope cannot slide over it. Rope A could be pulled without rope B noticing anything: The currents $\vec{F}_A$ and $\vec{F}_B$ do not have the same magnitudes anymore.

What can be done with pulleys?

A weight is to be hoisted by a motor, Fig. 7.7. The rope the weight is hanging from is attached to the motor over two pulley wheels. In all three parts of the rope, the magnitude of the momentum current is the same, but the direction is not.

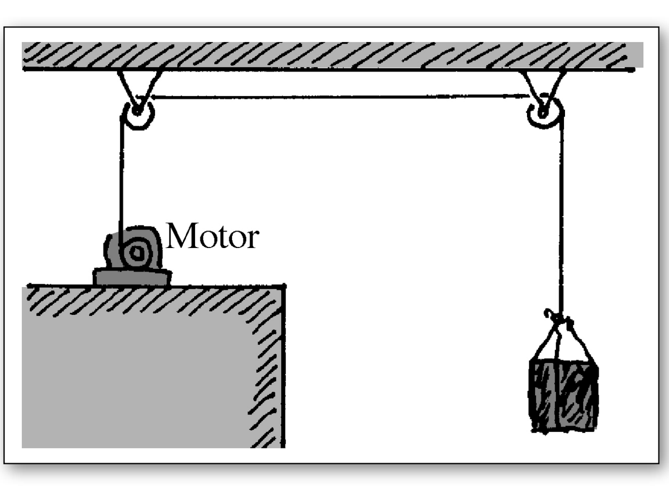


Fig. 7.7
The motor is underneath. In order to raise the load, the rope is looped around tow wheels.

This application of pulleys is so simple that we do not need to use any complicated physical considerations to understand it. However, there are more difficult applications.

Again a weight is lifted by a motor, Fig. 7.8. The rope moves differently than before though.

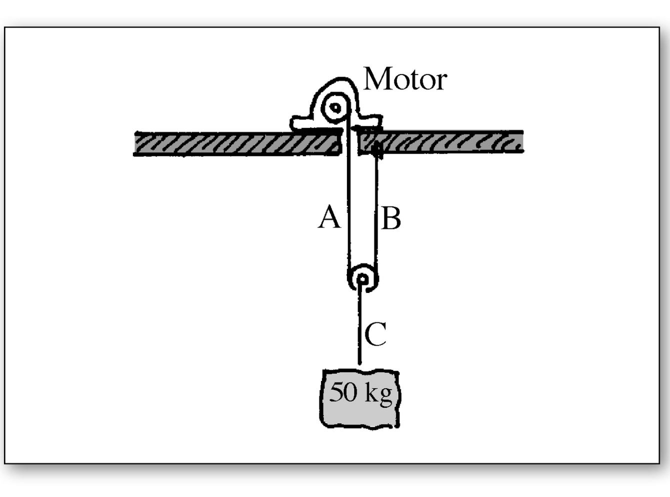


Fig. 7.8
The momentum current in the pull-rope is half that in the rope carrying the weight.

The momentum current flowing in rope C is

$F_C = m \cdot g = 50\ \text{kg} \cdot 10\ \text{N/kg} = 500\ \text{N}$.

What is the current in A and B?

According to our rule, the momentum currents in A and B are the same. The ropes run parallel to each other, so not only the magnitudes but also their directions are the same. Therefore:

$\vec{F}_A = \vec{F}_B$.

Since

$\vec{F}_A + \vec{F}_B = \vec{F}_C$

we have

$\vec{F}_A = \vec{F}_B = \vec{F}_C/2$.

Each of the ropes A and B has only 250 N flowing through it. This is interesting because in order to lift the weight, the motor only needs to be half as strong as it would need to be if the pulley wheel wasn't there.

This trick, used for reducing the momentum current in a rope, is even more effectively applied by using a pulley (or rather, an entire pulley block).

Fig. 7.9 shows a somewhat unusual but easy to understand version of a pulley block.

The bearing supports of the four pulleys are attached to the ceiling.

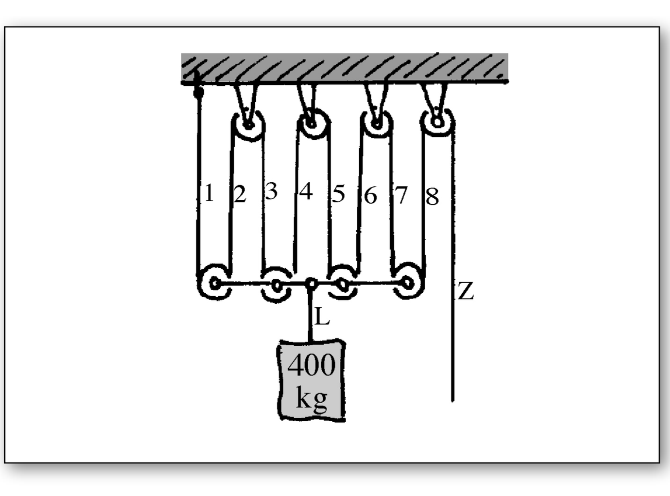


Fig. 7.9
An impractical but clearly arranged pulley block.

The ones below are attached to a rod. The weight is hanging from this rod. The first eight parts of the rope are numbered consecutively from 1 to 8. The traction rope has the letter Z and the weight is labeled L.

In order to raise the weight, Z must be pulled. What is the momentum current in Z?

We work step by step toward a solution. Rope sections 1 and 2 belong to one rope running over a wheel (wheel on the left, below), the currents in these parts must therefore be the same:

$\vec{F}_1 = \vec{F}_2$.

You see how it continues. We then have:

$\vec{F}_1 = \vec{F}_2 = \vec{F}_3 = \vec{F}_4 = \vec{F}_5 = \vec{F}_6 = \vec{F}_7 = \vec{F}_8 = \vec{F}_Z$.

Our next step is to consider the rod where the lower wheels rotate, as a node or junction. The momentum current F_L coming from the weight, flows into the rod, and the currents F_1 to F_8 flow out. The junction rule now tells us that:

$F_L = F_1 + F_2 + F_3 + F_4 + F_5 + F_6 + F_7 + F_8$.

Because the currents F_1 to F_8 are all the same, the following is valid:

$F_1 = F_2 = F_3 = F_4 = F_5 = F_6 = F_7 = F_8 = F_L/8$.

F_Z has the same magnitude as F_1 through F_8. Therefore

$F_Z = F_L/8$.

Only 1/8 of the momentum current flowing through the weight is needed to lift it. Lifting something with a pulley block is much easier than without it.

A real pulley block is hardly different from the one we have just investigated, Fig. 7.10. The upper pulley wheels are all on a common axis. Each wheel spins freely. The same is true for the lower pulley wheels.

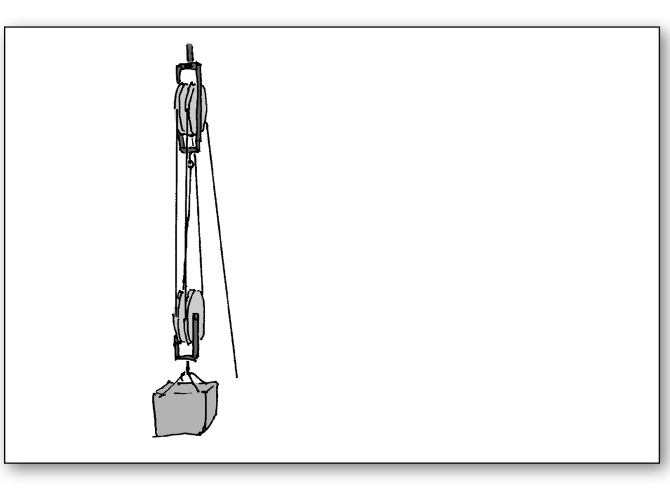


Fig. 7.10
Pulley block

These kinds of pulley blocks can be seen at freight harbors where cranes lift very heavy weights.

A pulley block is a very useful thing. We gain something: Out of a small momentum current we have created a large one.

However, as far as energy goes, we have not gained anything.

In the next section we will deal with the energy balance of a pulley block.

Exercises

1. Sketch a pulley block where the momentum current through the hook holding the weight is four times as large as in the pull rope.

2. What is the momentum current in the pull rope of the pulley in Fig. 7.11?

3. What is the disadvantage of the pulley block in Fig. 7.12?

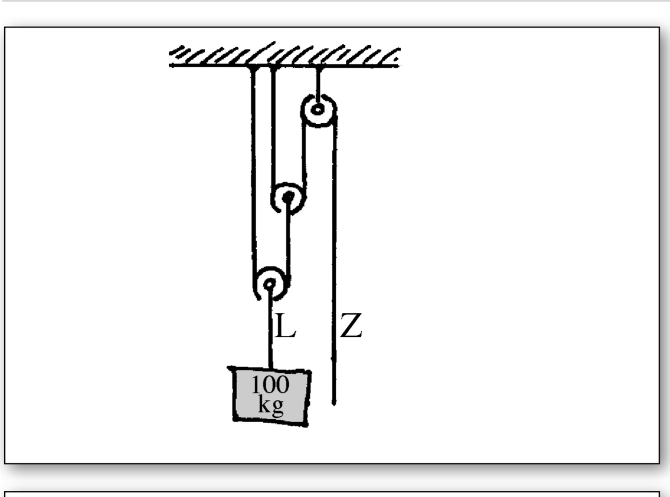


Fig. 7.11
For Exercise **2**

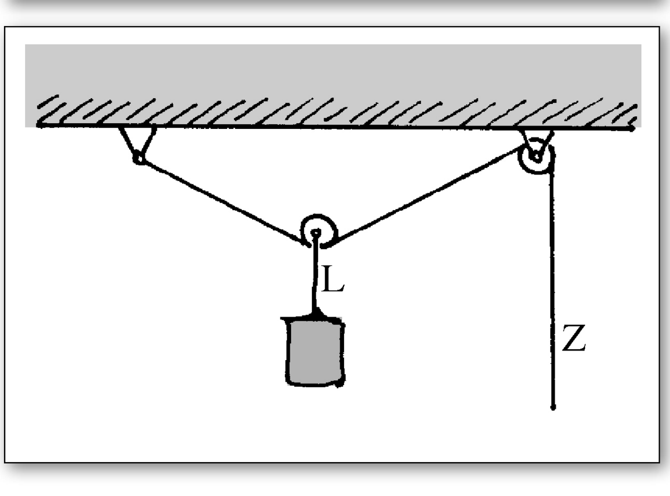


Fig. 7.12
For Exercise **3**

7.2 The balance of energy in a pulley block

The traction rope in Fig. 7.9 is pulled in such a way that its end moves the distance s_Z. According to our old formula, the amount of energy sent through the rope is:

$$E_Z = s_Z \cdot F_Z . \qquad (1)$$

Energy comes out of the pulley where the weight hangs from the rope. The amount is

$$E_L = s_L \cdot F_L . \qquad (2)$$

We want to compare E_Z and E_L. To do this, we use the relation

$$F_Z = F_L/8. \qquad (3)$$

We must also find the relation between s_L and s_Z. We ask: By what distance s_L does the weight move upward when the traction rope is pulled so that its end moves downward by the distance s_Z?

When Z is pulled, the rope sections 1 to 8 all shorten by the same amount. Each part contracts by $s_Z/8$. Each rope gets shorter by the distance s_L by which the weight is lifted. Therefore

$$s_Z = 8s_L. \qquad (4)$$

We now insert (3) and (4) into (1) and obtain:

$$E_Z = 8s_L \cdot F_L/8 = s_L \cdot F_L.$$

The amount of energy E_Z, which is put into the pulley through the traction rope Z, is equal to $s_L \cdot F_L$. According to equation (2), it is equal to the amount of energy E_L.

The energy we put in at Z comes out again at L. This doesn't surprise you, does it?

In other words, the price we pay for a small momentum current is a longer path. If we want to lift a weight 1 m, we must pull 8 meters of rope out of the pulley block.

Exercises

1. In Fig. 7.11, we lift a 100 kg weight 1 m with a pulley block. How many meters must the rope be pulled? How much energy is necessary?

2. The traction rope Z in Fig. 7.13 is moved 1 m upward. How much energy flows through the pulley block in the process?

3. Fig. 7.14 shows how a weight can be lifted by using two motors. The weight is 200 kg. The motor on the left pulls so that the traction rope Z_{left} moves at 0.2 m/s. The one on the right pulls so that rope Z_{right} moves at 0.4 m/s. What are the momentum currents in Z_{left} and Z_{right}? What are the energy currents in each of the ropes?

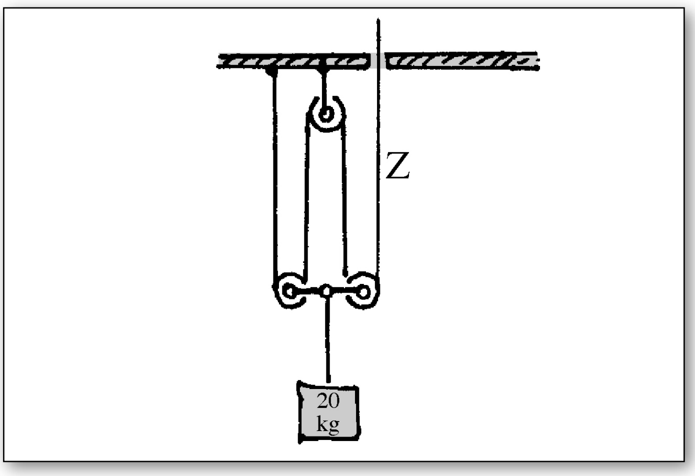


Fig. 7.13
For Exercise **2**

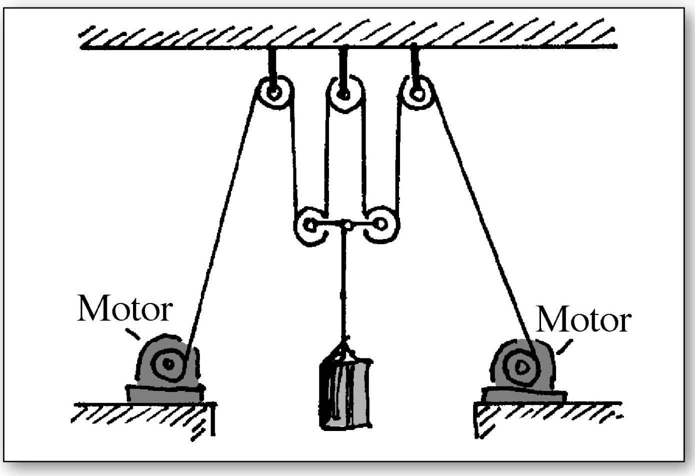


Fig. 7.14
For Exercise **3**

7.3 The law of the lever

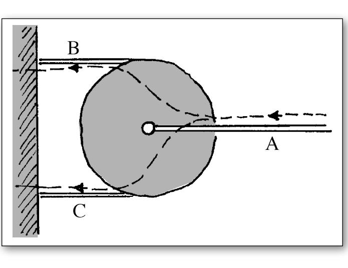


Fig. 7.15 The momentum current coming in at A branches at the wheel into two equal parts.

We know a lot about pulley wheels. We know that in Fig. 7.15, the momentum current flowing through A branches off into two equal parts. Because in this case nothing is moving, we could just as well be using a rod instead of a pulley wheel. Nothing would change about the momentum currents, Fig. 7.16.

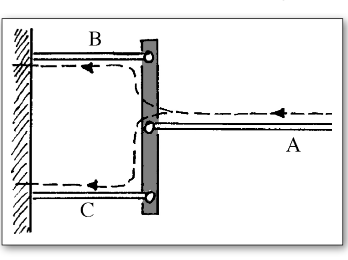


Fig. 7.16 The wheel can be replaced by a rod without the momentum current changing.

Here, too, the following is valid

$F_B = F_A/2$

and

$F_C = F_A/2$.

If this still isn't clear to you, test it experimentally.

We change the setup still more. We attach rope A asymmetrically instead of in the middle, as in Fig. 7.17. We call r_B and r_C the lengths of the *lever arms*. As before, the current which comes from A must be distributed over B and C. However, this time it does not divide into equal parts.

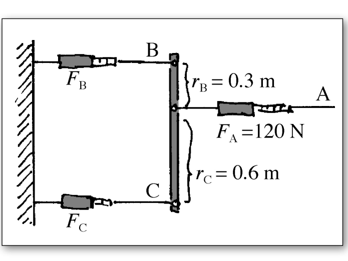


Fig. 7.17 The momentum current coming in through rope A branches into two parts of different values.

Direct measurement yields

$r_B \cdot F_B = r_C \cdot F_C$.

This can be changed into

$$\frac{F_B}{F_C} = \frac{r_C}{r_B}$$

In words:

The momentum currents are inversely proportional to the lever arms.

That is the *law of the lever*.

In the case of Fig. 7.17,

$r_B = 0{,}3$ m

and

$r_C = 0{,}6$ m.

Therefore

$$\frac{F_B}{F_C} = \frac{0.6 \text{ m}}{0.3 \text{ m}} = 2.0$$

This means that $F_B = 2 \cdot F_C$.

In addition, because

$F_B + F_C = 120$ N

it follows that

$F_B = 80$ N

und

$F_C = 40$ N .

The lever rule can be put into a simpler form. To do this we will once again describe the arrangement in Fig. 7.17 but in somewhat different words and with other letter symbols, Fig. 7.18.

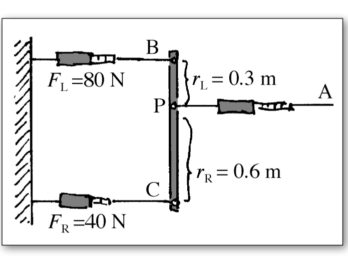


Fig. 7.18 As in Fig. 7.17; only the names have changed.

We have a rigid, unmoving rod. Momentum currents flow into the rod at three points. One of these points we call the *pivot*. We label it by P. The distances of the two other points from P are the lever arms. We refer to them as r_R and r_L. The strengths of the currents flowing over the outer inlets we call F_R and F_L. Up to now there is nothing new to us.

Now here is an important new concept: The products $r_R \cdot F_R$ and $r_L \cdot F_L$ are called *torques*. $r_R \cdot F_R$ is the *right torque* and $r_L \cdot F_L$ is the *left torque*.

Why do we use the names pivot and torque? What does our problem have to do with rotation? The rod is mounted at P so that it can rotate. Let's imagine that the rope below is not there. The upper one would set the rod in a rotation to the left. This is where the name left torque comes from. If we imagine the upper rope not to be there, then the rod would be set into a rotation to the right by the lower rope.

Using the new letter symbols, the lever rule looks like this:

$r_R \cdot F_R = r_L \cdot F_L$

and in words:

Right torque equals left torque.

If the lever rule is so formulated, it can be applied to the rod in yet another way, Fig. 7.17. To do so you should know that it doesn't matter where you set P.

Fig. 7.19 shows the same rod as in Fig. 7.17. However, here the pivot is set at the lower inlet. Rope A tries to turn the rod to the right at the new pivot and rope B tries to turn left. The lever arms are therefore

$r_R = 0.6$ m and $r_L = 0.9$ m.

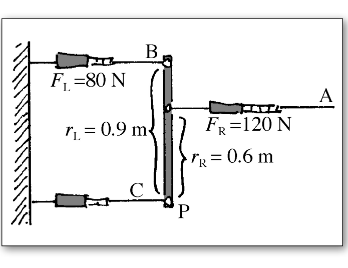


Fig. 7.19 Rope A would like to rotate the rod to the right around pivot P, rope B wants it to rotate to the left.

We already know the momentum currents. They are

$F_R = 120$ N and $F_L = 80$ N.

The right torque

$r_R \cdot F_R = 0.6 \text{ m} \cdot 120 \text{ N} = 72$ Nm

is indeed equal to the left torque

$r_L \cdot F_L = 0.9 \text{ m} \cdot 80 \text{ N} = 72$ Nm.

Notice that the unit for torque is the Nm (Newton-meter).

Finally, we set the pivot at the upper inlet, Fig. 7.20. Now rope C tries to turn to the right and rope A tries to turn left. The lever arms are correspondingly

$r_R = 0.9$ m and $r_L = 0.3$ m.

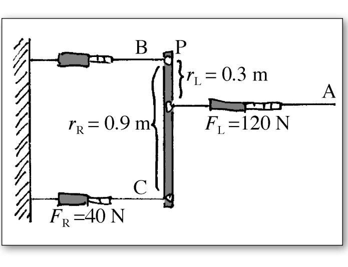


Fig. 7.20 Rope C tries to rotate the rod to the right around pivot P, rope A tries to rotate it to the left.

The corresponding momentum currents are

$F_R = 40$ N and $F_L = 120$ N.

The resulting right torque is

$r_R \cdot F_R = 0.9 \text{ m} \cdot 40 \text{ N} = 36$ Nm

and the left torque is:

$r_L \cdot F_L = 0.3 \text{ m} \cdot 120 \text{ N} = 36$ Nm.

Again, the torques are equal and the law of the lever has been fulfilled.

Example: A stressed rod

A heavy body (m = 80 kg) hangs from a horizontal rod, Fig. 7.21. The rod's weight is so small compared to that of the body that we do not need to take it into account. How strongly stressed are the supporting points of the rod?

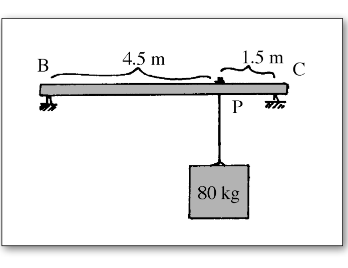


Fig. 7.21 The momentum current flowing from P to C is three times as strong as the one flowing to B.

We choose the point on the rod from which the object hangs to be the pivot.

The momentum current from the object to point P is

$F_P = m \cdot g = 80 \text{ kg} \cdot 10 \text{ N/kg} = 800$ N.

Support C tries to turn the rod to the left and support B tries to turn it to the right. Therefore the lever arms are:

$r_R = 4.5$ m and $r_L = 1.5$ m.

The lever rule yields

$$\frac{F_L}{F_R} = \frac{r_R}{r_L} = \frac{4.5 \text{ m}}{1.5 \text{ m}} = 3.0$$

Hence: $F_L = 3F_R$.

The current of 800 N flowing into the rod at P divides so that the momentum flowing to support C is three times that flowing to B.

In order to satisfy $F_R = F_R + F_L$, one must have $F_R = 200$ N
and
$F_L = 600$ N.

Example: Pliers

Pliers are made up of two levers connected by a hinge, Fig. 7.22. We will consider one of these, the lever 1. In order to cut a nail, the handles of the pliers are pressed together 20 cm from the hinge. A momentum current of 30 N flows into the handles. The cutting edges of the pliers are 3 cm from the hinge. We set P at the hinge so that

$r_R = 0.2$ m, $r_L = 0.03$ m and $F_R = 30$ N.

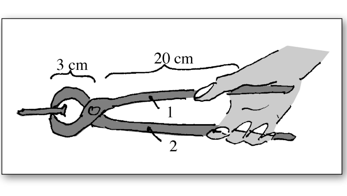


Fig. 7.22 Pliers are made up of two levers. Each of these has one short and one long arm.

The momentum current at the nail calculates to

$$F_L = \frac{r_R}{r_L} F_R = \frac{0.2 \text{ m}}{0.03 \text{ m}} \cdot 30 \text{ N} = 200 \text{ N}$$

A lever can make a large momentum current out of a small one.

The law of the lever can be used in completely different situations as well: The inlets do not need to be in a straight row and the three vectors of the momentum currents do not need to be parallel.

Fig. 7.23 shows a body with three momentum currents leading into it. We set the pivot at the support attached to the wall. The upper rope tries to create a turn to the left and the lower one attempts a rotation to the right.

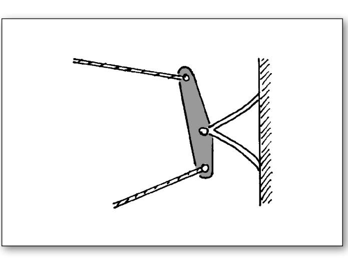


Fig. 7.23 The inlets of the momentum currents do not need to lie in a straight line, and the current vectors do not need to be parallel to each other.

What are the lever arms here? One draws a straight line through each of the two points where the ropes are attached so that their directions are the same as the corresponding current vectors, Fig. 7.24.

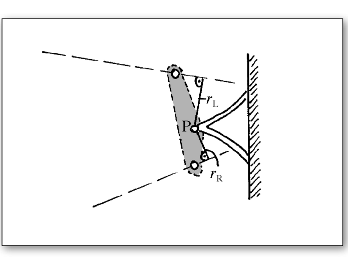


Fig. 7.24 As in Fig. 7.23, but with auxiliary lines and lever arms.

The lever arms are now simply the distances between the straight lines from pivot P.

Example: A relay arm

What is the momentum current in the horizontal rope in Fig. 7.25?

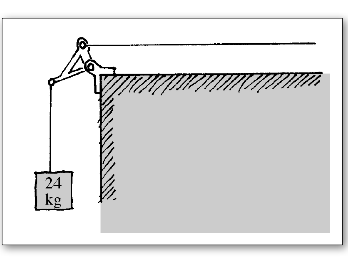


Fig. 7.25 Deflection lever

We arbitrarily choose the point where the relay arm is attached to the wall as our pivot, Fig 7.26. The horizontal rope tries to create a rotation to the right, the vertical one tries to create a rotation to the left around P. In Fig. 7.26, auxiliary lines and the lever arms are drawn. From the Figure we see that:

$r_R = 0.2$ m and $r_L = 0.4$ m.

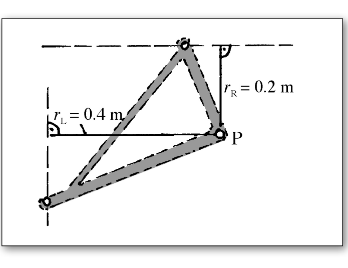


Fig. 7.26 Enlarged section of Fig. 7.25, with auxiliary lines and lever arms.

The momentum current in the vertical rope is:

$F_L = m \cdot g = 24 \text{ kg} \cdot 10 \text{ N/kg} = 240$ N.

Using the lever rule we have

$$\frac{F_L}{F_R} = \frac{r_R}{r_L} = \frac{0.2 \text{ m}}{0.4 \text{ m}} = 0.5$$

One could ask how strongly stressed the mounting is or in other words, what the momentum current at P is. Because the current vectors in both ropes are not parallel to each other, we cannot simply add up F_R and F_L. We must add up the vectors:

$\vec{F}_P = \vec{F}_R + \vec{F}_L$.

Exercises

1. Fig. 7.27 shows part of a vehicle brake. One pulls a lever on a rod. The lever is connected to a rope that pulls on the brakes attached to the wheels. What does the momentum current in the pulling rod need to be so that a current of 50 N flows into the brake rope?

2. The overhead crane in Fig. 7.28 spans the entire 12 m wide factory space. 9 tons is hanging from the crane (1 ton = 1000 kg). How strongly stressed are the two tracks on either side (in addition to the weight of the crane) when the weight is hanging in the middle? How strongly are they stressed when the weight hangs four meters from the track on the left?

3. The handles of the nutcracker in Fig. 7.29 are pressed together 15 cm from the nut. In order to crack the nut, a momentum current of 80 N must flow through it. How hard do we have to press?

4. A weight hangs from a horizontal pole. The pole is mounted at two places, Fig. 7.30. The pole presses down at A and presses up at B. This means that the momentum current vectors in A and B lie vertically, just as in the rope holding the weight does. How strongly stressed are points A and B? (What are the momentum currents that flow out of the pole at A and B?) Set the pivot in A in order to calculate the momentum current at B and set it in B to calculate the momentum current at A.

5. A pole supported by a rope holds a lamp weighing 8 kg, Fig. 7.31. What is the momentum current flowing through the rope? Set the pivot at the place where the pole is attached to the wall.

6. What is a screw wrench good for? Why doesn't one simply tighten the nut with one's hands?

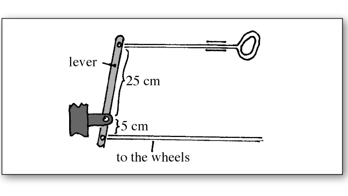


Fig. 7.27 For Exercise **1**

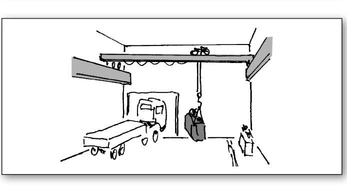


Fig. 7.27 For Exercise **2**

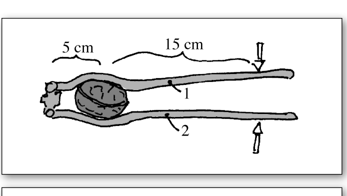


Fig. 7.29 For Exercise **3**

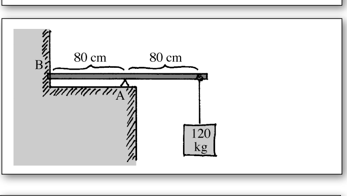


Fig. 7.30 For Exercise **4**

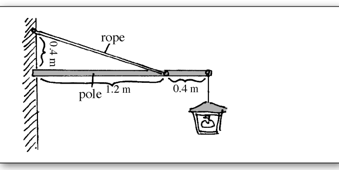


Fig. 7.31 For Exercise **5**

7.4 Equilibrium

In Fig. 7.32, everything is in order. A momentum current of 50 N flows from the right and 50 N flows from the left into the rod. At P, 100 N flow out. The lever rule

$r_R \cdot F_R = r_L \cdot F_L$

is obeyed.

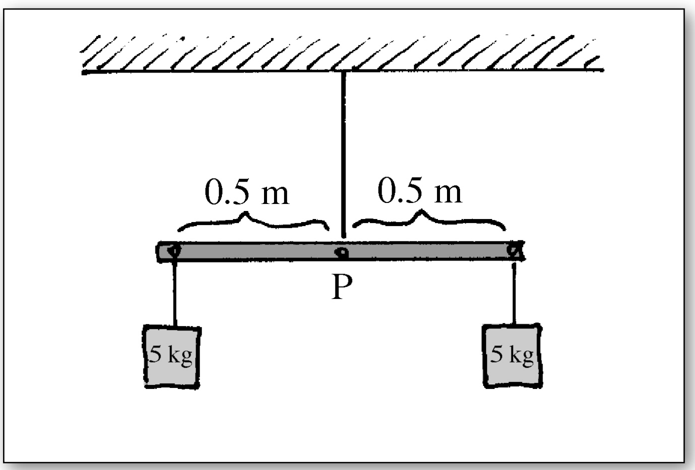


Fig. 7.32
The law of the lever is satisfied. The rod is in equilibrium.

We will now try to break the law of the lever. No one can stop us from hanging objects of two different weights from the rod, Fig. 7.33a. However, nature defends itself when we try to break the lever rule. As you probably predicted, Fig. 7.33b shows what actually occurs. The objects along with the rod are set in motion. In other words, the arrangement does not stay in *equilibrium*, as opposed to the arrangement in 7.32.

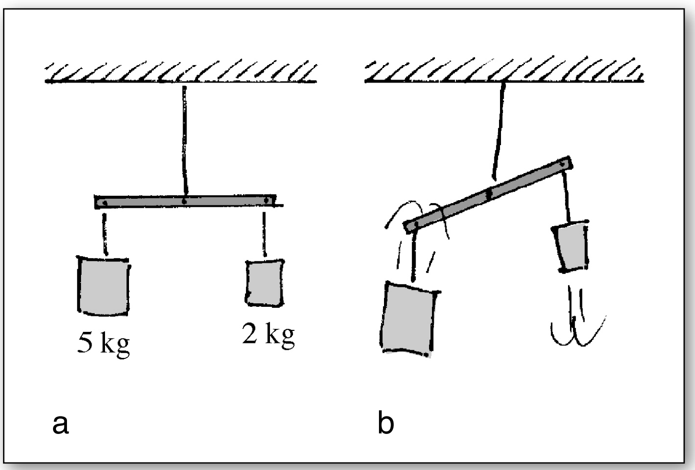


Fig. 7.33
(a) An attempt to break the law of the lever. (b) Bodies and rod are set in motion.

In Fig. 7.34, the lever rule is followed and the arrangement stays in equilibrium but this time with lever arms of different lengths.

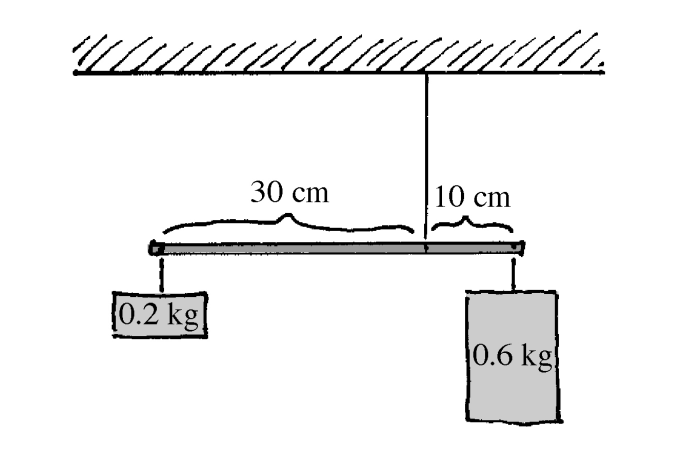


Fig. 7.34
Equilibrium with lever arms of differing lengths.

We can now make a more exact formulation of the lever rule:

A suspended body capable of rotation is in equilibrium when the right and left torques are equal.

Exercises

1. Calculate the right and left torques for the rod in Fig. 7.35. Is it in equilibrium?

2. A girl tries to lift a 500 kg stone by a lever in Fig. 7.36. Assume that half the mass of the stone is resting on the lever. Will the girl manage to do it? (She weighs 50 kg.)

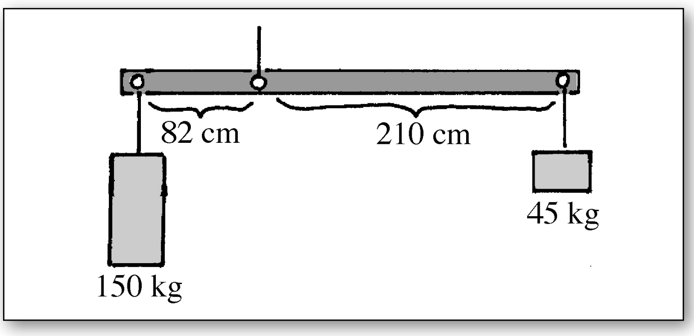


Fig. 7.35
For Exercise **1**

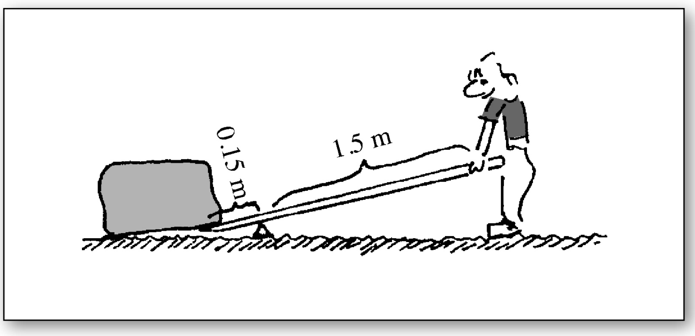


Fig. 7.36
For Exercise **2**

7.5 Center of mass

The rod in Fig. 7.37a is in equilibrium. Compare it to the situation in 7.37b. The only thing to change is that the balls don't hang from two strings anymore, but are attached to the rod. In combination with the rod, they make up one body, a kind of dumbbell.

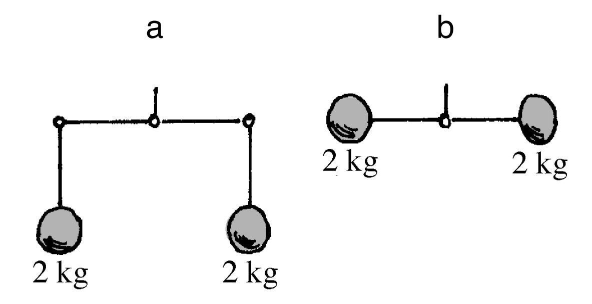


Fig. 7.37
(a) The rod is in equilibrium. (b) The dumbbell shaped body is in equilibrium.

This dumbbell is also in equilibrium. Two momentum currents are flowing into the ends of the dumbbell. (They come from the Earth and through the gravitational field). The momentum currents meet at the pivot and leave the dumbbell at that point.

In other words: We have suspended a body at one point so that it could rotate and this body is in equilibrium. It does not start to rotate by itself.

We turn the dumbbell a little, Fig. 7.38. What happens when we let it go? Nothing. It stays in its new position. It is in equilibrium again.

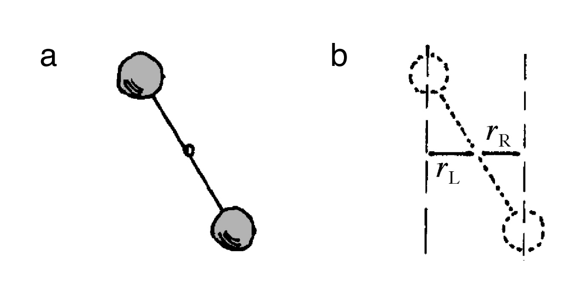


Fig. 7.38
By turning, the lever arms have shortened to half.

This can be seen in Fig. 7.38b. The inlets of both momentum currents from the Earth are where the two balls are. The current vectors lie vertically. That is why the auxiliary lines are vertical. The two lever arms r_R and r_L are shorter than in part 'a' of the figure. They have shortened by the same amount, though: r_R has shortened by half by turning and so has r_L.

F_R and F_L stayed the same during rotation, naturally. The torques $r_R \cdot F_R$ and $r_L \cdot F_L$ have reduced to half of their previous values. As before, this means that

$r_R \cdot F_R = r_L \cdot F_L$

is valid. The dumbbell has stayed in equilibrium. We can rotate the dumbbell any way we wish and it will always stay in equilibrium.

If the mounting at the pivot allows, we can also rotate it in the third dimension, Fig. 7.38, out of the plane of the drawing. This also changes nothing. It stays in equilibrium.

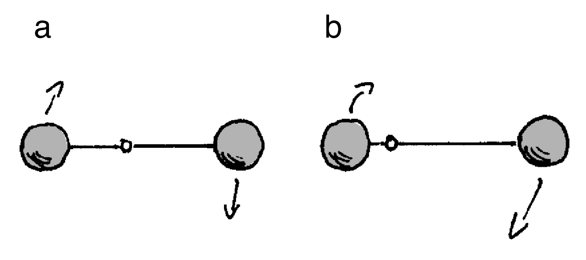


Fig. 7.39
Again, the body of Fig. 7.37b. Only the pivot point has changed.

We take the same dumbbell again, Fig. 7.39a, but with a different pivot this time. Of course it is no longer in equilibrium, nor is that the case in Fig. 7.39b. There is only one pivot for which it stays in equilibrium no matter what its orientation. This point is called the *center of mass*.

Not just dumbbell shaped bodies have centers of mass. Every object has a single center of mass. If the object is suspended at that point so that it can rotate, it is in equilibrium and stays there no matter how it is rotated.

An object's center of mass is often inside the object. How can a body be suspended to rotate around a point like this?

One drills a hole through the point of center of mass and puts an axle through the hole. The axle is mounted so it can rotate. The body is now in equilibrium, no matter how it is rotated, Fig. 7.40.

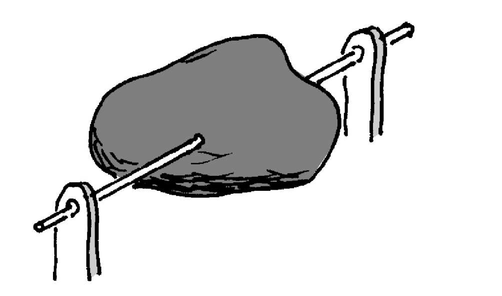


Fig. 7.40
The axis runs through the center of mass. The body is in equilibrium no matter what position it is in.

There are many possibilities for drilling such holes. It is unimportant how the holes are drilled, but they must pass through the center of mass, Fig. 7.41.

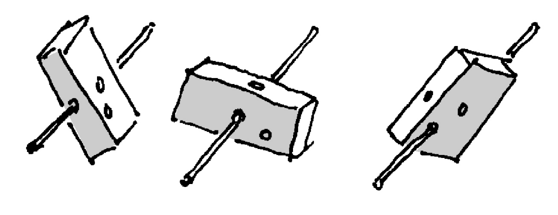


Fig. 7.41
Various axes through the center of mass. The body is always in equilibrium.

In summary:

> Every body has exactly one center of mass. If it is suspended to rotate around it's center of mass, the body stays in equilibrium, no matter what direction it is turned in.

If an object is highly symmetrical, the location of the center of mass is easy to predict.

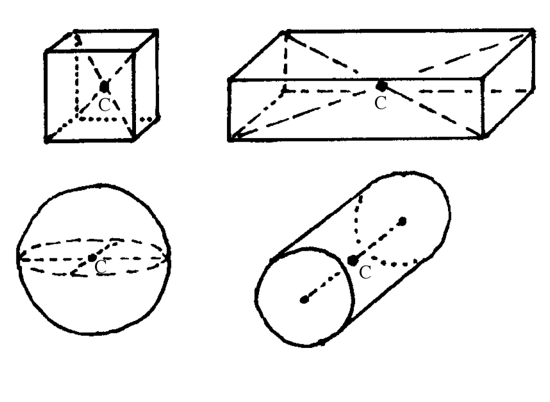


Fig. 7.42
The center of mass C in these bodies is the geometric center.

For a sphere, a cube, a cylinder or a cuboid it is simply the geometrical center, Fig. 7.42, provided that the mass in the body is distributed evenly. If half of a cube is composed of lead and half of aluminum, Fig. 7.43, its center of mass C is not in its geometrical center but shifted toward the part with the lead.

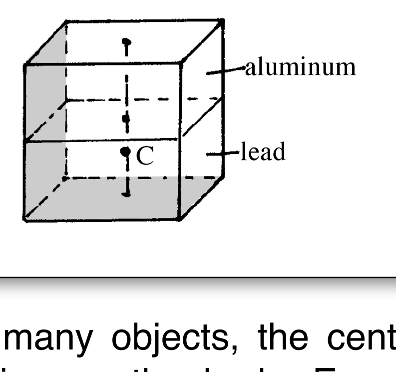


Fig. 7.43
The center of mass C is not the geometric center of the cube.

For many objects, the center of mass C lies outside the material making up the body. Examples would be a circular ring or a U shaped object, Fig. 7.44.

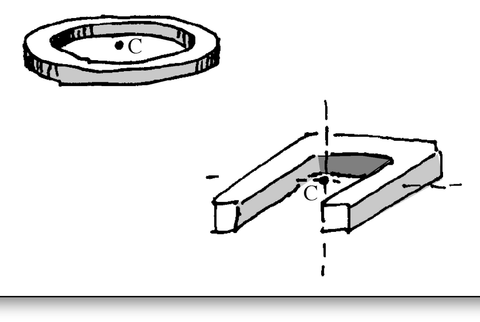


Fig. 7.44
In both of these cases, the center of mass C lies outside the body itself.

Exercises

1. Where is the center of mass of a bicycle wheel?
2. Where is the Earth's center of mass?
3. Take a few different objects and try to find their centers of mass by holding them between your middle finger and thumb so that they can rotate.
4. The Earth and the Moon can be considered a kind of dumbbell with the "rod" between them being the gravitational field. Where is the center of mass of this object? (Earth's mass: about 100 times the mass of the Moon, distance from Earth to Moon = 380,000 km).

7.6 Stable equilibrium

We suspend an object so that it can rotate and intentionally do not set the pivot at the center of mass. For simplicity, we again take a dumbbell for this. To prevent the pivot from coinciding with the center of mass C, we bend the dumbbell, Fig. 7.45.

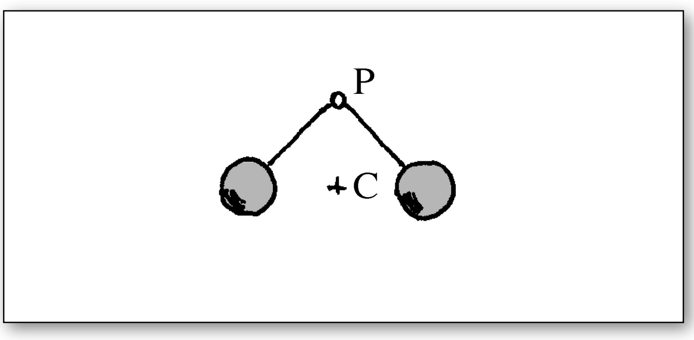


Fig. 7.45
The pivot point P does not coincide with the center of mass C. The body is in stable equilibrium.

What happens when the dumbbell is turned into the position shown in Fig. 7.46a? Your feelings surely tell you that it will not remain hanging like that: It will begin to rotate and gradually level off to the position shown in Fig. 7.45b.

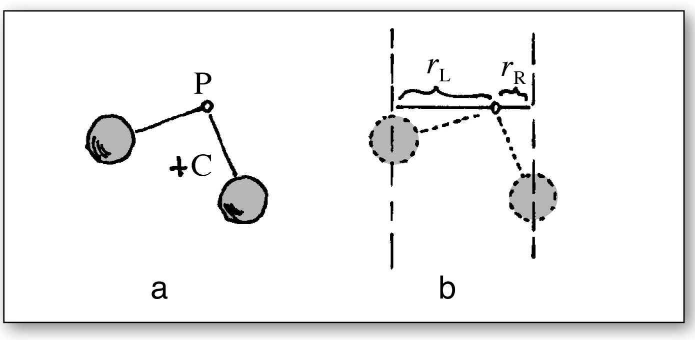


Fig. 7.46
(a) The body does not stay in this position. (b) One lever arm is longer than the other.

You do not need to rely upon your feelings to come to this conclusion, though. Fig.7.46b shows that the lever arm r_L is longer than r_R. Because the two spheres are of equal weight, this means that the left torque is greater than the right torque. For this reason, the dumbbell begins to rotate to the left. In the process of rotating, the lever arms change length. When the dumbbell has reached the position shown in Fig. 7.45, the two arms are equally long. This is the *position of equilibrium*. At first, the dumbbell will swing out past this position of equilibrium, but it will gradually settle down there.

What is the center of mass doing during all this? It is moving downward.

If we rotate the body out of the position of equilibrium, the position of the center of mass rises. It does not matter if we turn the body to the right or to the left. In equilibrium the center of gravity is obviously in the lowest possible position. Moreover, it lies directly underneath the pivot.

There is still another position of equilibrium for the dumbbell: when the center of gravity is directly above the pivot, Fig. 7.47. If it is rotated just slightly out of this position, it doesn't return to it by itself, but removes itself more and more from it and eventually settles down in the lower position of equilibrium. The upper position of equilibrium is *unstable* and the lower one is *stable*.

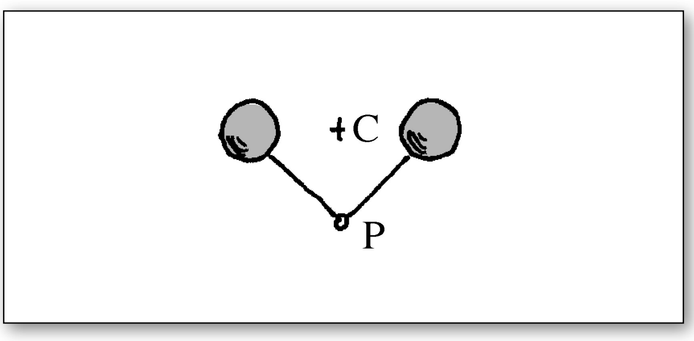


Fig. 7.47
Unstable equilibrium

A body is suspended so that it can rotate. If the pivot is vertically above the center of gravity, the body is in stable equilibrium. If it is rotated out of this position, it finds its way back there by itself.

We now have a very simple way of determining the center of mass of a body. One suspends the body from an arbitrary point, so that it can rotate, Fig. 7.48. It settles down so that its center of mass is vertically below the pivot. We then have a first straight line that the center of mass must be lying upon. We then suspend it again to rotate, but from another point, and let it settle down. Again, we obtain a straight line through the center of mass. The body's center of mass must be at the intersection of these two straight lines.

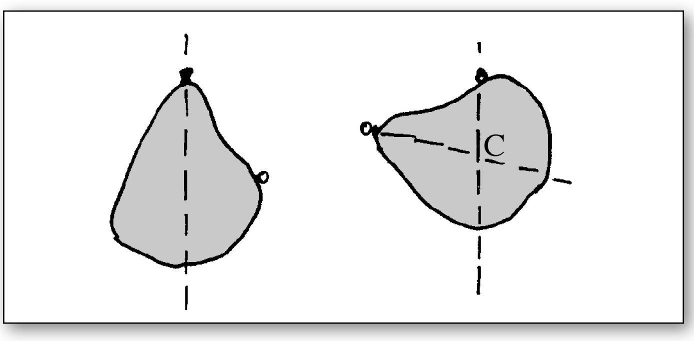


Fig. 7.48
The body adjusts in such a way that its center of mass lies vertically under the pivot point.

Exercises

1. Try to find the centers of mass for a few different objects by suspending them from two different points on or in them so they can rotate.

2. Two forks are stuck into a cork. The cork is mounted onto the point of a nail, Fig. 7.49. Why doesn't the cork with the forks fall down?

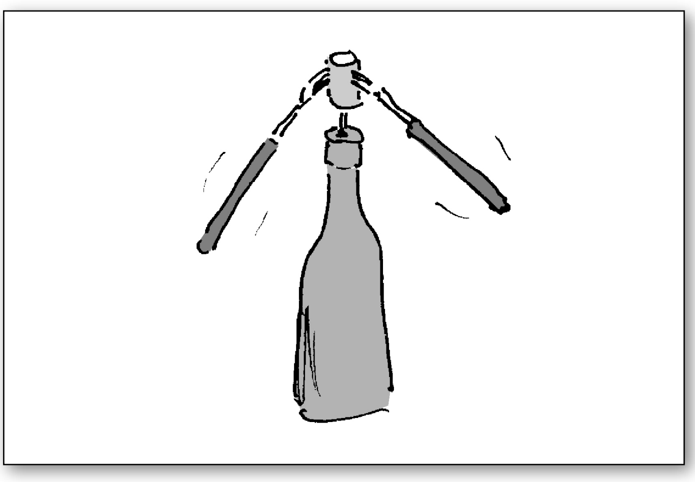


Fig. 7.49
For Exercise **2**

7.7 Center of mass and energy

It takes energy to bring a body out of a position of stable equilibrium. This is because the body's center of mass must be moved upward.

This is similar to how an object is lifted, Fig. 7.50. Also in that case, the center of mass is shifted upward, and this needs energy.

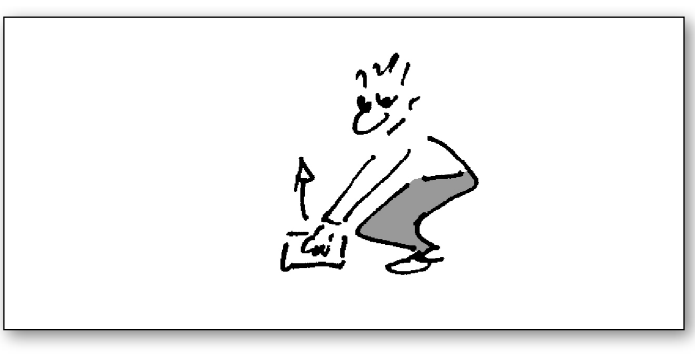


Fig. 7.50
It takes energy to move the center of mass of an object upwards.

Energy is needed to move the center of mass of a body upward.

This energy is stored in the gravitational field. When the body starts moving downward again, the gravitational field gives the energy back.

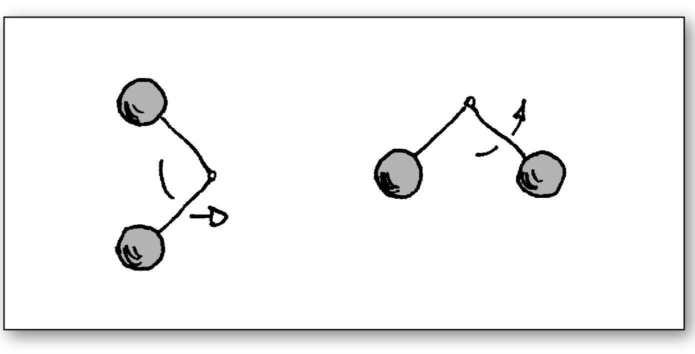


Fig. 7.51
The process taking place on the left happens by itself, the one on the right does not.

Why does the process shown on the left in Fig.7.51 happen on its own and the one on the right does not? Why does the process on the left in 7.52 happen on its own and the one on the right does not?

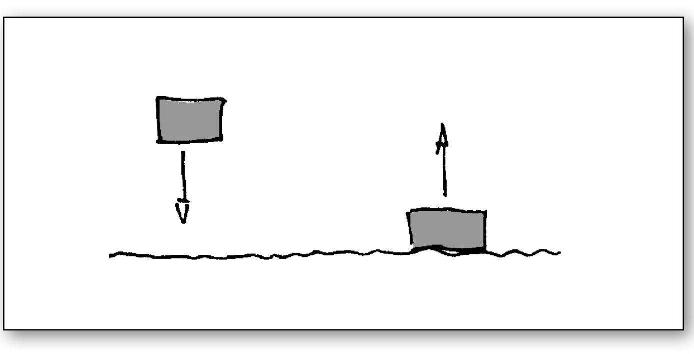


Fig. 7.52
The process on the left occurs by itself, the one on the right does not.

The reason for this is that it is always easier to get rid of energy than it is to obtain it. (Energy and money are similar in this way). The energy that is given up in the transition to the stable state of equilibrium is used to create heat. This process cannot be reversed. Heat cannot be destroyed. For this reason the transition from equilibrium to a state of non-equilibrium does not happen by itself. We must supply the energy from elsewhere.

We will use a couple of examples to observe the transition to a state of equilibrium.

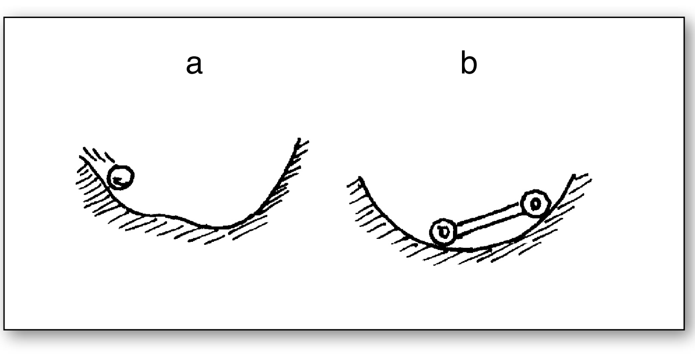


Fig. 7.53
(a) The sphere rolls to the deepest point of the depression. (b) The vehicle adjusts so that it is horizontal.

In Fig. 7.53a, a sphere rolls down to the lowest point. Its center of mass finds the lowest possible position. The vehicle in Fig. 7.53b shifts into a horizontal position. The wheels on the left need to move a bit upwards to achieve this. In the process, the center of mass moves downward, though. The crate in Fig. 7.54a cannot remain in that position. It tips to the left. In doing so, its center of mass moves downward. The object in Fig. 7.55a cannot remain like that because its center of mass can move further downward, Fig. 7.55b.

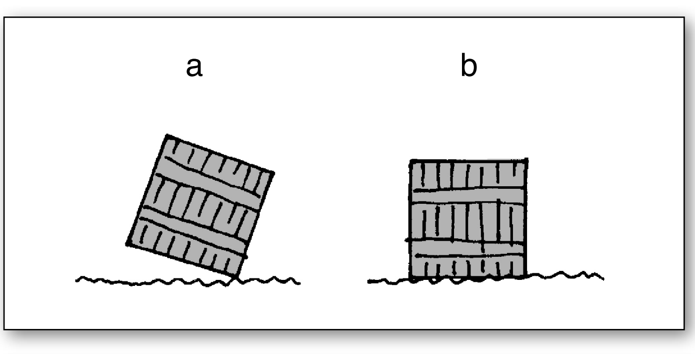


Fig. 7.54
The crate tips to the left. In the process, the center of mass moves downward.

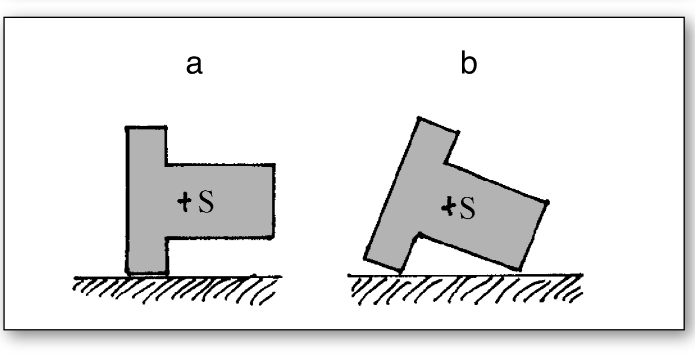


Fig. 7.55
The object tips to the right because thereby its center of mass lowers.

Sometimes one needs to lift a body's center of mass only slightly to put it into a position where it can move much further downward by itself. In other words: Very little energy needs to be introduced into the body to make a lot of energy come out of it.

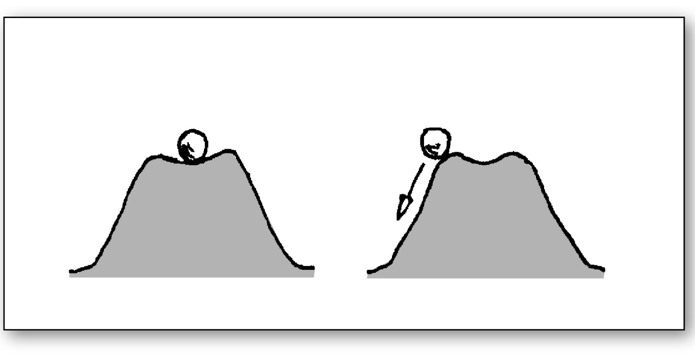


Fig. 7.56
Very little energy is needed to roll the ball over the edge.

Fig. 7.56 shows such a situation. Very little energy is needed to move the sphere up the small ridge. After that, it rolls by itself down the outer side of the mountain. Another well-known example is an object that is easily tipped, Fig. 7.57. In this case, as well, the center of mass need only be brought slightly upwards, bringing the vase into a position from where the center of mass goes much further all by itself.

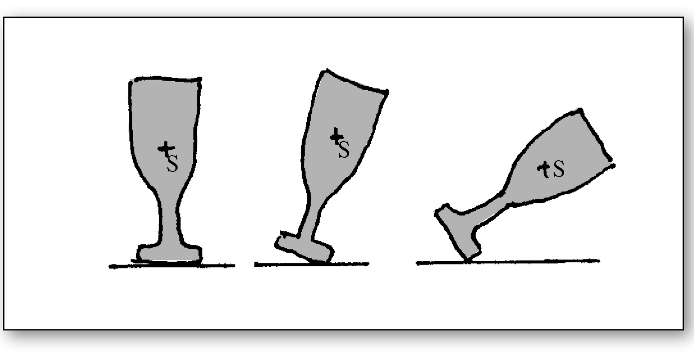


Fig. 7.57
Very little energy is necessary to make the vase fall over.

We now have a method for determining the mass of a body. Fig. 7.58 shows an old balance. The middle pivot lies somewhat higher than the two pivots holding the scales.

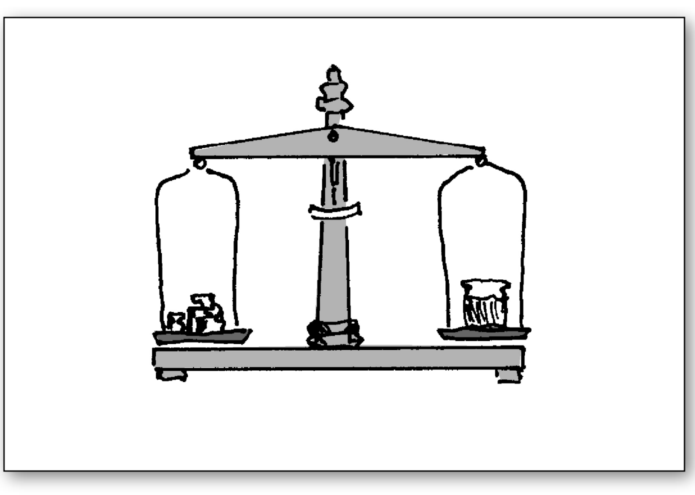


Fig. 7.58
A balance. The middle pivot point lies somewhat higher than the outer pivot points.

If the scales have equal weights on them, the balance eventually levels off and the bar holding them becomes horizontal. The center of mass is then at its lowest point.

There is a set of weights for scales: A set of bodies of known mass, from which various quantities of mass can be compiled. This is similar to different amounts of money that can be put together using different coins and bills.

In order to weigh an object, it is put onto one of the scales. On the other side, weights are placed so that the bar holding the two scales is horizontal. Now, the mass of the object is equal to the total mass of the weights.

Exercises

1. If a body's center of mass keeps its height when the body is moved, the body is in so-called indifferent equilibrium. In this case, the body is in a state of rest no matter where it is put. Give examples of this.

2. A bicycle that is not supported tips over. A car does not. Why?

3. Is the body in Fig. 7.59 in a stable state of equilibrium? If not, in which direction does it start moving?

4. Does the object in Fig. 7.60 tip over?

5. Balances can have arms of different length, Fig. 7.61. The weights are put onto the scale hanging from the longer arm. How does one find the mass of the object to be weighed? What advantage does this balance have over one with arms of equal length?

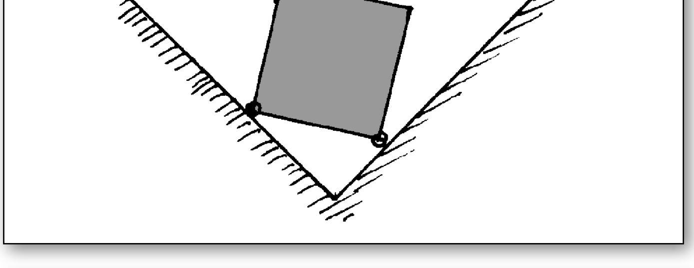


Fig. 7.59
For Exercise **3**

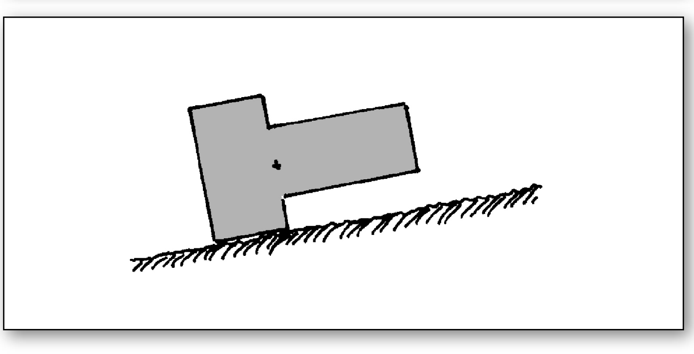


Fig. 7.60
For Exercise **4**

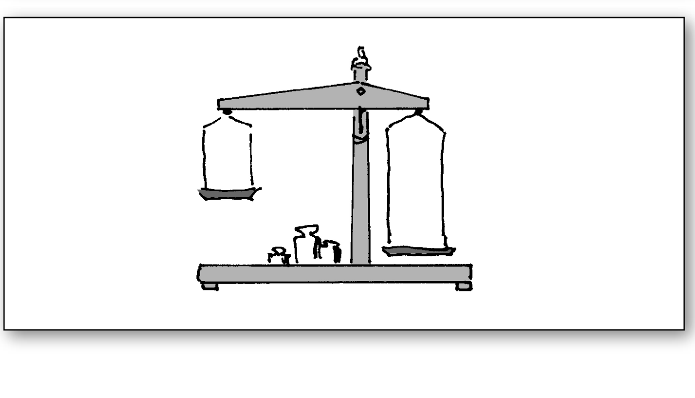


Fig. 7.61
For Exercise **5**

8

Angular momentum and angular momentum currents

This chapter deals with a special type of motion called rotational motion. It will become clear to you that rotational motion occurs in many situations and is especially important.

We will make an interesting discovery: Describing rotation is very similar to describing linear motion. One could say that there is an *analogy* between the corresponding fields of mechanics. This analogy allows us to save ourselves a lot of work.

8.1 Angular momentum and angular velocity

A wheel is mounted upon an motor shaft, the motor is turned on and the wheel rotates uniformly, Fig. 8.1. What does "uniformly" mean? With constant velocity, you might say. What is the velocity then? Point B at the edge of the wheel moves very fast, point A, near the axle, moves more slowly. This shows that there really is no uniform velocity at all. What we are looking for is a reasonable way to measure the velocity of a rotation.

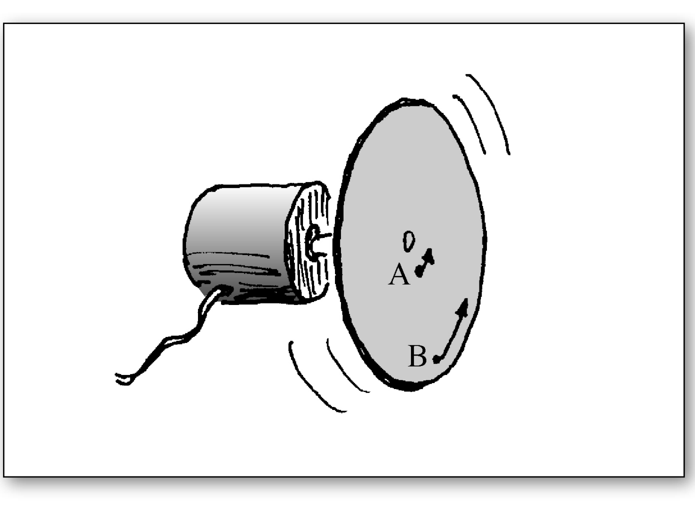


Fig. 8.1
Point B at the edge of the wheel moves faster than point A.

A sensible way of measurement is to use the angle which is "passed over" per second by a radius, Fig.8.2. The *angular velocity* is the ratio of the angle and the interval of time needed by the wheel to turn by this angle.

$$\text{angular velocity} = \frac{\text{angle}}{\text{time interval}}$$

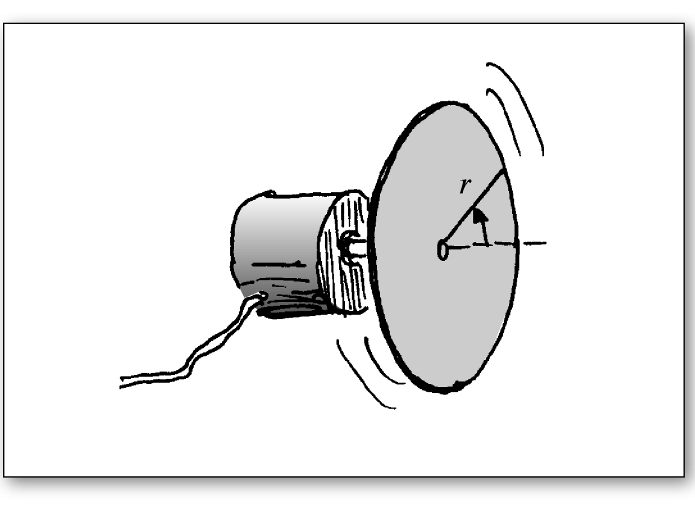


Fig. 8.2
The angular velocity is the angle covered by the radius *r* divided by the time span.

Various units measurements are used for the angle itself. The best known of these is the degree. For our purposes, though, it is more practical to take the full rotation of 360 degrees as the unit. We give the angular velocity in "rotations per second".

We consider a freely turning wheel on good ball bearings. For example, the wheel of an overturned bicycle, Fig.8.3. It turns at a determined velocity, i.e., at a certain number of rotations per second. We can ascertain the value of the angular velocity with a stopwatch. We have now described rotational motion of a wheel.

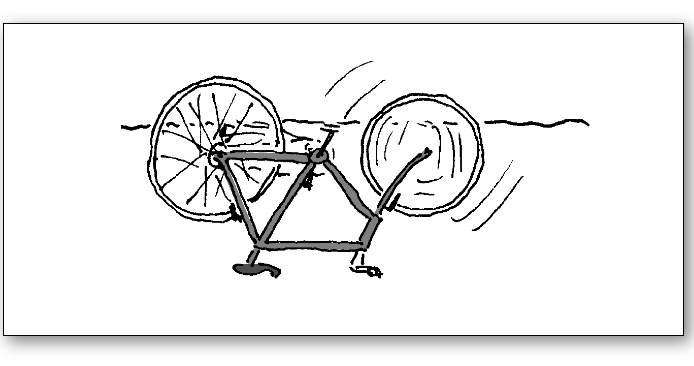


Fig. 8.3
The rotating wheel has a certain amount of angular momentum.

Angular velocity is to rotation what usual velocity is to linear motion. For the description of linear motion we have introduced a second quantity: momentum. It is a measure of the body's impetus.

In the same manner, it is possible to say our rotating wheel has impetus: something that is put in when it is set in rotational motion and that comes out again when the wheel brakes. This type of impetus is called *angular momentum*.

Angular momentum and usual (linear) momentum are not the same. If the wheel in Fig. 8.4a had usual momentum, it would move like the one in Fig. 8.4b.

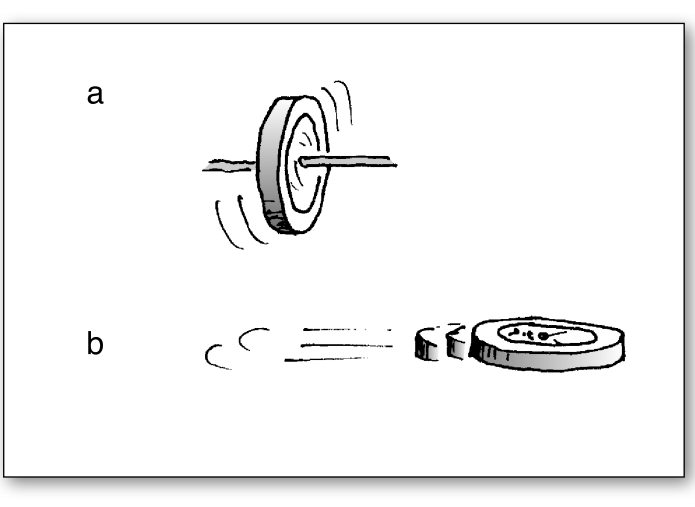


Fig. 8.4
(a) The wheel has angular momentum. (b) The wheel has linear momentum.

Let us investigate the characteristics of angular momentum. What does it depend upon? What paths does it take in different processes?

Two identically built wheels rotate at different angular velocities, Fig. 8.5. Which of the wheels has more angular momentum? The faster one, of course.

> The greater the angular velocity, the more angular momentum is contained in a body.

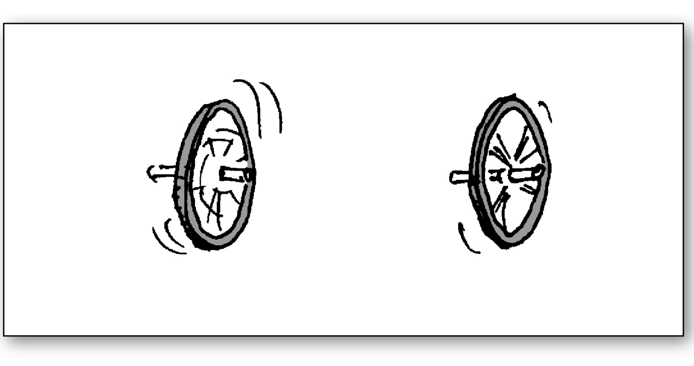


Fig. 8.5
The wheels rotate at different velocities. Which one has more angular momentum?

The two wheels in Fig. 8.6 have the same form, but they are made of different materials. One of them is very light and the other is very heavy. They are set in motion so that they rotate at the same angular velocity. Which one has more angular momentum? It is the heavier one of course.

> The greater the mass of a body, the more angular momentum it contains.

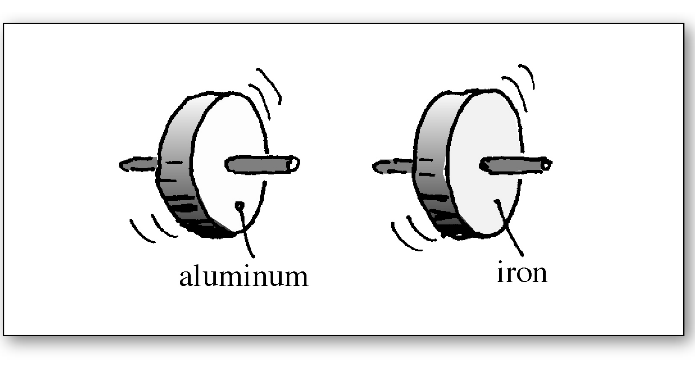


Fig. 8.6
The wheels have the same form but different weights. Which one has more angular momentum?

Two bodies can have the same mass, rotate at the same velocityv and nevertheless contain different quantities of angular momentum. We will see how this is possible when we have a bit more experience in dealing with angular momentum.

We now consider a simple experiment, Fig. 8.7. We need two wheels, the axle of one of them is fixed to a table and the other one can be carried around. The two wheels can be connected to each other by a kind of friction clutch where one wheel takes the other along with it.

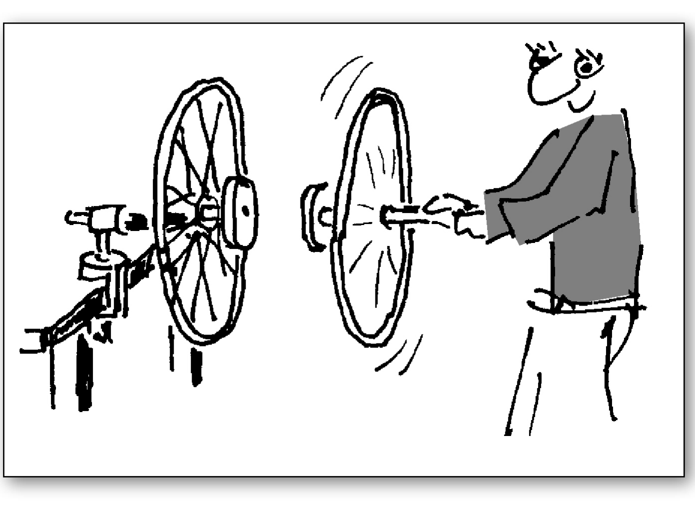


Fig. 8.7
As soon as the clutch discs touch, the angular momentum begins to flow from the wheel on the right to the one on the left.

At first the two wheels are separate. One of them is made to rotate, the other is not. The clutch discs are brought into contact with each other. What happens?

The rotating wheel becomes slower and the other, which was not rotating before, starts to rotate. After the clutch discs have slipped relative to each other for a while, the wheels reach the same angular velocity.

That was the observation. What is the explanation? What happened with angular momentum during this process?

The angular momentum stored in the wheel that was turning at the beginning, was reduced. The angular momentum of the wheel that was not turning at the beginning, increased. Angular momentum must have been gone from one to the other.

> Angular momentum can go from one body to another.

The angular momentum that was stored in only one wheel at the beginning, distributed evenly between the two wheels.

> Angular momentum can be distributed over several bodies.

Again, we consider a single wheel fixed to an axle. The axle has good ball bearings. The wheel is set in rotational motion, it is charged with angular momentum. Now one grasps the rotating axle and 'brakes' the motion, Fig. 8.8. After a while, the wheel comes to rest. Where did the angular momentum go?

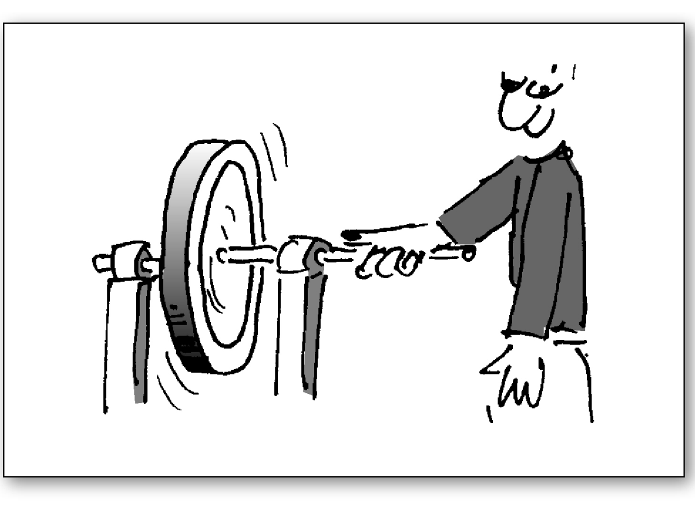


Fig. 8.8
The angular momentum flows into the ground.

This is very similar to a situation you are already familiar with when an automobile moving straight along, brakes. The momentum of the car flows into the ground in exactly the same way that the angular momentum flows into the ground.

The same would have happened if we had not stopped the wheel deliberately. The angular momentum would have flowed through the ball bearings to the ground, but more slowly.

You see what wheel bearings are good for: They should hold an axle without allowing angular momentum to flow into the ground.

> If a wheel has bad bearings so that it comes to rest on its own, the angular momentum is flowing out of it and into the ground.

We go back now to the experiment with the two wheels, Fig.8.7. We set the wheel attached to the table in rotational motion. We then also set the movable wheel in rotational motion, but in the opposite direction. We set the number of rotations per second equal for both wheels.

Again we bring the two wheels into contact by clutch discs. How does the final state look this time? Both wheels stand still. The explanation? There was angular momentum before, but where is it now?

At the beginning, each wheel had a quantity of angular momentum not equal to zero. If the quantity of angular momentum of one wheel is given the opposite sign of the other, then the total amount of angular momentum at the beginning was already zero. We can conclude from the experiment that:

> Angular momentum can have positive and negative values.

We can arbitrarily choose which value we wish to be positive and which is said to be negative. How do we reach such a decision, though? A practical possibility would be the *right-hand-rule*, Fig. 8.9:

> One grasps the rotational axis with the right hand so that the bent fingers point in the direction of the rotation. If the thumb then points in the positive *x*-direction, the angular momentum is positive. If it points in the negative *x*-direction, the angular momentum is negative.

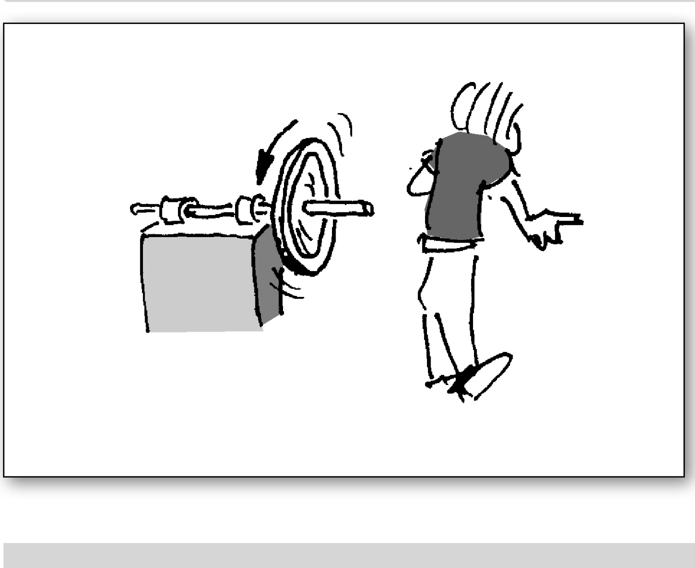


Fig. 8.9
The right-hand rule

> ***Exercise***
>
> Find the rules written in bold for linear motion in section 3.2 of this book that correspond to the rules written in bold in the section we have just completed. Put them into a table to compare them.

8.2 Angular momentum pumps

Angular momentum flows out of a rotating wheel by itself. It flows over the bearings (which are never perfect) and into the ground. In order to put angular momentum into the wheel, effort is needed. The wheel will not start turning by itself.

A wheel can be charged with angular momentum by hand turning it with a crank. Another possibility would be to use a motor, Fig. 8.10.

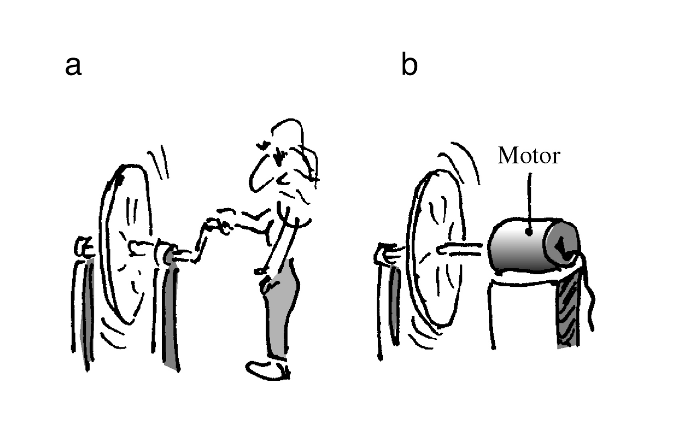


Fig. 8.10
8.10. (a) The person acts as a pump for angular momentum. (b) The motor acts as an angular momentum pump.

In both cases, something is used that forces the wheel to be charged with angular momentum: an *angular momentum pump*. In the first case, the person works as an angular momentum pump, in the second case it is the motor.

Where does the angular momentum pump get its angular momentum? It is the same as with linear momentum: It can be gotten from the Earth. An experiment shows this clearly.

We set the *x*-axis vertically upwards. We need a revolving chair and a large wheel with good bearings that can be comfortably held by its axis. The person doing the experiment stands next to the revolving chair, holds the wheel so that the axis is vertical and sets it in motion. Then he sits on the chair, Fig. 8.11, and slows the wheel down until it stops rotating. One sees that in the process, the chair with him on it begins to rotate. The explanation: During the process of braking, angular momentum flows out of the wheel and into the person and the revolving chair. It flows no further than this. It could not flow into the ground because the chair is insulated from the ground by its bearings.

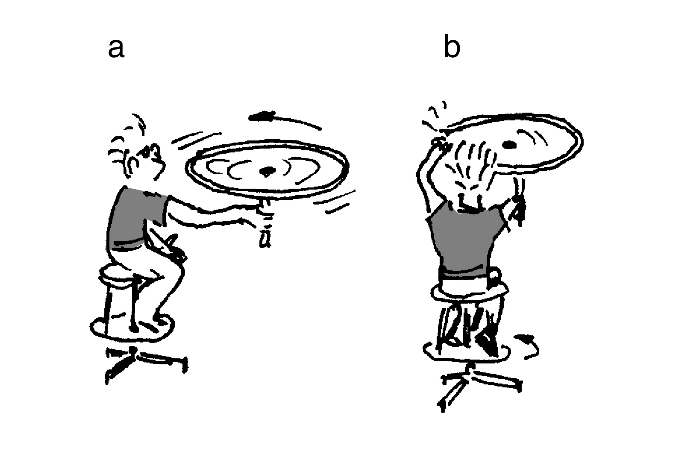


Fig. 8.11
(a) Only the wheel has angular momentum. (b) Angular momentum flows out of the wheel into the person and chair.

If the person's feet are on the floor while he is braking the wheel, the angular momentum flows directly into the ground.

Now we will try a variation of this experiment. The person sits in the revolving chair and holds the wheel, Fig. 8.12. Chair and wheel are at rest. The person now sets the wheel in motion. What happens? When the wheel starts rotating, the chair with the person on it begins turning as well. However, it rotates in the opposite direction of the wheel.

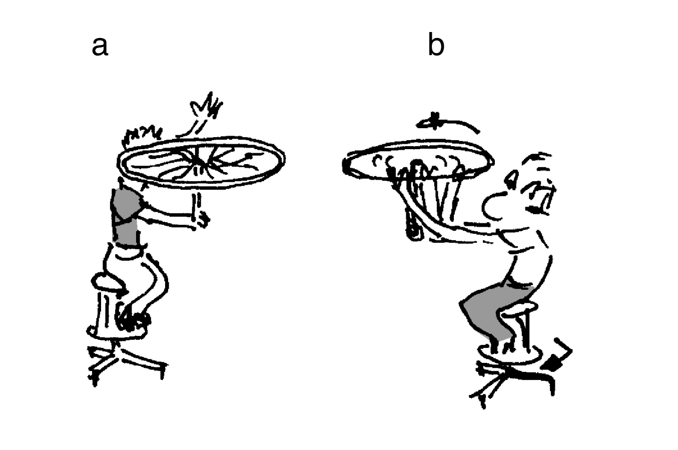


Fig. 8.12
(a) Wheel, person, and chair without angular momentum. (b) Angular momentum is pumped out of the person and chair into the wheel.

Obviously the person pumped angular momentum out of the chair and out of himself and into the wheel. The person and the chair now have negative angular momentum.

If the person puts his feet on the ground while charging the wheel, the chair will not rotate. The angular momentum must have been pumped directly out of the Earth and into the wheel.

Exercise

The person in Fig. 8.13 holds a rotating wheel in each hand, with the axes pointing upward. The wheels are identical. They have the same amount of angular momentum but their directions of spin are opposite to each other. The person brakes the wheels simultaneously while sitting on the revolving chair. What happens? What would happen during braking if the two wheels had rotated in the same direction before?

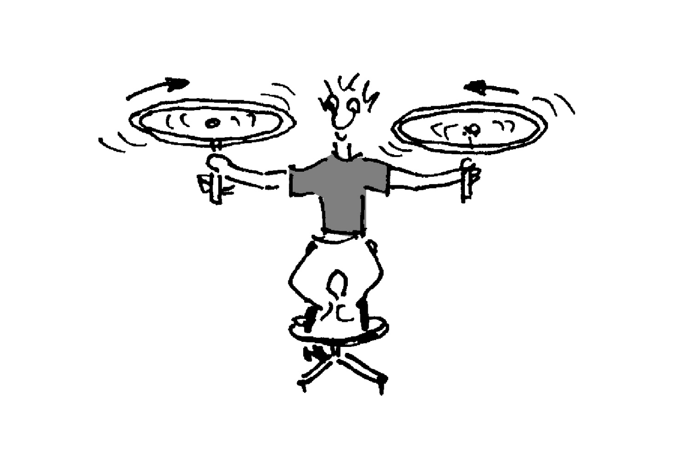


Fig. 8.13
For the exercise

8.3 Flywheels

A rotating wheel contains angular momentum. It is a storage device for angular momentum. Some wheels have the sole purpose of storing angular momentum. These are called *flywheels*.

What are flywheels be used for? Steam engines and combustion engines (automobile engines) do not pump angular momentum evenly, but intermittently. An automobile engine produces about 50 thrusts of angular momentum per second. There are short time intervals between these thrusts when it is not 'pumping'. It has a flywheel in order to override these idle times. While the engine is running, part of the angular momentum goes into the flywheel. It comes out again during the pauses. In this way, the engine shaft creates a more or less constant current of angular momentum.

How can the maximum amount of angular momentum be put into a flywheel? We have already seen that the faster a body rotates, and the heavier it is, the more angular momentum it contains. Therefore, a flywheel must rotate fast and have a large mass.

We will consider a simple and somewhat crude way of comparing amounts of angular momentum. The body we wish to investigate sits upon an axis with good bearings, Fig. 8.14. We take the axis between our thumb and forefinger and brake as strongly as we can. It takes a certain interval of time for the body to come to a stop. The more angular momentum it has, the longer it takes for all of the angular momentum to flow out.

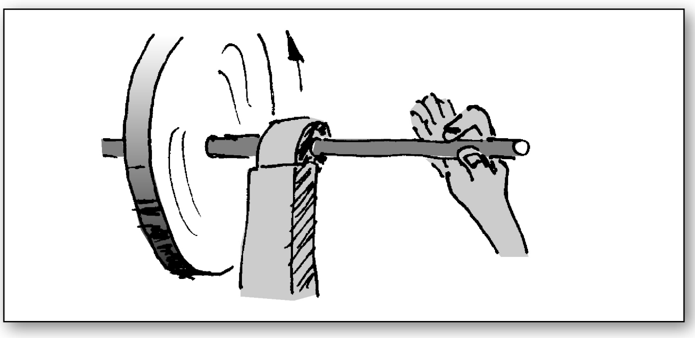


Fig. 8.14
The longer it takes for the rotating body to come to rest, the more angular momentum it contains.

We will now compare two rotating bodies in each of the following cases.

1. The bodies are built identically. One rotates quickly and one slowly. Of course it takes longer to bring the quickly rotating one to a stop than it does the other one because it contains more angular momentum.

2. The bodies rotate at the same angular velocity but have different masses. It takes longer to brake the heavier one than it does to brake the lighter one because the heavier one has more angular momentum.

3. We now compare two bodies with the same mass and the same angular velocity. The only difference is that a part of the mass of one body is further out than the other's, Fig. 8.15. The result is noticeable: It takes longer to slow down the one whose mass is further out. It contains more angular momentum than the other one.

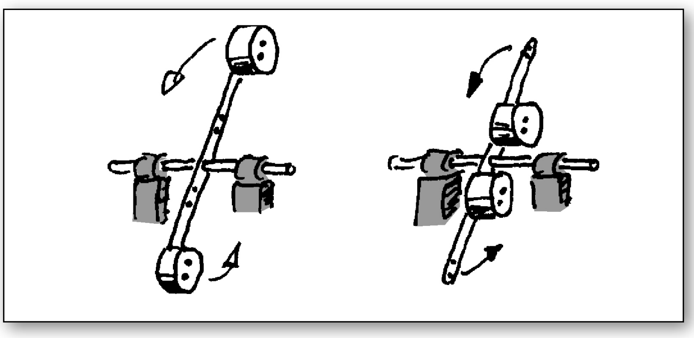


Fig. 8.15
The body whose mass is further out, contains more angular momentum.

We have found a new relationship:

> The further out the mass of a body is, the more angular momentum it contains.

We have discovered a rule that must always be taken into account when constructing a flywheel: The mass must be as far out as possible. A flywheel with a large storage capacity would look like this: A large and heavy ring with thin spokes attached to its hub, Fig. 8.16.

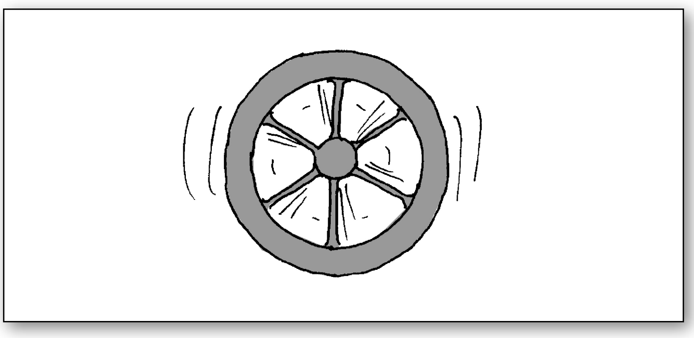


Fig. 8.16
Flywheel. The spokes hold a heavy ring.

Exercises

1. Wheels can have different functions. Storing angular momentum is just one of these. What else are wheels used for? Name several different uses for them.

2. Name some examples of uses for flywheels.

3. It is not possible to store an unlimited amount of angular momentum in a flywheel just by making it rotate faster and faster. Why not?

8.4 Angular momentum conductors

Fig. 8.17 shows how a flywheel is charged with angular momentum. The angular momentum is drawn from the ground by the motor. It then flows over the *drive shaft* and into the flywheel. We see that drive shafts serve to transport angular momentum. They are *angular momentum conductors*.

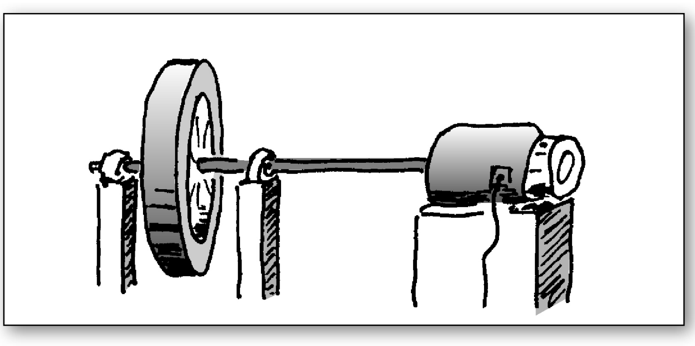


Fig. 8.17
The angular momentum flows through the drive shaft to the flywheel.

What is the property of the drive shaft that enables it to conduct angular momentum? What material must it be made of? The only requirement is that it be a solid material. Any kind of solid bar can be used as an angular momentum conductor.

> Solid materials conduct angular momentum.

We will now look at a couple of other devices that have to do with transporting angular momentum.

A bearing serves to hold an drive shaft in place so that no angular momentum flows into the ground.

> Bearings serve as angular momentum insulators.

Fig. 8.18 shows a clutch. The connection between the motor and the flywheel can be opened or closed by a lever.

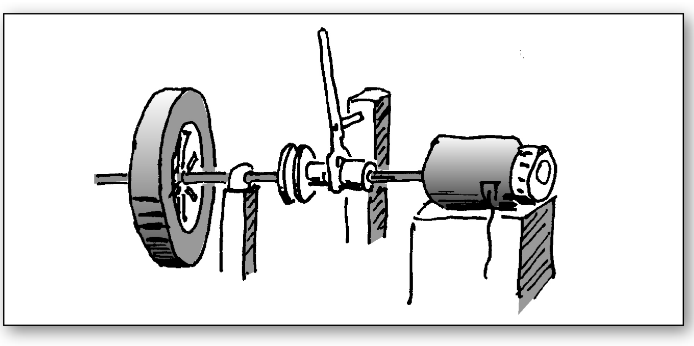


Fig. 8.18
The connection between motor and flywheel can be interrupted by the clutch.

> An angular momentum conductor can be interrupted by using a clutch.

Every automobile has a clutch. It can be found between the motor and the gearbox, Fig. 8.19. When the clutch pedal is pressed on (far left in a car) it takes the car out of gear and the connection between the motor and the gearbox is interrupted.

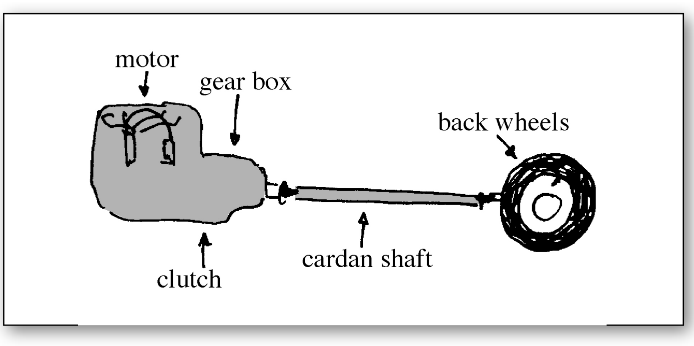


Fig. 8.19
Part of the propulsion system of an automobile.

One needs to declutch before changing gears. If this is not done, the strong angular momentum current from the motor to the tires will damage the gearing mechanism.

We allow angular momentum to flow through a drive shaft to a flywheel. Does it make any difference to the drive shaft whether or not an angular momentum current is flowing? Does the drive shaft "feel" the angular momentum current? Does it matter if it flows right to left or left to right?

If it is a thick shaft, you cannot tell by looking at it. Therefore, we will use a bendable, elastic object for this, possibly a plastic ruler, Fig. 8.20a. How does the ruler react when an angular momentum current flows through it? It twists because it is under a special kind of stress. We call this *torsion stress*. Even a solid object where no twisting can be seen when angular momentum is flowing through it, is under torsion stress.

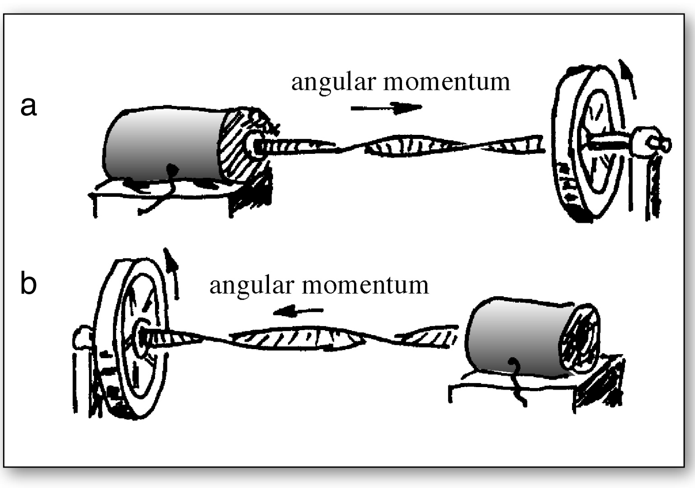


Fig. 8.20
(a) Angular momentum flows from left to right. (b) Angular momentum flows from right to left.

The direction of twist depends upon the direction of the flow of angular momentum. In Fig. 8.20a, the wheel is charged with positive angular momentum. This means that the angular momentum in the ruler is flowing from left to right.

The wheel in Fig. 8.20b also has positive angular momentum flowing into it. In this case it is coming from the right and is flowing to the left. What is the difference between the two rulers?

The edges of both rulers make a spiral. As you might already know, there are two types of spirals: right spirals and left spirals, Fig. 8.21. A right spiral is the kind that looks like a corkscrew, or a normal screw thread. A left spiral makes a so-called left-handed thread, or a mirror image of a corkscrew.

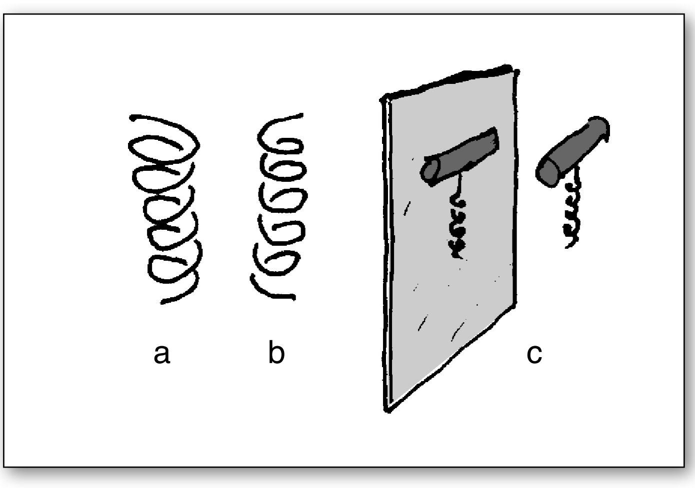


Fig. 8.21
(a) Spiral to the right. (b) Spiral to the left. (c) Corkscrew and mirror image.

Now, back to our angular momentum currents. In Fig. 8.20a, angular momentum flows from left to right. The ruler is twisted like left-handed thread. In Fig. 8.20b, the angular momentum flows from right to left. The ruler is twisted like a right-handed thread.

> Angular momentum to the right:
> conductor is twisted like a left-handed thread;
> Angular momentum to the left:
> conductor is twisted like a right-handed thread.

Exercises

1. Design an experiment to show whether or not water conducts angular momentum.

2. Design an experiment to prove that magnetic fields conduct angular momentum.

3. Air conducts almost no angular momentum. Describe an experiment or name a device that shows that air does conduct angular momentum a little bit.

4. Drive shafts are angular momentum conductors. A car has a number of drive shafts. They have different names for each function. Name some of these. What purpose do they serve?

5. Why are some drive shafts thicker and some thinner?

8.5 Angular momentum circuits

Fig. 8.22 shows how a coffee grinder is constructed. A real coffee grinder is a little more compact, but essentially, it looks like the one in the figure.

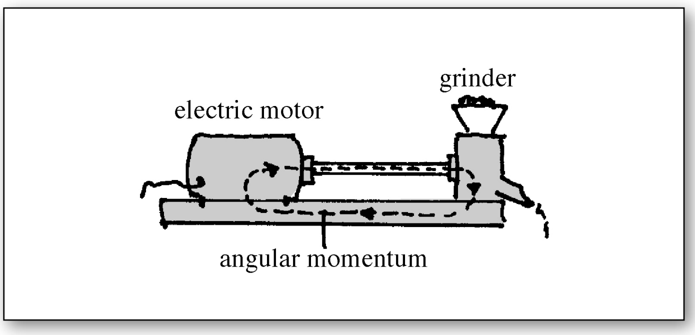


Fig. 8.22
Coffee grinder. The angular momentum flows in a closed circuit.

The grinder is powered by an electric motor. The motor pumps angular momentum through an axle to the grinder. Does the angular momentum increase in the grinder? No, because it would need to rotate faster and faster which it does not do.

Where does the angular momentum go? It must flow out of the grinder. This is no surprise. After all, there is a lot of friction between the rotating inner part of the grinder and the unmoving outer part of it. Friction is like a very bad bearing, meaning one that lets angular momentum flow off easily.

This means we have a closed angular momentum circuit: The motor pumps angular momentum out of the casing of the device, through the shaft and to the grinder. From there it goes to the casing of the grinder. It flows from there back to the motor.

Of course the motor as well as the grinder must be well mounted to the casing.

The situation is very similar to the turbine and generator in a power plant, Fig. 8.23.

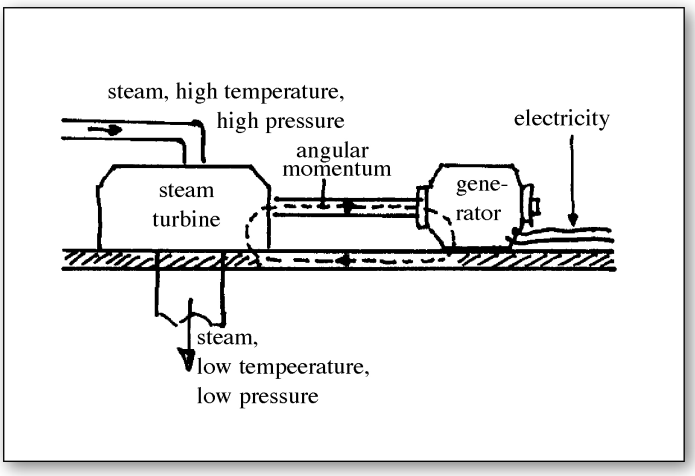


Fig. 8.23
Turbine and generator in a power plant. The angular momentum flows in a closed circuit.

Fig. 8.24a shows how someone drills a hole in a board. Angular momentum flows out of the Earth, through the man, over the drill and into the board. From there it flows over the vise and back into the ground.

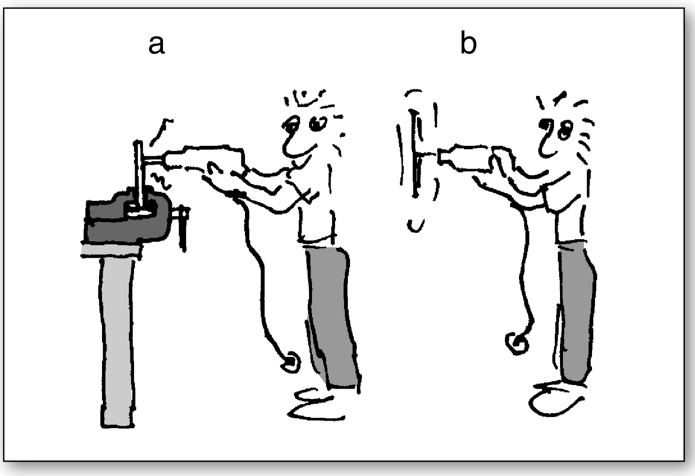


Fig. 8.24
(a) The angular momentum current circuit is closed. (b) The angular momentum current circuit is interrupted (open).

Fig. 8.24b shows what happens when the angular momentum circuit is not closed. The board was taken out of the vise. The angular momentum cannot flow off anymore. The motor runs but it doesn't pump anymore. The board rotates but it doesn't get faster; it does not get any new angular momentum.

Exercises

1. What path does the angular momentum take in an electric fan?

2. Someone sharpens a pencil. What path does the angular momentum take?

8.6 Angular momentum as an energy carrier

We will now consider the coffee grinder again, but will take a new viewpoint: We will perform an energy balance. The motor receives energy and gives it to the grinder. What are the energy carriers? Electricity is the carrier by which the energy reaches the motor.

It is now clear how it continues. In addition to energy, angular momentum flows between the motor and the grinder. In this case, angular momentum must be the energy carrier, Fig. 8.25.

Angular momentum is an energy carrier.

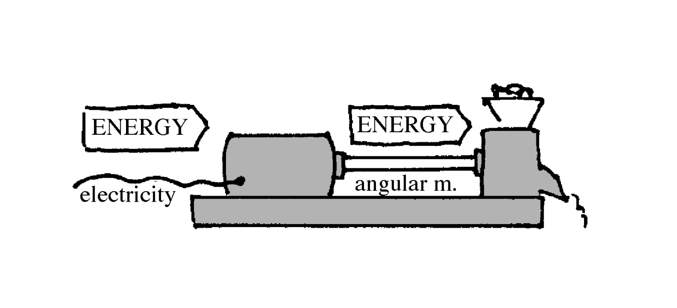


Fig. 8.25
Energy gets with the carrier angular momentum to the grinder.

In other words: In an electric motor, the energy is transferred from the carrier electricity to the carrier angular momentum. Energy travels from the motor to the grinder along with angular momentum. Within the grinder, the energy leaves the angular momentum and the angular momentum travels back to the motor via the casing.

Fig. 8.26 shows the flow diagram of a hydroelectric plant.

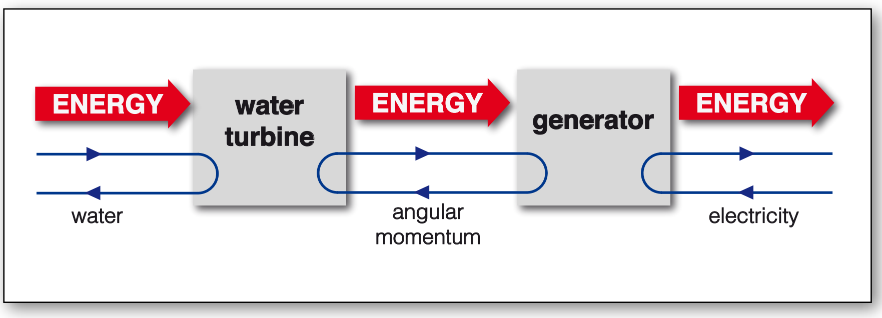


Fig. 8.26
Flow diagram of a hydroelectric power plant.

We will now apply the balance of angular momentum and energy to a flywheel. The flywheel in Fig. 8.27 is charged with angular momentum. This means that the motor pumps angular momentum out of the ground, over the axle and into the flywheel.

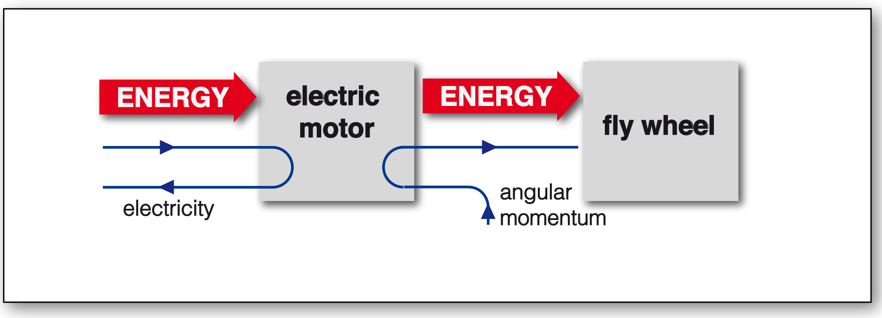


Fig. 8.27
A flywheel is being charged with energy and angular momentum.

By now we have realized that not only angular momentum but also energy flows through a rotating axle. Where does this energy go? Because the flywheel has no outlet for the energy, the energy must also accumulate in it. The flywheel stores angular momentum and energy simultaneously.

A rotating flywheel, meaning one that has been charged with angular momentum and energy, can be used to drive a dynamo, for example, Fig. 8.28.

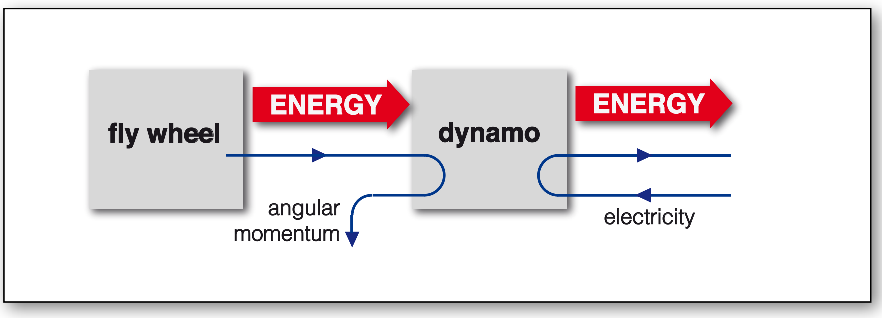


Fig. 8.28
A flywheel is driving a dynamo.

Of course you have seen toy cars with flywheel drives. They just need to be pushed strongly once across the floor. In the process, the flywheels get charged with angular momentum and energy. The cars can then travel a while on their own using the energy in the flywheel.

Exercises

1. Sketch the flow diagrams of a water turbine, a windmill, a water pump and the blades of a ventilator.

2. Name some energy sources that give up energy with the carrier angular momentum. How do you recognize them?

3. Name some energy recipients that receive energy with the carrier angular momentum.

4. How do you recognize devices which receive energy from a person by the carrier angular momentum?

9

Compressive and tensile stress

9.1 The relation between pressure and momentum current

A block K is clamped between two walls by a spring F, Fig. 9.1. A momentum current flows through this setup. Whenever a momentum current flows, the conductor is under *mechanical stress*: Either compressive or tensile stress. You remember our rule that momentum flow to the right means compression and momentum flow to the left means tension.

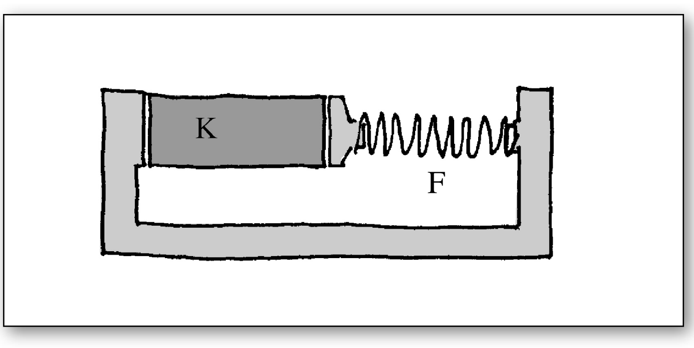


Fig. 9.1
Block K is under compressive stress.

Let us consider the stress in our block. Because the momentum current is distributed over the entire block, every part of it is under compressive stress. Every part of it 'feels' the pressure, Fig. 9.2.

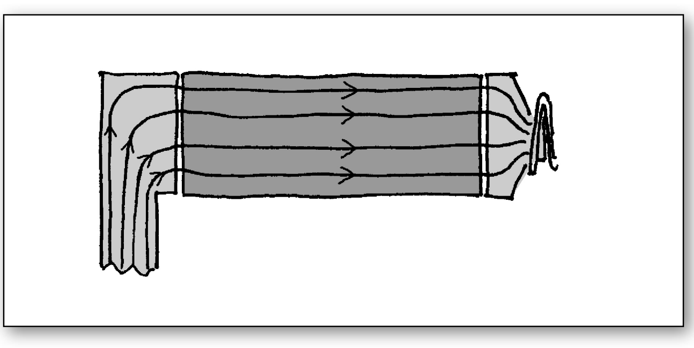


Fig. 9.2
The momentum current distributes over the entire cross section of the block.

In Fig. 9.3, we compare blocks K1 and K2. The two springs are totally identical and the same momentum currents flow in each of them. Let's assume that this current is 200 Hy/s = 200 N. Block K2 has a larger cross sectional area than K1. The momentum current distributes over a larger surface here. The *momentum current per surface area* is therefore smaller. In block K1

$$\frac{200}{25}\ \text{Hy/s} = 8\ \text{N}$$

flow through each square centimeter of the surface area, and

$$\frac{200}{100}\ \text{Hy/s} = 2\ \text{N}$$

flows through each square centimeter of the cross sectional area of K2.

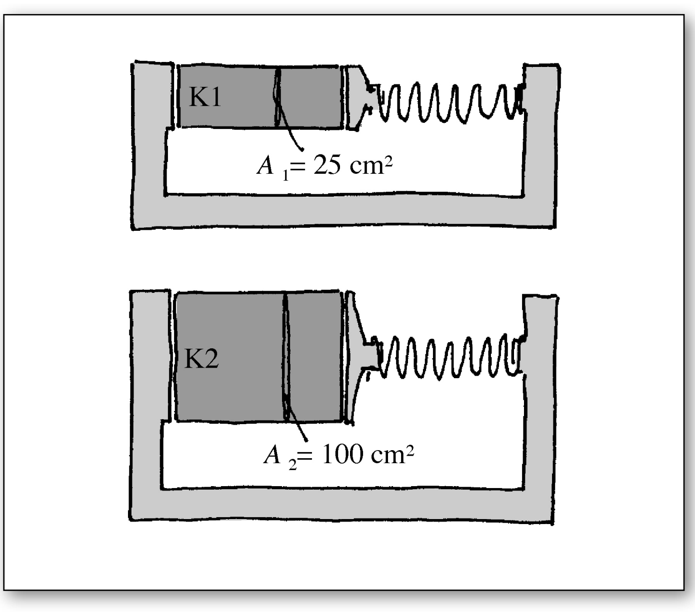


Fig. 9.3
The momentum currents in K1 and K2 are the same. The momentum current per cross sectional area (pressure) is greater in K1 than in K2.

This means that any given part of the material of K1 'feels' a higher pressure than a corresponding piece of K2.

We see that in order to characterize the mechanical stress at a point somewhere inside a body, the momentum current per surface area can be used. This quantity, i.e., the ratio of the momentum current to the surface through which it flows, is called pressure. It is the same quantity we have already been introduced to in a different way.

Because pressure is expressed by p, we have

$$p = \frac{F}{A}\ .$$

If the momentum current is given in Newtons (N) and the surface area in m², the resulting unit for pressure is N/m². This unit is called a Pascal, abbreviated to Pa. Therefore

$$\text{Pa} = \frac{\text{N}}{\text{m}^2}$$

1 Pa is a very small pressure. Larger units are often used instead

1 kPa = 1000 Pa

and

1 MPa = 1 000 000 Pa

or the bar:

1 bar = 100 000 Pa.

Now back to our blocks. The pressure, or the compressive stress, in block K1 is

$$p_1 = \frac{F}{A_1} = \frac{200\ \text{N}}{0.0025\ \text{m}^2} = 80\,000\ \text{Pa} = 80\ \text{kPa}\ .$$

The result for block K2 is

$$p_2 = \frac{F}{A_2} = \frac{200\ \text{N}}{0.01\ \text{m}^2} = 20\,000\ \text{Pa} = 20\ \text{kPa}$$

(The areas A_1 and A_2 must be expressed in m² for the resulting pressure unit to be Pa.)

In Fig. 9.4, a momentum current of 200 N flows in the negative direction through body K. In calculating the quantity p, this is taken into account by putting a minus sign in front of the current value. Therefore

$$p = \frac{-200\ \text{N}}{0.01\ \text{m}^2} = -20\,000\ \text{Pa} = -20\ \text{kPa}$$

A negative stress value means tensile stress.

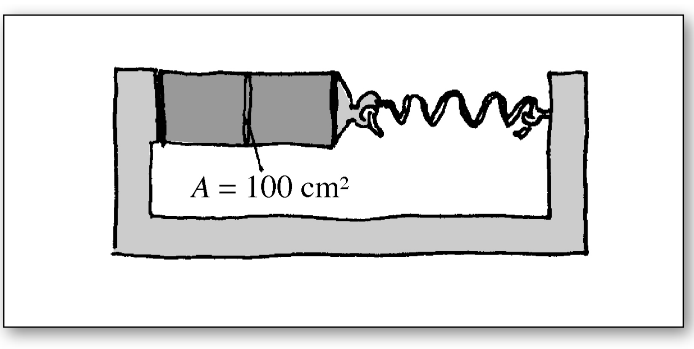


Fig. 9.4
The block is under tensile stress. The pressure is negative.

Summary:

Pressure equals momentum current per surface area.

Exercises

1. A car is being towed. Fig. 9.5 shows the hook on the car being towed, a piece of metal wire and a piece of plastic rope knotted to it. A momentum current of 420 N flows into the car. Calculate the stress in the ropes at locations 1, 2, and 3. Watch for the algebraic sign: compressive or tensile stress?

2. The ropes in Fig. 9.6 have a cross section of 1.5 cm². The crate has a mass of 12 kg. Calculate the tensile stress at locations 1, 2, and 3.

3. You push a thumbtack into a wooden board. Estimate what the pressure is at the middle of the nail halfway up. What is the pressure at the tip of the thumbtack?

4. Estimate the compressive stress at the tip of a nail being hit by a hammer.

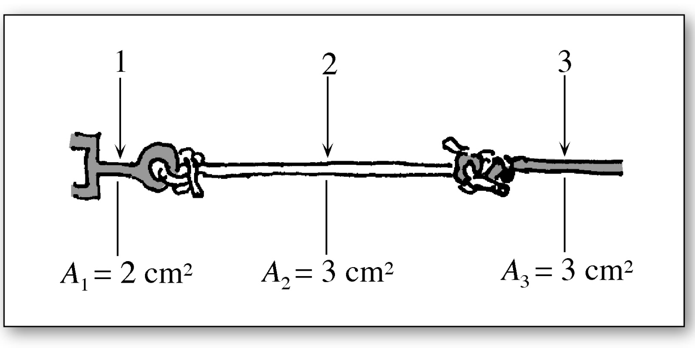


Fig. 9.5
For Exercise **1**

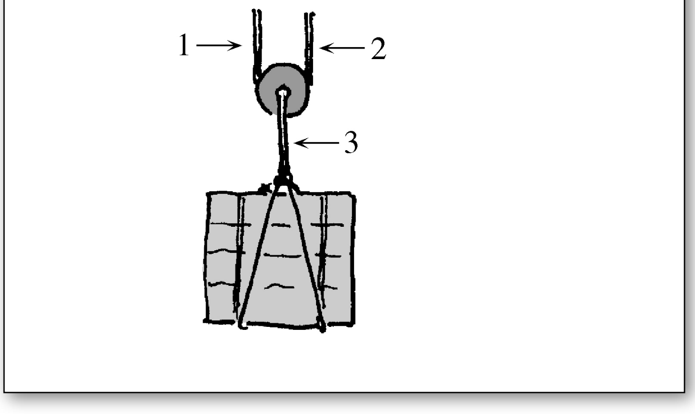


Fig. 9.6
For Exercise **2**

9.2 Stress in three directions

We wish to put a body under both compressive and tensile stress. You might think that this cannot be done, that a body can only be under either compressive or tensile stress and that to put a body under both is impossible. We will try anyway and find we are successful.

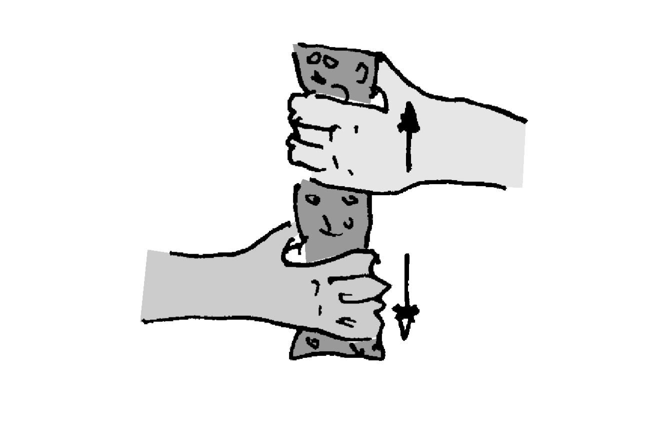


Fig. 9.7
The sponge's interior is under tensile stress vertically and under compressive stress horizontally.

We take the object, a sponge, for example, in both hands and press our fingers together. At the same time, we pull apart with our hands, Fig. 9.7. The interior of the sponge actually feels both compressive and tensile stress: compressive stress in the horizontal direction and tensile stress in the vertical direction. Fig. 9.8 shows a similar situation. Block K is under tensile stress in the horizontal direction and under compressive stress in the vertical. It is, of course, possible to put it under tensile stress in both directions and under compressive stress in both directions. The compressive and tensile stresses in the horizontal and vertical directions can also have different values.

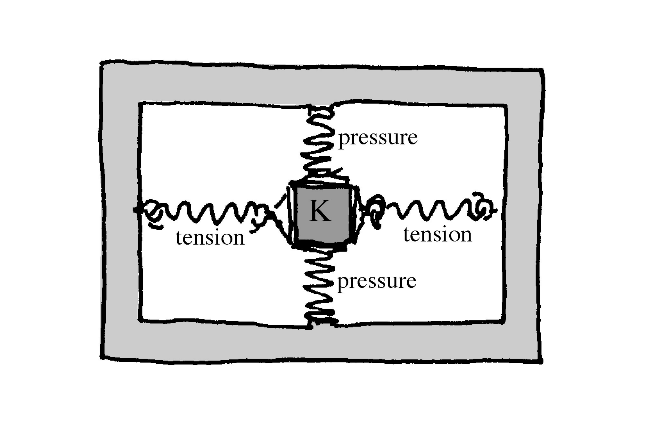


Fig. 9.8
The block is under compressive stress vertically and, horizontally, under tensile stress.

In the case of Fig. 9.9, the horizontal pressure has the value

$$p_1 = \frac{50\ \text{N}}{0.01\ \text{m}^2} = 5000\ \text{Pa} = 5\ \text{kPa}$$

and the vertical one is

$$p_2 = \frac{300\ \text{N}}{0.015\ \text{m}^2} = 20\,000\ \text{Pa} = 20\ \text{kPa}$$

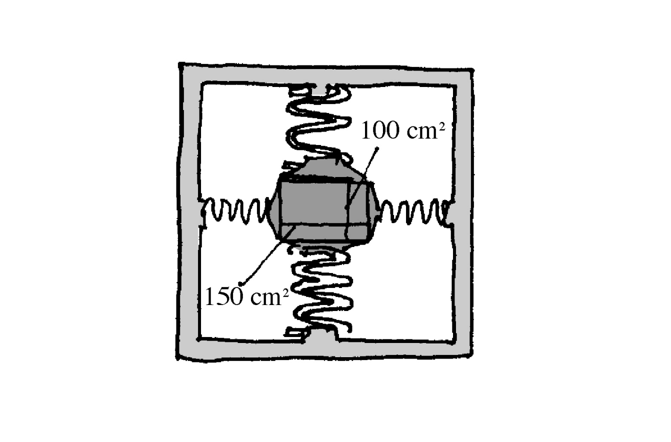


Fig. 9.9
The pressures differ vertically and horizontally.

Finally, it is possible to put the block under compressive or tensile stress in a third direction as well, Fig. 9.10. For example, this could be

p_1 = 5000 Pa

p_2 = 2000 Pa

p_3 = 40 000 Pa.

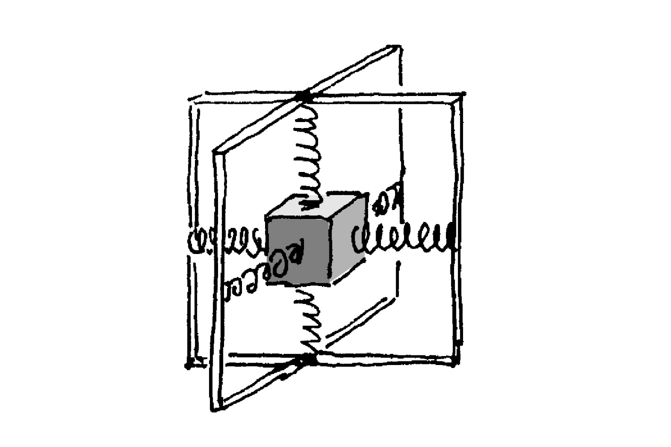


Fig. 9.10
Pressure can be given in three mutually perpendicular directions.

You might ask whether it is possible to just continue in this way, to create more and more compressive stress values in more and more spatial directions. Why not five different compressive or tensile stresses in five different directions, Fig. 9.11? Because it is not possible. It is difficult to show the proof of this so we will just have to accept it:

> It is possible to fix independent compressive or tensile stress values in three mutually perpendicular directions.

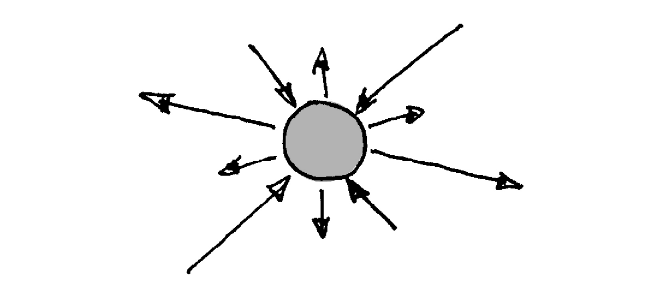


Fig. 9.11
No more than three pressures are allowed in three dimensions (in two dimensions, only two).

As soon as you try to change the stress in a fourth direction, the stresses in the first three directions automatically change.

This statement is valid for every point inside of a body. However, mechanical stress can change from location to location. In the case of the pressed sponge in Fig. 9.7, the stresses in the middle are different from those at the upper and lower ends of it.

If the pressure in three mutually perpendicular directions has the same value, say 12 kPa, then the pressure is equal to 12 kPa in all other spatial directions.

Every material has only a certain tolerance for compressive or tensile stress. It is often so that a material can be put under higher compressive stress than under tensile stress.

Concrete, for example, tolerates a compressive stress of about 50 MPa, but only 1/20th of this value as tensile stress. Sometimes a concrete support is expected to take tensile stress at certain points. Fig. 9.12 shows a concrete support mounted at both ends and carrying a load in the middle. This is a typical situation. The concrete on the upper part of the support is under horizontal compressive stress. The lower part is under horizontal tensile stress. Since it cannot tolerate the high tensile stress it is reinforced by steel. Steel tolerates high tensile stress.

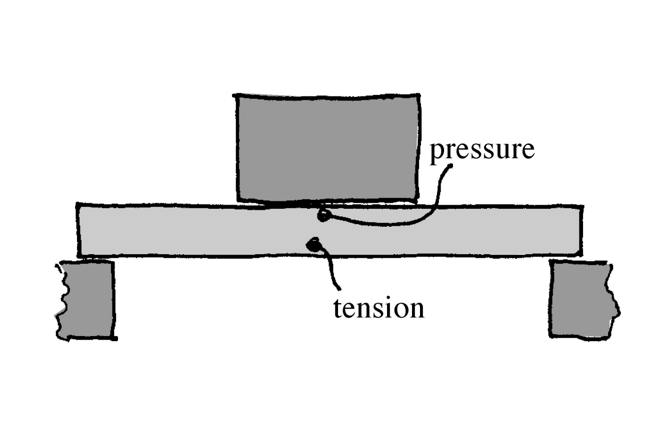


Fig. 9.12
The upper part of the support is under compressive stress in the horizontal direction; in the lower part, there is tensile stress.

For the same reason, some synthetic materials are reinforced with carbon fibers in order to raise their tensile strength. Such materials are used in the manufacturing of skis, diving boards for swimming pools, and for gliders.

There are many materials that cannot tolerate the same stress in different directions. A good example of this is wood. Pine can tolerate a tensile stress of 10 MPa in the direction of its grain, but only 1/20th of that in the perpendicular direction.

Exercises

1. Name some materials which have high tolerance for tensile stress but low tolerance for compressive stress.

2. Name some materials having high tolerance for compressive but low tolerance for tensile stress.

3. Name some materials that tolerate strongly different compressive and tensile stresses in different directions.

9.3 Pressure in liquids and gases

Up to now we have only considered mechanical stress in solid objects. (A sponge is a 'solid' object because it is neither liquid nor gaseous.) Now we wish to put a liquid, e. g. water, under pressure. At first we will do this awkwardly and try to proceed as we did with the block in Fig. 9.1. We press from above in the middle of the water, Fig. 9.13. The result is as expected: the water moves sideways.

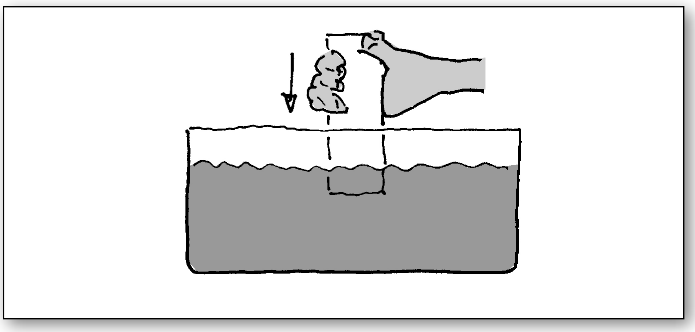


Fig. 9.13
Water cannot be put under pressure in this manner. It gets out of the way sideways.

We then try a new approach and contain the water in such a way that it cannot move sideways, Fig. 9.14. If the piston has a cross sectional area of $A = 5\ \text{cm}^2$ and the momentum current is $F = 200$ N, a pressure of

$$p = \frac{F}{A} = \frac{200\ \text{N}}{0.0005\ \text{m}^2} = 400\,000\ \text{Pa} = 0.4\ \text{MPa}$$

results in the horizontal direction.

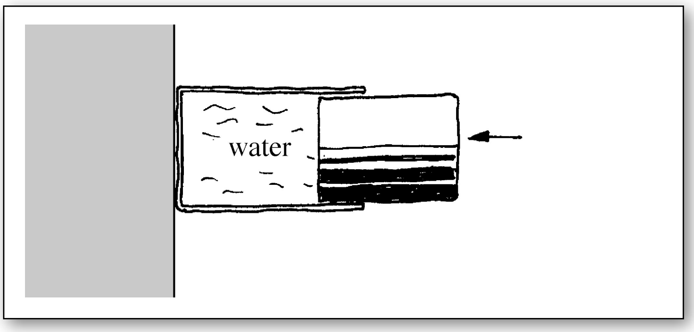


Fig. 9.14
The piston is under pressure only horizontally, but the water is under pressure in all directions.

The water tries to move in the direction transverse to the direction of pressure, thus creating a compressive stress in this transverse direction. This stress has the same value as the one in the direction of the piston. In every other direction, as well, the pressure has the same value.

The experiment shown in Fig. 9.15 shows this very clearly.

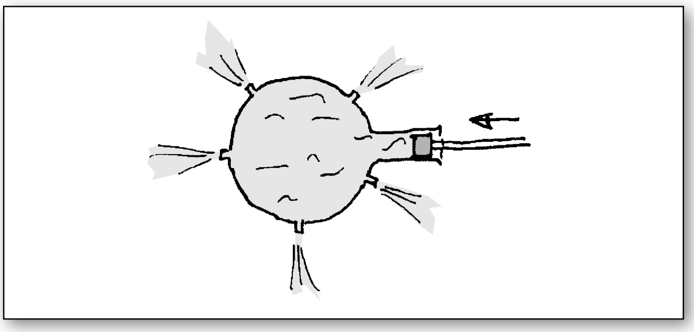


Fig. 9.15
Water sprays in all directions because there is pressure in all directions.

> At any location in a liquid, the pressure is the same in every direction.

The same holds for gases because they also move sideways if they are not prevented from doing so.

9.4 Density

We consider a body of uniform composition. It has a mass *m* and a volume *V*. The ratio of mass to volume is the density *ρ*:

$$\rho = \frac{m}{V} \ .$$

An object made of iron has a greater density than one made of wood, since a cubic meter of iron has a greater mass than a cubic meter of wood. Density is independent of the object's size and shape because it is the mass divided by its volume. It depends only upon the material it is made up of. It is a *material property.*

This is why we do not need to say that the iron object has a density of 7800 kg/m^3. It is enough to say that iron has this density.

In order to have something concrete to think about here, we have spoken about solid bodies. The ratio *m/V* can just as well be used to characterize a liquid or a gas.

There are a lot of other material characteristics that can be expressed as numerical values such as electric conductivity, thermal conductivity, the ability to be heated, the ability to be magnetized, color, absorptivity…

9.5 Hydrostatic pressure

Remember: the pressure at any given location in a liquid is the same in all *directions*. However, this does not mean that the pressure must be the same at every *location*. We will get to know a situation where the pressure changes from place to place.

Fig. 9.16 shows a cylindrical container filled with water. The pressure of the water increases from top to bottom. We wish to measure the pressure at a distance h from the water's surface.

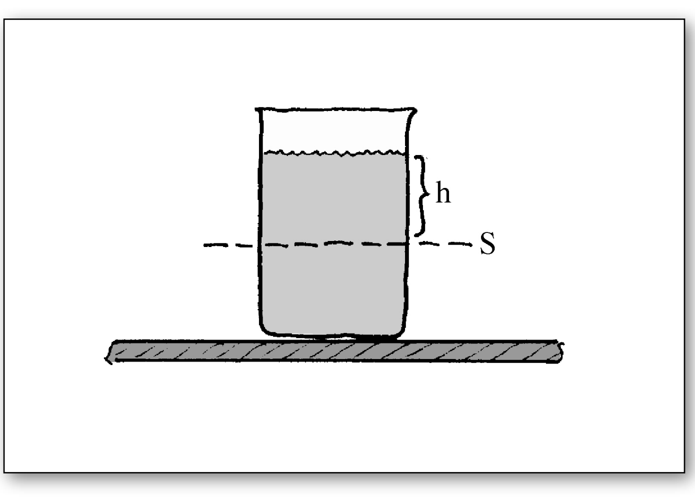


Fig. 9.16
Pressure increases from top to bottom.

In our minds, we make a cut S through the water. The surface area of the cut is A. Our first question is how strong the momentum current through this area is.

This momentum current comes from two different sources:

1. Because the pressure of the air above the water's surface is p_{air}

$p_{air} = 1$ bar = 100 000 Pa,

a momentum current of

$F_1 = p_{air} \cdot A$

flows into and through the water.

2. According to our old equation

$F = m \cdot g$ (1)

a momentum current flows through the gravitational field into every part of the water, and it flows down through the water as well (m = mass, g = strength of gravitational field). The entire momentum current flowing into the water from above flows through our cross section. To calculate this momentum current we have to use the mass of the water above the cut in Equation (1):

$F_2 = p_{above} \cdot A$ (2)

The mass m_{above} can be calculated easily. To do this, we solve the relation

$\rho = \frac{m}{V}$

for m and put for ρ the density of the water and for V the volume of that part of the water which is above the cross section:

$m_{above} = \rho_{water} \cdot V_{above}$.

Now we have

$V_{above} = A \cdot h$.

Therefore,

$m_{above} = \rho_{water} \cdot A \cdot h$.

Inserting this into equation (2) yields

$F_2 = m_{above} \cdot g = \rho_{water} \cdot A \cdot h \cdot g$.

The total momentum current F is comprised of both F_1 and F_2:

$F = F_1 + F_2 = p_{air} \cdot A + \rho_{water} \cdot A \cdot h \cdot g = (p_{air} + \rho_{water} \cdot h \cdot g) \cdot A$.

We can now calculate the pressure at the height of the cross section by using $p = F/A$:

$$p = \frac{F}{A} = \frac{(p_{air} + \rho_{water} \cdot g \cdot h)A}{A} = p_{air} + \rho_{water} \cdot g \cdot h$$

The pressure at distance h from the water's surface is therefore:

$p = p_{air} + \rho_{water} \cdot h \cdot g$ (3)

Since the momentum current is made up of two parts, also the pressure is made up of two parts:

– of the air's contribution p_{air} above the water;

– of the contribution $p_S = \rho_{water} \cdot h \cdot g$, which has its origins in the weight of the water. The water below feels the weight of the water lying above it. This contribution p_S is called the *hydrostatic pressure* of the water.

The calculated pressure is actually a vertical pressure. Because we have considered a liquid here, the same pressure must rule in the horizontal directions as well.

Of course these considerations are valid for other liquids as well, not just for water. It is only necessary to use the density ρ of the liquid being considered in place of the density of water (ρ_{water}). In general we have

> The hydrostatic pressure in a liquid is:
>
> $p_S = \rho \cdot g \cdot h$

Let us calculate the hydrostatic pressure in water numerically. We set

$\rho = 1000$ kg/m³ und $g = 10$ N/kg

and obtain

$p_S = 1000 \text{ kg/m}^3 \cdot 10 \text{ N/kg} \cdot h = 10\,000 \cdot h \cdot \text{N/m}^3$

If the distance to the water's surface is

$h = 10$ m,

then

$p_S = 100\,000 \text{ N/m}^3 = 100\,000 \text{ Pa} = 1$ bar.

The total pressure is

$p = p_{air} + p_S = 1 \text{ bar} + 1 \text{ bar} = 2$ bar.

At 10 m below the surface of the water, the pressure is 1 bar higher than at its surface. At 20 m depth, it is 2 bar higher, etc.

Fig. 9.17 shows the pressure as a function of the depth h. The zero point of h is at the water's surface.

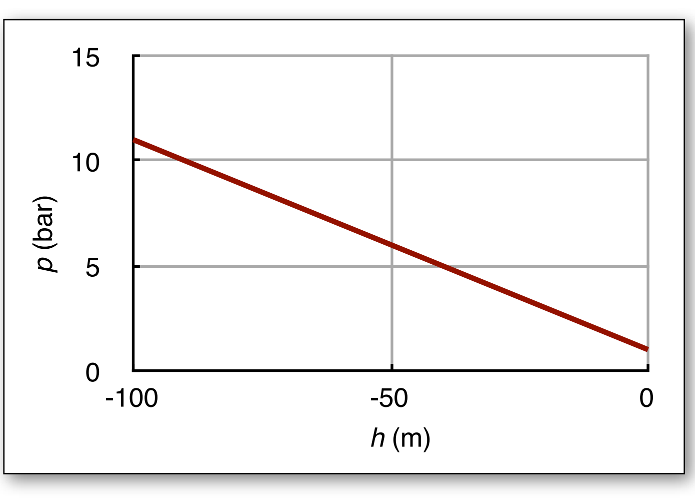


Fig. 9.17
Pressure as a function of the depth of water. The zero point of depth lies at the water's surface.

Just as the pressure of water decreases upwardly, air pressure also decreases upwardly from the Earth's surface. The pressure of the air around us is the hydrostatic pressure of the air. In this case though, the reduction in pressure with altitude is not linear. We cannot calculate air pressure with altitude using the formula

$p_S = \rho \cdot g \cdot h$

any longer, because the density of the air decreases with altitude. Fig. 9.18 shows the (hydrostatic) pressure of the air as a function of altitude. Notice that the height axes of Fig. 9.17 and 9.18 have different units.

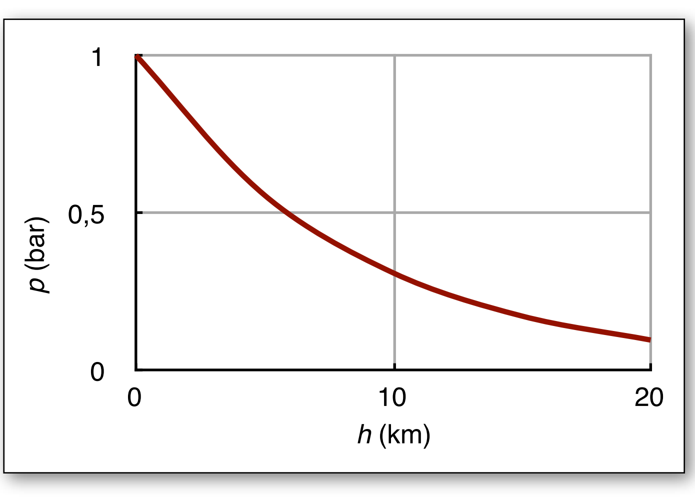


Fig. 9.18
Air pressure as a function of altitude. The altitude is counted positive in the upward direction.

Fig. 9.19 shows the pressure as a function of the height above and below the ocean's surface. The vertical scale begins 100 m below the water's surface. At this point, the pressure is 11 bar. At the surface, the pressure reduces to 1 bar. This is the hydrostatic pressure of air at sea level. This pressure decreases as one goes on moving upward. However, it decreases slowly because air has a low density.

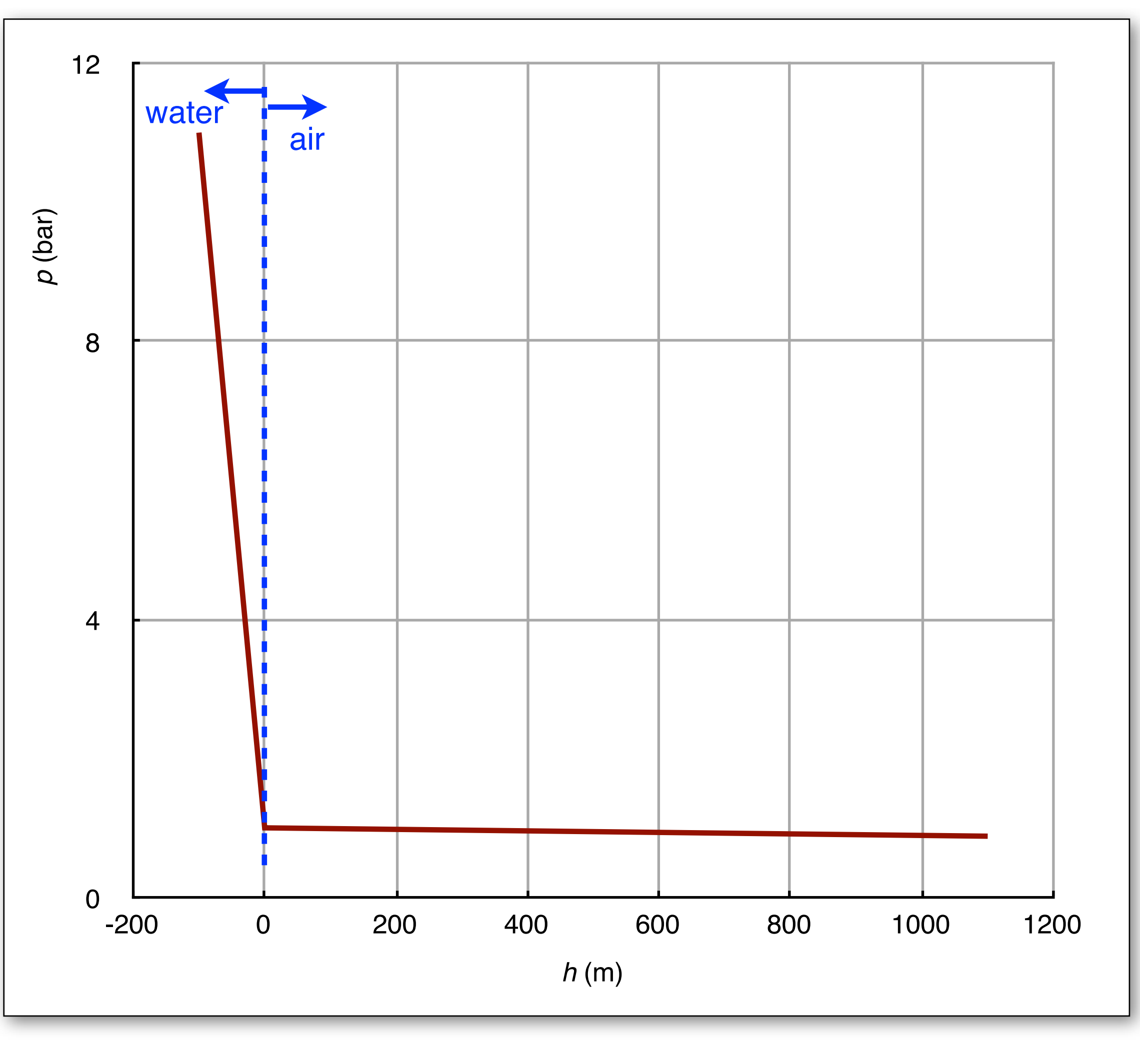


Fig. 9.19
Pressure as a function of the altitude above and below the ocean's surface. The zero point lies at the ocean's surface.

Exercises

1. What is the hydrostatic pressure of water at the bottom of a swimming pool of 4 m depth? What is the total pressure?

2. At its lowest point, the ocean is 11 000 m deep. What is the pressure there?

3. What is the pressure at the bottom of the container in Fig. 9.20? The piston dividing the two liquids can be moved easily. It is so small and light that its influence upon the pressure at the bottom can be ignored. Mercury's density is 13,550 kg/m³.

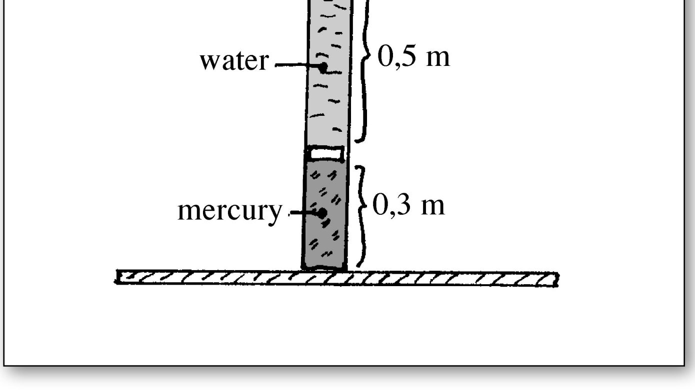


Fig. 9.20
For exercise **3**

9.6 More complicated containers

In order to find the formula

$p_S = \rho_{water} \cdot g \cdot h$

we considered a container with vertical walls. We might conclude that our formula holds only for this type of container.

Fig. 9.21a shows a container made up of two parts. The parts are connected by a pipe. What is the hydrostatic pressure at A and what is it at B? We apply our formula and find that

at A: $p_{S,A} = \rho_{water} \cdot g \cdot h_A$,

at B: $p_{S,B}' = \rho_{water} \cdot g \cdot h_B'$.

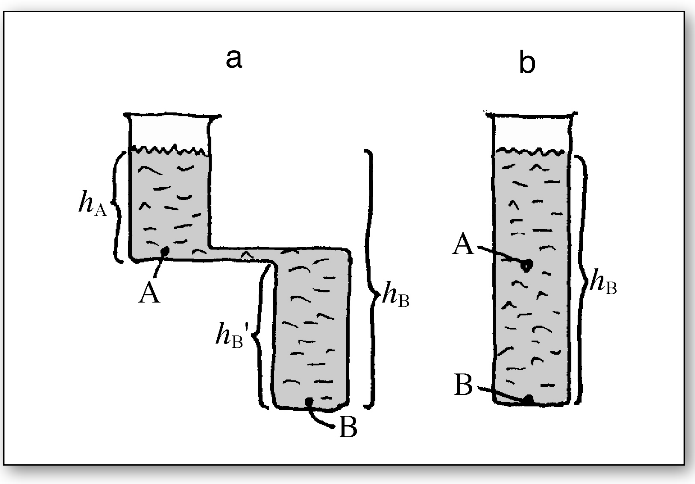


Fig. 9.21
The pressure at point B in the container on the left is the same at that at point B in the one on the right.

The pressure values which we have calculated, are correct. However, it must understand what kind of pressure has been calculated. For the moment, we will ignore the fact that air presses down with 1 bar upon the surface of the water.

$p_{S,A}$ is the hydrostatic pressure caused by the water in the upper container at location A. What about $p_{S,B}'$? $p_{S,B}'$ is the hydrostatic pressure at B as a result of the water in the lower container. However, we must also take into account that the water in the container above presses upon the water in the container below through the thin pipe connecting them. This means that the pressure in B is not only determined by the water in the lower container. Because of the connecting pipe, it is influenced by the water in the upper one as well.

Instead we can say that the pressure at B is the hydrostatic pressure of all of the water, that in the higher container plus that in the lower container:

$p_{S,B} = p_{S,A} + p_{S,B}' = \rho_{water} \cdot g \cdot (h_A + h_B') = \rho_{water} \cdot g \cdot h_B$.

The pressure at B is the same we would find if we had a unique container of height h_B, Fig. 9.21b. In other words: The height h which we must use in the equation

$p_S = \rho_{water} \cdot g \cdot h$

is the vertical distance to the water’s surface. It does not matter if the surface lies above the considered point or if it is displaced from it, and it does not matter how large the surface is.

At any point in the water at a given depth below the surface, the pressure is the same. This is shown in Fig. 9.22.

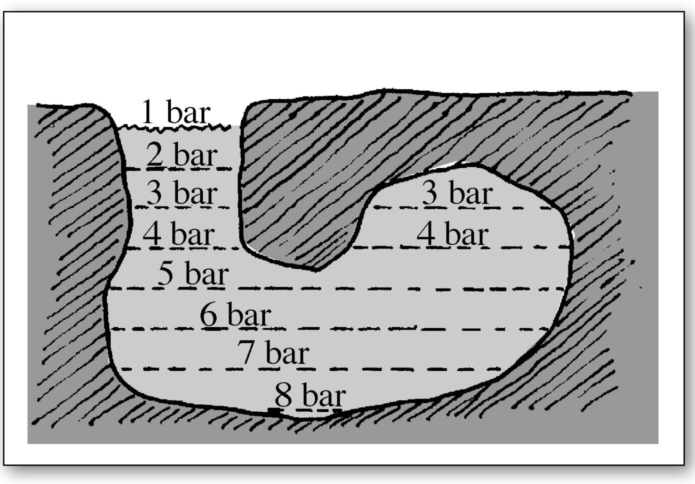


Fig. 9.22
In a continuous amount of fluid, the pressure is the same at all points on a horizontal plane.

The fact that the pressure in a liquid is the same everywhere at a certain depth holds only if there are no currents in it. If the liquid (or gas) flows, the pressure is no longer constant because a current is produced by a pressure difference.

We conclude:

> In liquids and gases at rest, the pressure is the same at every point on a horizontal level.

This principle agrees with something everyone has seen: In connected containers, the water levels are the same, Fig. 9.23.

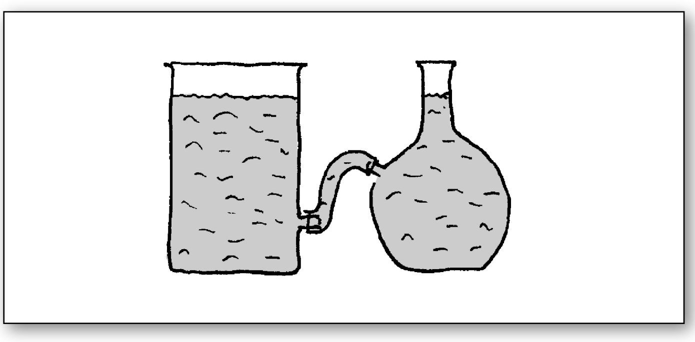


Fig. 9.23
In communicating containers, the surfaces of the liquid have the same heights.

The hydrostatic pressure of both water surfaces is 0 Pa, meaning the values are the same. Our rule tells us that the surfaces must be lying at one and the same level.

Exercises

1. What happens if the valve in Fig. 9.24 is opened? Why?

2. In Fig. 9.25, there is water in the container on the left and alcohol on the right. They border between the liquids lies in the horizontal pipe. The level of the alcohol in the container on the right is higher than the water level in the container on the left. Why? What is the level difference? ($\rho_{water} = 790$ kg/m³)

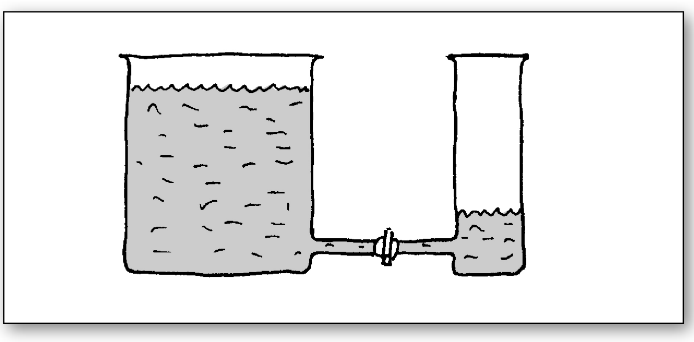


Fig. 9.24
For exercise **1**

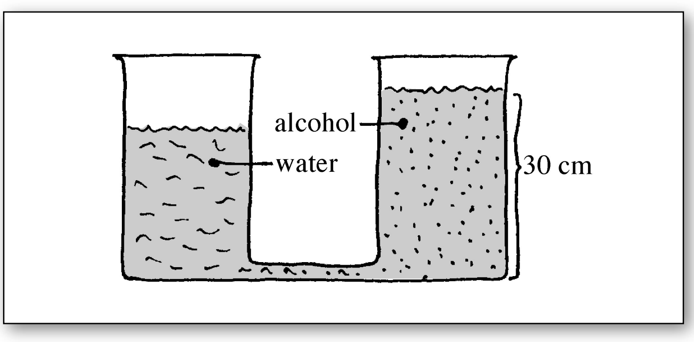


Fig. 9.25
For exercise **2**

9.7 Buoyant force

A ball is pressed down into water. One feels that the ball "wants" to come up. Why is this so? The water is pressing upon all sides of the ball. However, water pressure increases with depth, so the pressure at the bottom of the ball is stronger than at its top. The result is the ball being pushed upward, Fig. 9.26.

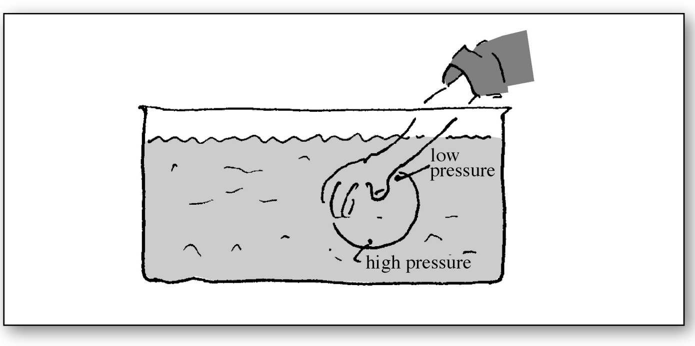


Fig. 9.26
The pressure is higher at the underside of the ball than on it's upper side.

Not only very light objects (like a ball) experience a buoyant force, but every other body put into a liquid does as well.

A piece of iron is hung from a scale and then immersed in water, Fig. 9.27. The reading of the scale decreases. The water pushes the piece of iron upward, and it seems to become lighter.

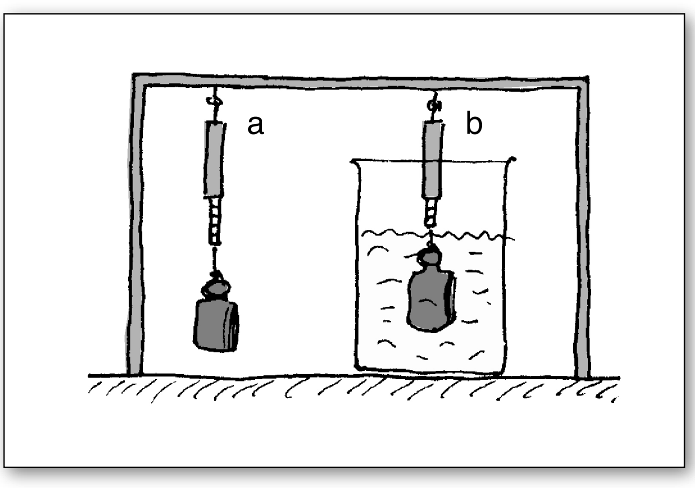


Fig. 9.27
A piece of iron hanging from a scale is immersed in water. It appears to be lighter.

A momentum current of

$F = m_{iron} \cdot g$

flows through the gravitational field into the piece of iron. In Fig. 9.27a, the momentum arriving in the body leaves again through the string. In Fig. 9.27b, only a part of this momentum current flows off through the string. The rest of it flows into the water. The strength of the momentum current flowing through the water is called *buoyant force*. We will calculate it.

In order to do this, we first consider the water without any object immersed in it, but we imagine the space the object would occupy in it. (You could imagine this part of the water being separated from the rest by a thin plastic bag.) This 'body of water' is suspended, it does not float to the surface nor does it sink to the bottom. This means that it does not receive or lose any net momentum.

The total momentum current of

$F_{in} = m_{water} \cdot g$

flowing in (the object receives it through the gravitational field) flows off again through its surface into the surrounding water and from there, into the Earth. Therefore, the momentum current F_{out} flowing away, must be equal to

$F_{out} = m_{water} \cdot g$

We now replace our phantom body made of water with the original one of iron. A momentum current of

$F_{in} = m_{iron} \cdot g$

flows into the iron body.

The same current flows away from it as before because the pressure distribution on the surface of the iron body is the same as on the surface of the phantom water body. Therefore, a current of

$F_{out} = m_{water} \cdot g$

flows away. This time the net current is not zero, but

$F_{net} = F_{in} – F_{out} = (m_{iron} – m_{water}) \cdot g$ (1)

The inflowing momentum current is reduced by $m_{water} \cdot g$ as a result of the immersion. This is the buoyancy force F_A we are looking for:

$F_A = m_{water} \cdot g$.

We remember what m_{water} means: It is the mass of our phantom body made of water, or in other words, the mass of the water displaced by the iron body.

These considerations are also valid when our immersed body is not made of iron and our liquid is not water. In Equation (1), we use m_K instead of m_{iron} for the mass of the body, and m_{liq} instead of m_{water} for the liquid. We obtain:

$F_{net} = (m_K – m_{liq}) \cdot g$ (2)

The buoyant force is the part subtracted from $m_K \cdot g$, so that:

$F_A = m_{liq} \cdot g$.

We can also express it this way:

> The apparent mass is smaller than the actual mass by the mass of the liquid displaced.

We will change Equation (2) to a more convenient form. We replace the two masses m_K and m_{liq} with the help of the equation

$m = \rho \cdot V$,

which results from

$\rho = m/V$.

Therefore

$m_K = \rho_K \cdot V$, (3)

and

$m_{liq} = \rho_{liq} \cdot V$. (4)

The volume V of the body is the same as that of the displaced liquid.

Introducing Equations (3) and (4) into (2) yields:

$F_{net} = (\rho_K – \rho_{liq}) \cdot V \cdot g$.

This equation tells us that the net momentum current is positive when ρ_K is greater than ρ_{liq}. It is negative when ρ_K is smaller than ρ_{liq} . A positive net momentum current into a body means that the body begins to move downward: It sinks. A negative net momentum current means that it begins moving in the negative direction, meaning upward: It rises, it floats. Only when $\rho_K = \rho_{liq}$, is F_{net} 0, and the body remains suspended.

> $\rho_K > \rho_{liq}$: The body sinks.
> $\rho_K < \rho_{liq}$: The body floats.
> $\rho_K = \rho_{liq}$: The body is suspended.

Exercises

1. Calculate the buoyant force F_A of a piece of iron having a volume of 5 cm³, that is totally submerged in mercury. Does the iron float or sink? By how many grams does the mass of the piece of iron seem to be reduced? (Density of iron: 7900 kg/m³, density of mercury: 13,550 g/m³.)

2. A granite boulder with a mass of 150 tons lies at the bottom of the ocean (Density of granite: 2600 kg/m³). What is the buoyant force? How much smaller does its mass appear to be than it would be on dry land?

3. A stone with a density of 2400 kg/m³ lies at the bottom of a swimming pool. In the water, it "weighs" 1.4 kg. What is its real mass?

4. The density of wood is less than that of water. This is why a piece of wood submerged in water rises to the top. When a piece of the wood protrudes out of the water, it stops rising. Why?

5. A ship weighs 1500 tons. What is the mass of the displaced water?

6. First, a ship sails down a river and then out to sea. The density of seawater is somewhat greater than the density of the river water. What consequences does this have for the ship?

9.8 Tensile stress in gases and liquids

When you drink soda through a drinking straw, you have the feeling of pulling on the soda, Fig. 9.28. How else could it rise through the straw? When you start to drink, there is still air in the straw. You suck and the soda rises. It looks like you could also pull on the air.

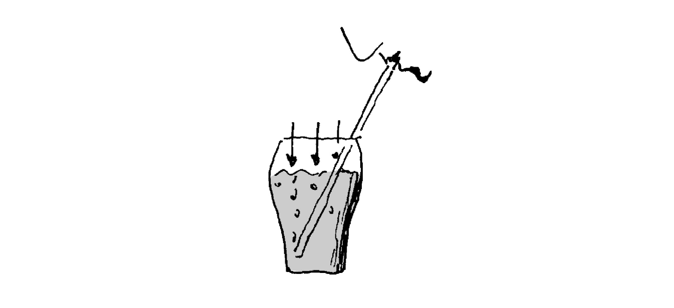


Fig. 9.28
The soda in the drinking straw is pushed upward, not drawn upward.

We will see that this is an erroneous conclusion. You cannot pull on either air or on the soda. More generally:

Gases and liquids cannot be put under tensile stress.

Why do we draw the wrong conclusion here? How does the soda get into our mouths if not by pulling?

The air in the cylinder in Fig. 9.29a is under normal pressure: $p = 1$ bar. The air outside, though, is under pressure of 1 bar as well. Although the air inside presses against the piston, we do not need to hold it in place because the air pressure from outside presses exactly as strongly and balances out the pressure from inside.

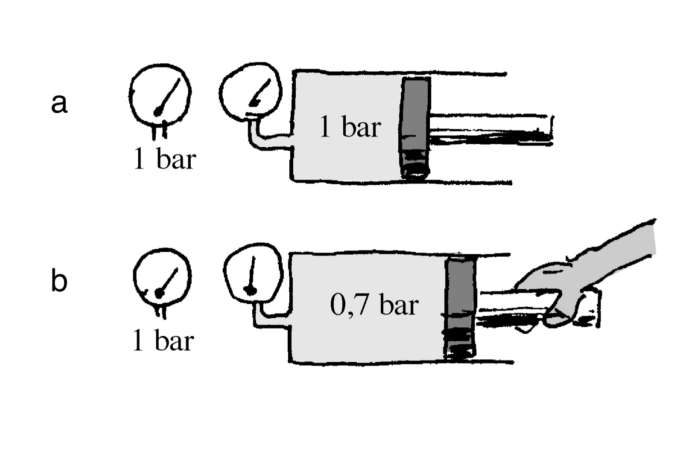


Fig. 9.29
Air presses from inside and from outside on the piston. (a) The pressure is the same inside and outside. (b) The pressure outside is greater than inside.

We now pull the piston a little to the right and hold it there, Fig. 9.29b. The pressure inside decreases, but of course, the pressure outside does not. We have the impression of the piston being pulled left and we need to hold onto it. Actually, it is only being pressed to the left by the air outside. In the interior, the pressure remains positive, but it is smaller than before. There is no tensile stress.

You could think that it might be possible to produce a tensile stress inside if you pull far enough, if the piston is moved far enough to the right, Fig. 9.30. The experiment shows that this doesn't work. The pressure decreases, but more and more slowly. It never reaches the value of 0 bar. It always remains positive.

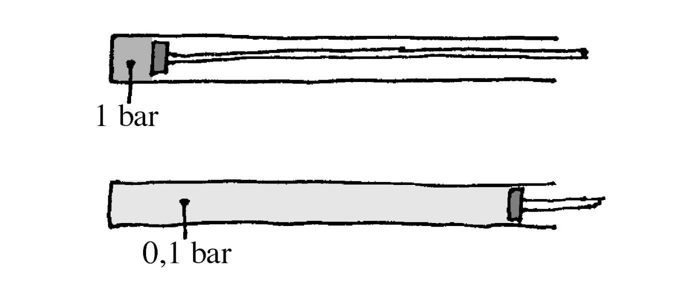


Fig. 9.30
No matter how far the piston is pulled outward, the pressure in the cylinder stays positive.

Instead of pulling a piston, a vacuum pump could be attached to the cylinder, Fig. 9.31. The pressure decreases as the pump runs and only after all the air has been pumped out does the pressure reach the value 0 bar. Pumping more does not produce a negative pressure. No wonder: When all the air is gone, nothing more is there to be under tensile stress.

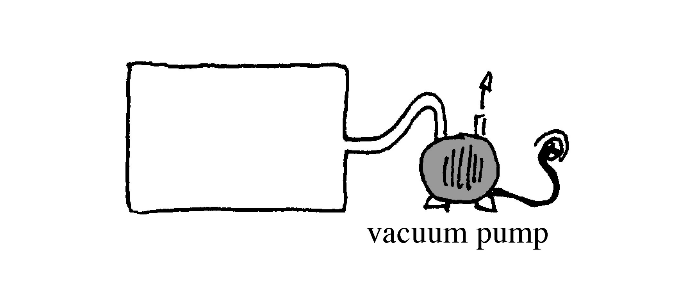


Fig. 9.31
A vacuum pump cannot create negative pressure either.

Something else happens when one tries to pull on a liquid, water for example, Fig. 9.32. The piston cannot be pulled outward as easily as it would be with a gas. Again, the reason is not a tensile stress inside, but the outside air pressure upon the piston. If one pulls hard enough and overcomes the outside air pressure, the piston will move indeed. However, it can be seen that the liquid does not expand like the air does in Fig. 9.29. Rather, a bubble forms, a space where there is no liquid water. This cannot be an air bubble. Where would the air come from? Actually, this space is almost totally empty. A close investigation shows that it is not quite empty: There is a slight amount of water vapor (water in the form of gas) in it.

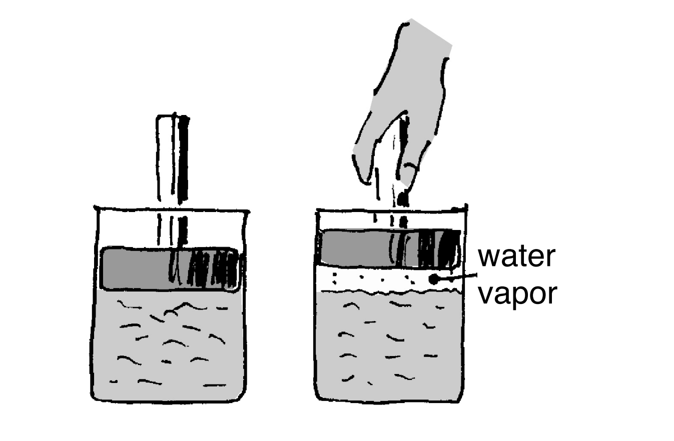


Fig. 9.32
There is water in the cylinder. When the piston is pulled, a bubble containing water vapor forms.

Now back to our soda. By sucking on a drinking straw, one removes air from it. The pressure in the air in the straw decreases. The air pressure outside presses upon the soda and it rises in the straw. The soda is not pulled up the straw, it is pushed up.

A suction pump functions similarly to this, Fig. 9.33. The pump appears to pull the water upward. Actually, it is only lowering the pressure at the top so that the outside air pressure can push the water upward.

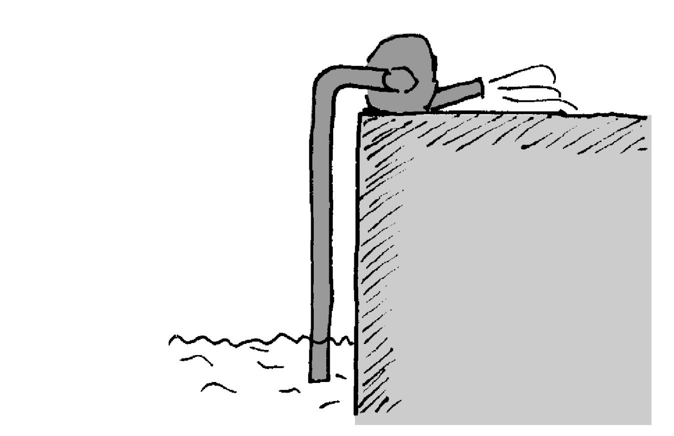


Fig. 9.33
The pump reduces the positive pressure at its inlet. The air over the water's surface pushes the water upward through the pipe.

Exercises

1. A glass is pressed under water with its opening downward. Why doesn't it fill up with water?

2. A glass is held under water so that it fills with water. It is then pulled out of the water with the opening pointing downward, Fig. 9.34. Why does the water stay in the glass?

3. In Fig. 9.35, what is the pressure of the water at A? What is the pressure at B? What happens if the faucet is opened?

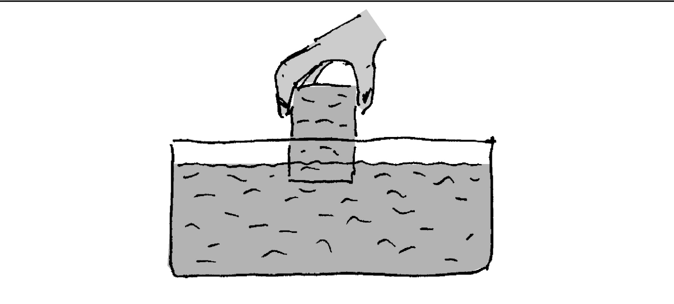


Fig. 9.34
For exercise **2**

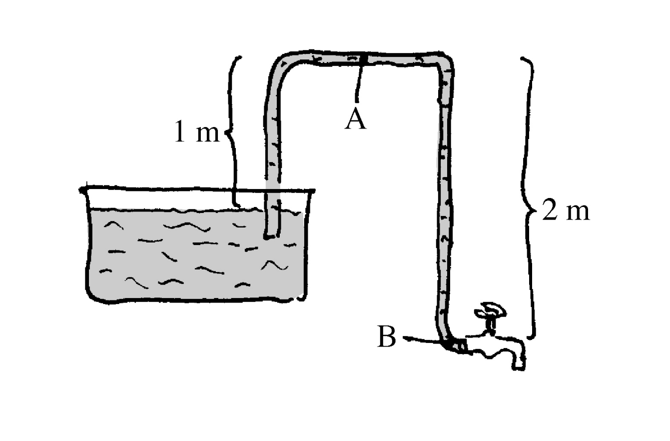


Fig. 9.35
For exercise **3**

9.9 Hydraulic transport of energy

We are now able to understand why hydraulic machines are so practical.

We will first investigate the simplest hydraulic transport of energy imaginable, Fig. 9.36. This is a pipe with a piston at each end and a liquid between them. You can think of water as the liquid, but oil is normally used. The advantages of oil are obvious: It freezes at much lower temperatures than water, and it lubricates the pistons.

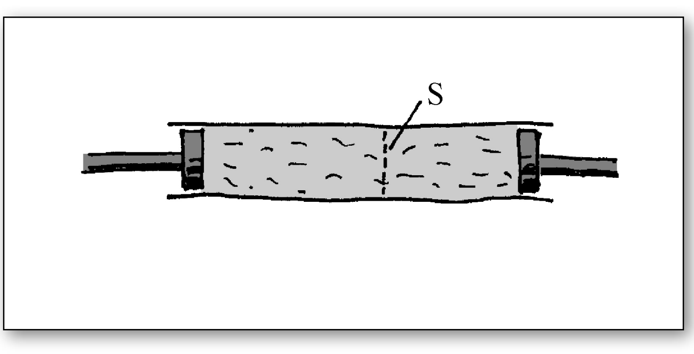


Fig. 9.36
Energy transport with a flowing liquid

If the piston on the left is pushed to the right, the piston on the right moves as well. If the right-hand piston is able to move freely, almost no energy is transported. The small amount of energy put into the left side is necessary to overcome friction.

If the piston on the right is driving something, moving the one on the left becomes more difficult. Energy is put in on the left and it comes out again on the right.

We wish to calculate the strength P of the energy current from left to right. This means we will express it by easily measured quantities. We wish to find the energy current flowing through the imagined cross-section S. How many Joules are flowing through this area per second?

The liquid moving in the pipe is equivalent to a rod, and the energy current for a rod is

$P = v \cdot F$

Here, v is the velocity with which the rod moves, and F is the momentum current in it.

This formula is also valid for hydraulic liquids. The velocity at which the liquid moves is v. F can be expressed by the pressure p and the cross sectional area A of the pipe:

$F = A \cdot p$

The energy current is then

$P = v \cdot A \cdot p$

This formula is very useful because it can be used to calculate the energy current at every location in a pipe of any form and diameter.

We consider a pipe that is somewhat more complicated: A pipe whose cross sectional area increases from A_1 to A_2, Fig. 9.37. We also compare the energy currents at locations 1 and 2. At 2, the liquid flows more slowly than at 1. This means that v_2 is smaller than v_1. It is not difficult to find the relation between v_1 and v_2.

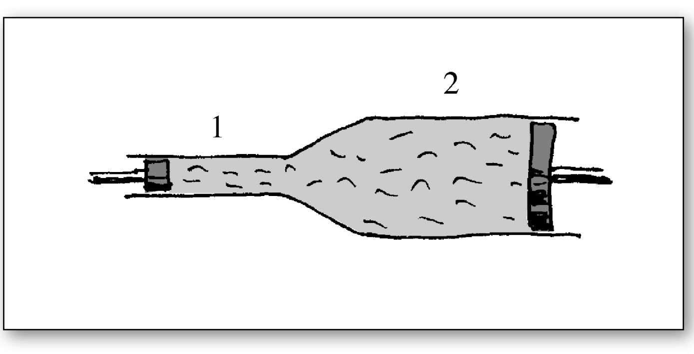


Fig. 9.37
Energy transport in a pipe with an increasingly large cross sectional area.

If the piston on the left is moved a distance Δx_1, a liquid volume of

$\Delta V_1 = A_1 \cdot \Delta x_1$

is pushed to the right. The piston on the right must now make space for the same amount of liquid that the piston on the left displaced. This means that

$\Delta V_1 = \Delta V_2$

or

$A_1 \cdot \Delta x_1 = A_2 \cdot \Delta x_2$. (1)

Now, if the left hand piston is moved at a constant rate and the time needed to push it a distance Δx_1 is Δt, then the velocity of the left hand piston is v_1

$v_1 = \frac{\Delta x_1}{\Delta t}$ (2)

and the velocity v_2 of the right hand piston is

$v_2 = \frac{\Delta x_2}{\Delta t}$ (3)

We divide both sides of equation (1) by Δt and obtain

$\frac{\Delta x_1}{\Delta t} \cdot A_1 = \frac{\Delta x_2}{\Delta t} \cdot A_2$.

Using Equations (2) and (3), we get

$v_1 \cdot A_1 = v_2 \cdot A_2$.

We now multiply both sides of this equation by the pressure p of the liquid which is the same on the right and the left:

$p \cdot v_1 \cdot A_1 = p \cdot v_2 \cdot A_2$ (4)

We saw before that the energy current is

$P = v \cdot A \cdot p$ (5)

In Equation (4), the strength of the energy current on the left is the one at section 1, and on the right, it is the one at section 2. The equation tells us that both are equal: The energy put in on the left comes out on the right. This surely doesn't surprise you.

The energy current is the same everywhere in the pipe. The place where the cross sectional area is smaller, the fluid velocity is greater and where the cross sectional area is larger, the fluid velocity is smaller. The energy current, though, is the same everywhere.

Equation (5) shows us how to transport a lot of energy (per second). We need a high pressure, a large cross sectional area for the pipe, and a high velocity. All of these factors have their limitations, though. If the pressure is too high, the pipe or hose will break. A large cross sectional area can be impractical because the hose would be too clumsy to handle. High velocity has the disadvantage of large energy losses by friction.

For these reasons, one seeks a compromise where the disadvantages are acceptable. We consider a typical example: The energy transfer from the pump to the arm of a power shovel. The pressure in the power shovel's hoses is about 150 bar = 15 MPa, the cross sectional area of the hose is 6 cm² = 0.0006 m² and the flow velocity is 0.5 m/s. The resulting energy current is

$P = v \cdot A \cdot p = 0.5\ \text{m/s} \cdot 0.0006\ \text{m}^2 \cdot 15\ \text{MPa} = 4500\ \text{W}$.

It is possible to transport energy easily by using hydraulic equipment. Such equipment also has other advantages. We will look at one of them.

The momentum current in piston 1 in Fig. 9.38 is

$F_1 = A_1 \cdot p$,

and in piston 2, it is

$F_2 = A_2 \cdot p$.

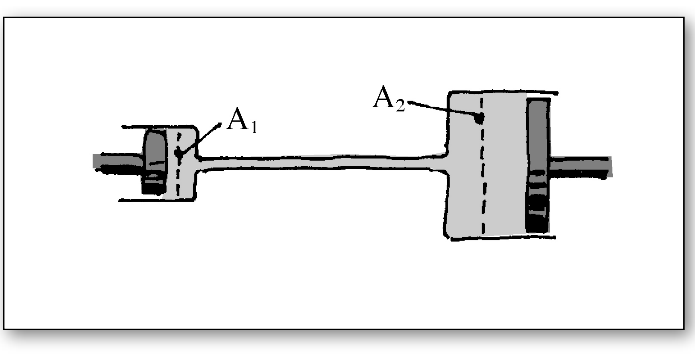


Fig. 9.38
A stronger momentum current flows in the piston on the right than in the one on the left.

Combining the equations, we obtain

$\frac{F_1}{F_2} = \frac{A_1}{A_2}$

By pushing on the left hand piston, a momentum current is produced in the right hand piston which is stronger by a factor of A_2/A_1. This effect can be used to lift heavy charges. The car in Fig. 9.39 is lifted by hand but with the help of hydraulics. Don't forget, though, energy is not gained as a result of this. Lifting the car is easier with hydraulics than it would be if you tried it the direct way, Fig. 9.40, but it takes longer.

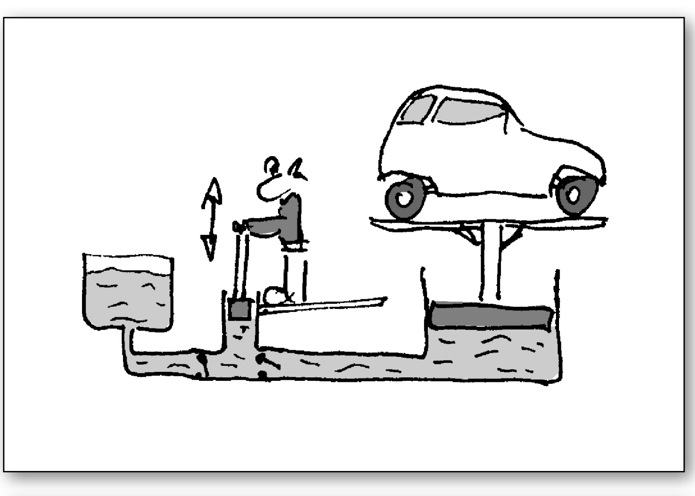


Fig. 9.39
It is easy to lift a car by hand.

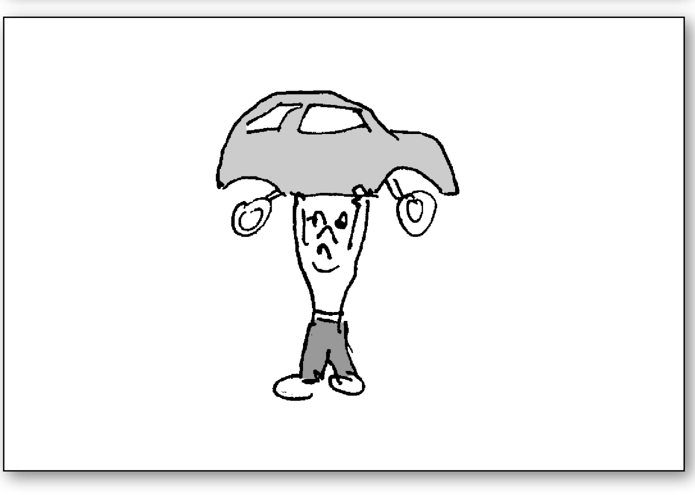


Fig. 9.40
It is difficult to lift a car by hand.

Exercises

1. There is a pressure of 150 bar in the hose of a power excavator's hydraulic system. The cross section of the hose is 5 cm², and the velocity of the hydraulic oil is 20 cm/s. How much energy is being transported? How strong is the momentum current in the hose?

2. At the entrance to a water turbine there is a pressure of 80 bar. The diameter of the pipe is 1 m. The turbine receives 12 MJ of energy per second. How fast is the water flowing in the pipe?

10

Entropy and entropy currents

The second main area of physics that we will now turn to is thermodynamics – the science of heat. The name already gives away what the subject is all about: the phenomena having to do with an object being hotter or colder. Similarly to how we accounted for momentum in mechanics, we will learn to count quantities of heat in thermodynamics, i.e., we will make use of a law of balance for heat.

Thermodynamics is important for understanding natural phenomena as well as technical devices and machines.

Life on Earth is possible only because of the huge current of heat coming from the Sun. Climate and weather on Earth are determined essentially by thermal processes. (By 'thermal' we mean having to do with thermodynamics.)

Countless machines function according to the principles of thermodynamics. Some of these are automobile engines, steam turbines in power plants, and heat pumps in refrigerators.

Heat loss and the sources of heat in heating systems of houses can be quantitatively described by using thermodynamics.

Heat also plays an important role in chemical reactions.

Thermodynamics deals with different phenomena than mechanics does. For this reason, thermodynamics uses different physical quantities. However, this does not mean that we can forget about mechanics completely when working with thermodynamics. For one, there are quantities that are common to both subjects such as energy and energy currents. There are also principles, relationships and rules in mechanics that have corresponding ones in thermodynamics. It is not necessary to begin learning all over again in order to understand thermal physics.

10.1 Entropy and temperature

As always, when we begin with a new field of physics, we must get to know our most important tools: the physical quantities we will be working with. In mechanics we began with two quantities to describe the state of motion of a body. These were velocity and momentum. Similarly, we can use two quantities to describe the thermal state of a body.

You already know one of these quantities, *temperature*. It is abbreviated by the Greek letter called Theta and looks like this: ϑ. Temperature is measured in the unit °C (degrees Celsius). The sentence “The temperature is 18 degrees Celsius” can be shortened to ϑ = 18°C.

The second quantity is also known to you, but by another name than the one used in physics. This is what in everyday language we call ‘quantity of heat’ or just ‘heat’. We will make a simple experiment to show the difference between quantity of heat and temperature, Fig. 10.1. Glass A contains 1 l of water at 80°C. We pour half of this water into another empty glass B. What happens with the temperature and the amount of heat? The temperature of the water in glasses A and B is the same as it was in glass A before it was poured. However, the amount of heat was distributed over the water in both glasses after pouring. If there were 10 units of heat in glass A at the beginning, afterwards there are 5 units in A and 5 units in B.

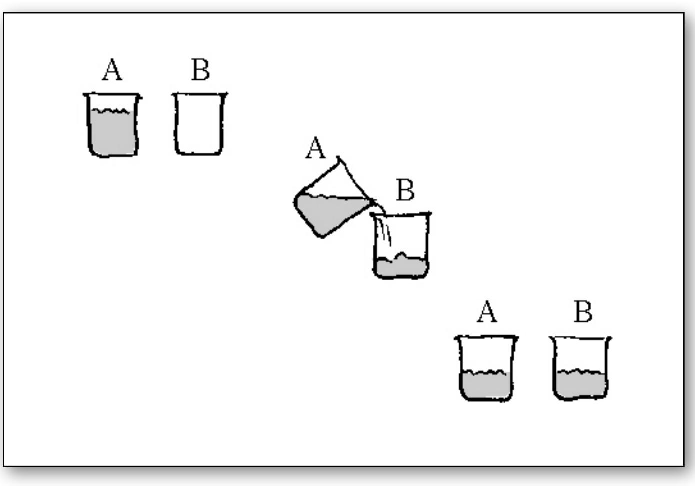


Fig. 10.1
Half of the water in container A is poured into container B.

We see that temperature describes the state of being warm (or cold) of a body, independent of its size. The quantity of heat is what is *contained* in the body.

What we call “quantity of heat” in everyday language, has a special name in physics. It is called *entropy*. The symbol used for entropy is *S* and its unit is the Carnot, abbreviated to Ct. If a body contains 20 Carnot of entropy, it can be written like this:

S = 20 Ct.

This unit is named after Sadi Carnot (1796 – 1832), a physicist who made important contributions toward the discovery of entropy.

In the following, when we are investigating the properties of entropy, just remember that it deals with what we call heat in everyday language.

We compare the two water glasses in Fig. 10.2. Both contain the same amount of water. The water in the glass on the left is hot, having a temperature of 70°C. The water in the glass on the right is cool. It has a temperature of 10°C. Which glass contains more entropy? (In which glass is more heat?) Of course, it is the one on the left.

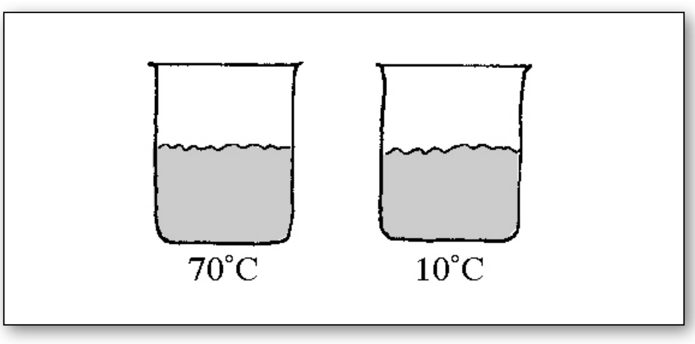


Fig. 10.2
The water in the glass on the left contains more entropy than the one on the right.

> The higher the temperature of an object, the more entropy is contained in it.

Now we will compare the water glasses in Fig. 10.3. In this case, the temperatures are the same, but the mass of the water is different in each glass. Which glass contains more entropy? Again, it is the one on the left.

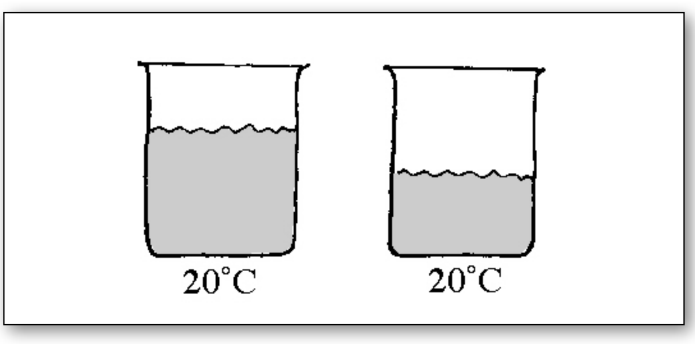


Fig. 10.3
The water in the glass on the left contains more entropy than the one on the right.

> The greater the mass of an object, the more entropy is contained in it.

Which of the glasses in Fig. 10.4 contains more entropy cannot be determined yet.

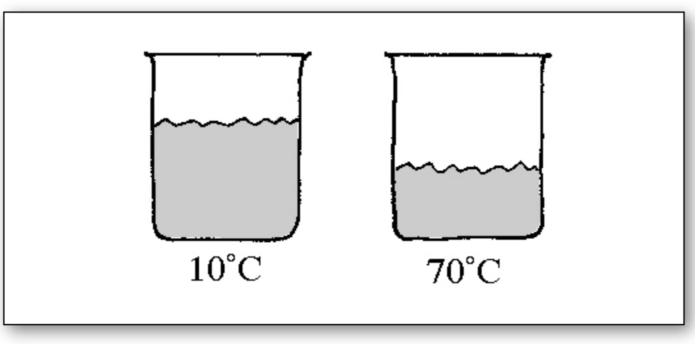


Fig. 10.4
It is not easy to decide which of these glasses contains more entropy.

We will consider an experiment like the one in Fig. 10.1. In glass A, there is 1 l of water with 4000 Ct of entropy. We pour 1/4 of the water, or 250 ml, into the other empty glass B. How much entropy is in A after pouring? How much is in B? Entropy is distributed according to the ratio of the amounts of water. This means that glass B received 1000 Ct, and 3000 Ct stayed in A.

Exactly what is 1 Carnot? Is this a lot or a little entropy? 1 Carnot is a very handy unit: 1 cm^3 of water at 25˚C contains 3.88 Ct. A general rule:

> 1 cm^3 of water at normal temperature contains about 4 Ct.

Exercises

1. The air in a room A having a volume of 75 m^3 has a temperature of 25°C. The air temperature in another room B having a volume of 60 m^3 is 18°C. Which room contains more entropy?

2. There is 3900 Ct of entropy contained in the coffee in a full coffee pot. Coffee is poured into three cups. Each cup then has the same amount of coffee and the pot is half full. How much entropy is contained in the pot after pouring into the three cups? How much is contained in each cup?

10.2 Temperature difference as driving force for an entropy current

We place a container A with hot water into a container B with cold water, Fig. 10.5. We will observe what happens and then explain our observations.

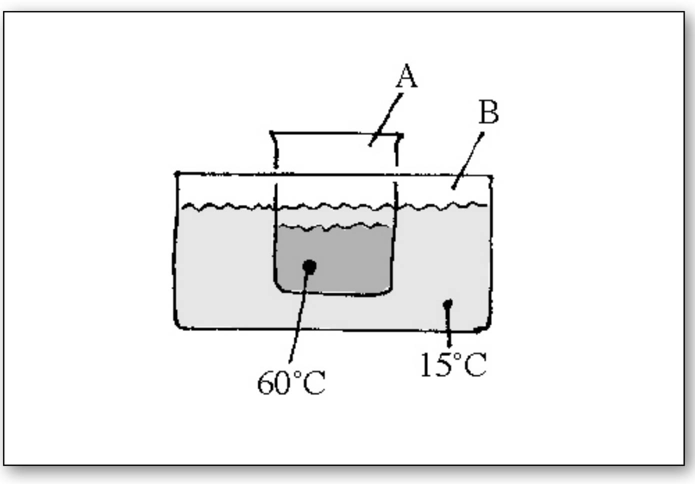


Fig. 10.5
Entropy flows out of the inner container A into the outer container B.

First the observation: The temperature of the water in container A decreases, and the temperature in B increases. The temperatures approach each other and finally become equal. The temperature of B increases, but not beyond that of A.

Now the explanation: Entropy flows from A to B and does so until the temperatures become equal.

This experiment can be repeated with other types of containers, Fig. 10.6a and b. The water temperature will always be the same in both containers in the end. In the case of Fig. 10.6a, the end temperature is closer to the initial temperature of B, in Fig. 10.6b it is closer to the initial temperature of A. In every case, the final result is that

$\vartheta_A = \vartheta_B$.

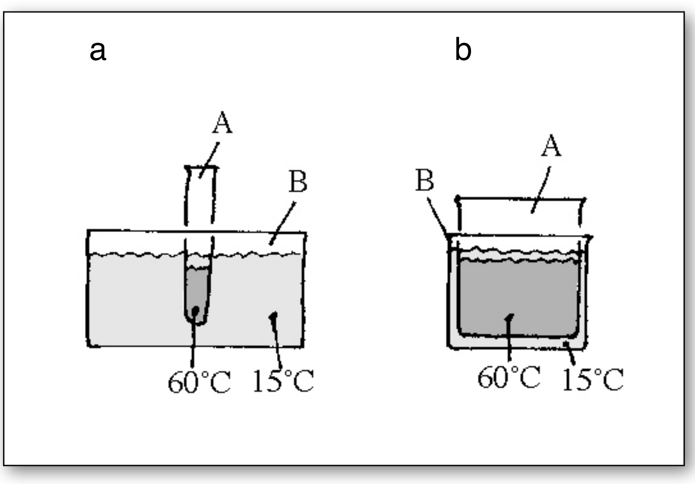


Fig. 10.6
In both cases, entropy flows from the inner to the outer container.

Of course it is possible to begin with the inner container A having the lower temperature and the outer container B having the higher one. In this case, as well, the temperatures approach each other and finally reach the same value. We conclude that:

> Entropy flows by itself from places of higher temperature to places of lower temperature.

This sentence is certainly familiar to you. If you look back in this book you will find two more versions of it. (We will meet it again further on, as well.) A temperature difference $\vartheta_A - \vartheta_B$ can be understood as the driving force for an entropy current.

> A temperature difference is the driving force for an entropy current.

It is now easy to understand why the entropy currents finally stopped flowing in Figs. 10.5 and 10.6. As soon as the temperatures became equal, the driving force for the entropy current vanished.

The state of equality of temperatures reached at the end is called *thermal equilibrium*.

You have a cup of tea in front of you. The tea is still too hot to drink so you wait for it to cool off. What actually happens in cooling? Because the temperature of the tea is higher than that of the air or the table, an entropy current flows from the tea into the environment. Does the environment become warmer because of this? To be precise: yes, it does. However, the entropy that comes from the tea distributes so widely and is so diluted that we hardly notice it.

Touch various objects in your classroom. Some feel cool: the metal of the furniture, pillars made of concrete. Others seem to be less cool to the touch, for instance, a wooden chair. Others seem almost warm: a woolen glove or a piece of styrofoam. The temperature of a metal object appears to be lower than one made of wood. This statement should get your attention. We just stated that "Entropy flows by itself from a place of higher to a place of lower temperature." According to this, entropy should continually flow from the wooden parts to the iron parts of a chair in the classroom. By doing so, the iron would become warmer and the wood cooler until....? Until the temperatures became equal.

Before we speculate any further, we will determine the temperatures of various objects in the room by measuring them. This way, we do not have to rely only upon our feelings. The result is surprising. All of the temperatures are the same. Iron, wood, and styrofoam all have the same temperature assuming that they were in the room long enough for their temperatures to have become equal.

Only in winter do the objects that are higher up in the room have a slightly higher temperature than the ones below. This is due to the heated air rising upward. The adjustment to thermal equilibrium is constantly disturbed by heating. In summer, equilibrium is generally easy to accomplish. We come to the following conclusion: Our feeling for 'warm' and 'cold' misled us. How this happens, and that we are actually not misled, will become clear in the next section.

Exercises

1. (a) Entropy goes from the hot plate into the pot when you cook something. Why? (b) The pot is put on a mat on the table. Entropy then goes from the pot into the mat. Why? (c) A cooled soda bottle is placed upon a table. The place on the table where the bottle is standing becomes cold. Why?

2. A big metal block A has a temperature of 120°C, a smaller block B, made of the same metal, has a temperature of 10°C. The blocks are brought into contact with each other so that the entropy from one of them can flow into the other one. From where to where does it flow? Is the final temperature closer to 120°C or 10°C?

3. You have a small hot metal block and a larger cooler block in front of you. (a) Can you say which one contains more entropy? (b) You bring the two blocks into contact with each other. What happens with temperature and entropy? (c) At the end, which block contains more entropy?

10.3 The heat pump

The fact that entropy flows by itself from an object of higher to an object of lower temperature does not mean that it cannot flow in the opposite direction as well, meaning from cold to hot. It can, but not by itself. To achieve this, you must force it: An entropy pump is necessary. The customary name for such a device is *heat pump*.

Nowadays every house has a heat pump. It is part of the refrigerator and serves to transport the entropy from inside to outside. Before we take a closer look at a refrigerator, we must first get to know some principal aspects of heat pumps.

Like any other pump, a heat pump has two connections for whatever is being pumped: an intake (or inlet) and an outlet. A water pump has an intake and an outlet for water and a momentum pump has an inlet and an outlet for momentum. Correspondingly, a heat pump has an intake and an outlet for entropy, Fig. 10.7. The inlet and the outlet are both made up of coiled metal pipes through which a liquid or a gas flows. In this way, entropy is transported into or out of the pump.

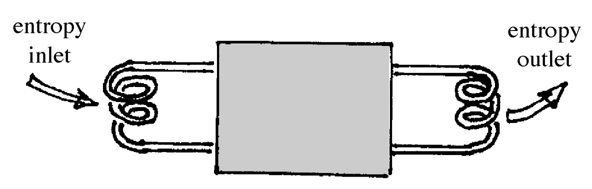


Fig. 10.7
The heat pump has an inlet and an outlet for entropy.

A heat pump transports entropy from a place of lower to a place of higher temperature.

Cooling an object means that entropy is removed from it; heating an object means that entropy is introduced into it. Fig. 10.7 shows that a heat pump can be used to cool as well as to heat. In fact, heat pumps are used for both purposes.

We look more closely at the refrigerator, Fig. 10.8. The heat pump itself is at the bottom toward the back of the refrigerator. The entropy outlet can also be seen at the back. It is the coiled pipe taking up most of the backside of the refrigerator. A metal grating is placed between the pipes in order to better help the entropy to go into the air. As long as the refrigerator is running, these coiled pipes stay warm and we can tell that entropy is flowing out of the refrigerator. The intake for entropy is inside the refrigerator. It is a coiled pipe inside the walls of the freezer compartment.

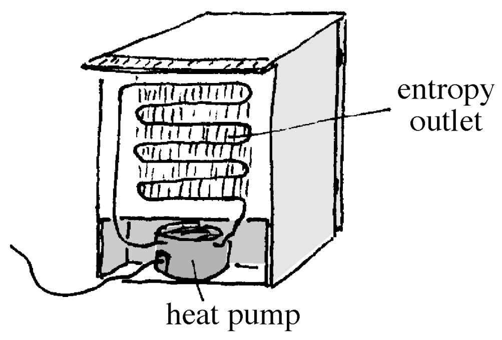


Fig. 10.8
A refrigerator seen from behind, showing the heat pump and the coils, through which the heat leaves the refrigerator.

Some houses are heated with a heat pump. Entropy is taken from the air outside or from a stream or river flowing nearby. The water in some indoor swimming pools is also heated in this way.

An air-conditioner is another device that makes use of a heat pump. An air-conditioner sets a certain temperature and a certain humidity inside a building. One of its functions is to cool the air inside. It does so by use of a heat pump. Fig. 10.9 shows a simple climate control unit that can only cool the air inside a room.

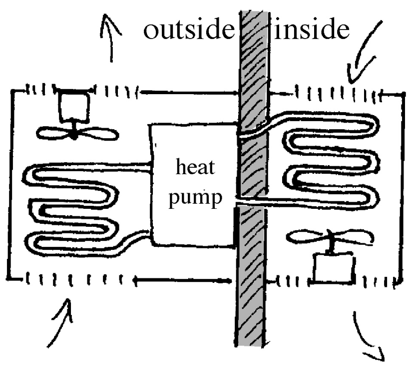


Fig. 10.9
A simple air conditioner. The fans inside and outside are there to improve heat exchange with the air.

Exercises

1. Examine the refrigerator you have at home. Look for the heat pump and the intake and outlet for the entropy. Hold your hand on the coiled pipe of the entropy outlet.

2. What happens to the entropy when the refrigerator door is left open for a long time?

10.4 Absolute temperature

How much entropy can be pumped out of an object? How much entropy does it contain?

We must be clear about the fact that these are two different questions.

If only positive entropy exists, only as much entropy can be pumped out of the object as there is in it. In the same manner, only as much air can be pumped out of a container as it contains to begin with.

It would be different if there were such a thing as negative entropy. Then it would be possible to take out entropy even when the entropy content were at zero Carnot. For example, if a further 5 Ct were pumped out, the resulting entropy content would equal minus 5 Carnot. That this is conceivable, we know from our observations of momentum. Momentum can be taken out of a body at rest (a body with momentum of zero Huygens). Its momentum then becomes negative.

We will now replace the questions we asked at the beginning with another one: Is there such a thing as negative entropy? (It might be said that negative entropy is what is called 'coldness' or 'quantity of coldness' in everyday language).

Actually, the answer to this question is easy to find. All that is needed is a very good heat pump. One takes an object, a brick for example, and pumps the entropy out of it for as long as possible. Let us try it with the refrigerator. The brick's temperature sinks to perhaps –5°C. It can go no further because the refrigerator's heat pump cannot do more. More entropy can be extracted from the brick if it is put into the freezer compartment: The temperature decreases down to –18°C. Better and more expensive heat pumps do exist and even lower temperatures can be achieved with them. These kinds of heat pumps are called refrigerating machines.

Some types of refrigerating machines are able to bring the temperature of our brick down to –200°C. Air is in liquid form at these temperatures. Such machines are used to liquefy air. There are refrigerating machines that can take even more entropy out of our brick. Further decrease of the temperature is proof of this. It is possible to keep decreasing to –250°C, and then to –260°C, etc.

With even greater effort, the temperature can fall all the way down to –273.15°C. At that point, however, it stops. No matter how great the effort made, this is the lowest the temperature can go.

The explanation for this is simple:

1) At this temperature, our brick does not contain any entropy anymore.

2) Entropy cannot have negative values.

The lowest temperature that an object can have is –273.15°C. The object contains no entropy at this temperature.
At ϑ = –273.15°C, S = 0 Ct.

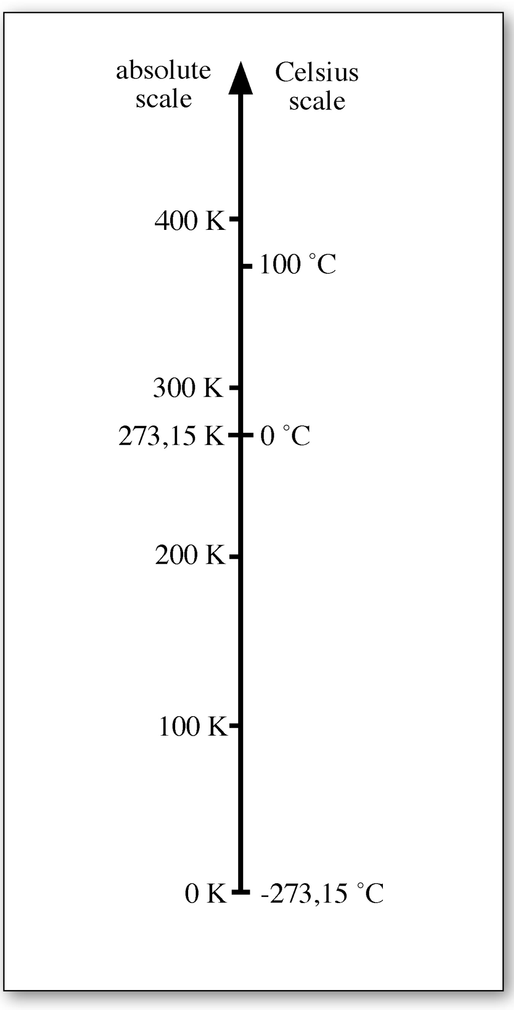


Fig. 10.10
The Celsius scale and the absolute temperature scale.

After discovering that a lowest possible temperature exists, it was considered sensible to introduce a new temperature scale. This new *absolute temperature scale* is shifted with respect to the Celsius scale so that its zero point is at –273.15°C. The symbol for absolute temperature is T and its unit is Kelvin, abbreviated to K. Fig. 10.10 shows the relation between the two scales. Notice that a temperature difference of 1°C equals a temperature difference of 1 K.

The boiling temperature of water on the Celsius scale is

ϑ = 100°C.

And on the absolute scale, it is

T = 373.15 K.

On the absolute scale, zero is at –273.15°C. The unit of absolute temperature is the Kelvin.

Fig. 10.11 shows the relation between the temperature of and the amount of entropy contained in a piece of 100 g of copper.

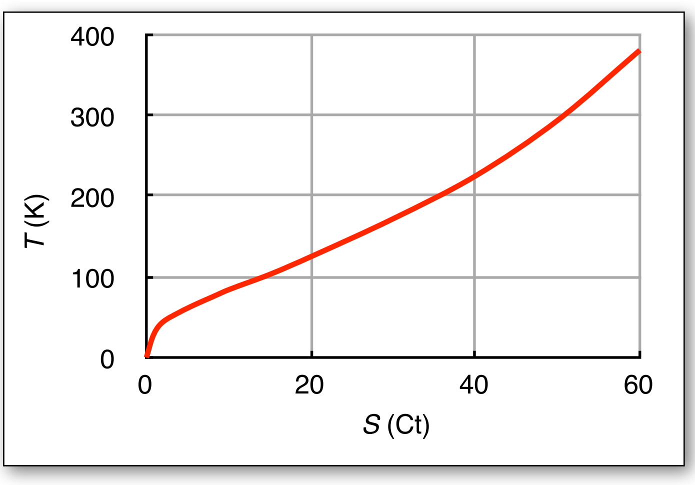


Fig. 10.11
Temperature as a function of the entropy content for 100 g of copper.

Exercises

1. Convert the following Celsius temperatures into absolute temperatures:

0 °C	(melting point of water)
25 °C	(standard temperature)
100 °C	(boiling point of water)
– 183 °C	(boiling point of oxygen)
– 195,8 °C	(boiling point of nitrogen)
– 268,9 °C	(boiling point of helium)
– 273,15 °C	(absolute zero point)

2. Convert the following absolute temperatures into Celsius temperatures:

13,95 K	(melting point of hydrogen)
20,35 K	(boiling point of hydrogen)
54,35 K	(melting point of oxygen)
63,15 K	(melting point of nitrogen)

3. How much entropy does 1 kg of copper at a temperature of 20°C contain? Use Fig. 10.11 to answer the question.

10.5 Entropy production

A heat pump can be used to heat a room: Entropy is brought in from outside the house. In fact, most room heaters don't do this. They burn fuels, for example heating oil, coal, wood or natural gas. Burning is a chemical reaction by which the combustible fuel and oxygen change into other substances, mostly carbon dioxide and water in gaseous form. Where does the entropy that is given off by the flames come from? It was neither in the fuel nor in the oxygen at the beginning because both of them were cold. Apparently, it comes into being during burning. *Entropy is created in the flames*, Fig. 10.12.

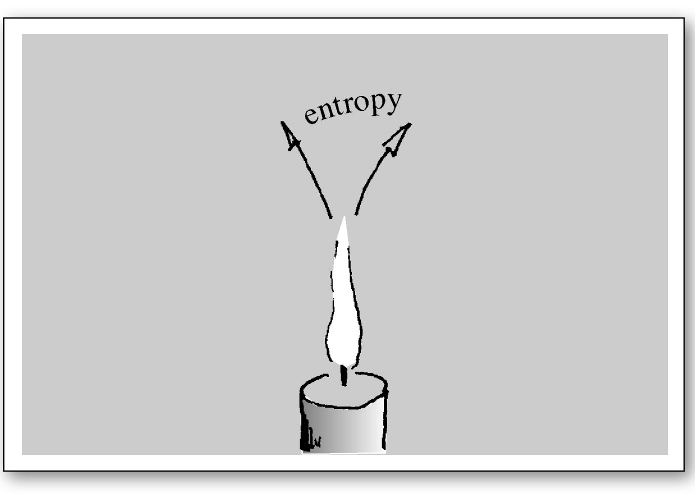


Fig. 10.12
Entropy is produced in the flame

Another type of heating is electrical. A strong electric current is sent through a thin wire and the wire is heated up. *Entropy is created in the wire*, Fig. 10.13. Many electrical appliances operate on this principle. Some examples of these would be hot plates, irons, immersion heaters, night storage heaters, hair-dryers, and light bulbs.

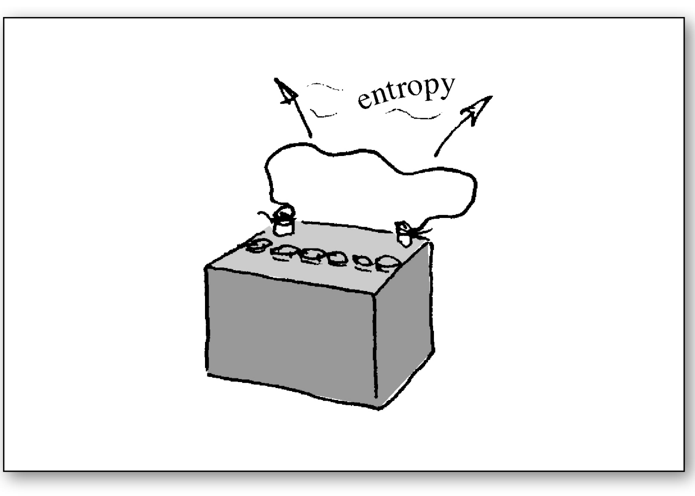


Fig. 10.13
An electric current flows through the wire, producing entropy.

You know another way of creating entropy, namely, mechanical friction. If you slide down a rope hanging from the ceiling, you feel the creation of entropy in a very unpleasant way. It is noticeable when one tries to drill with a dull drill, or uses a dull saw. *Entropy is produced at the contact surface of two objects rubbing against each other.*

In all of these processes, entropy is newly created and not brought from somewhere else.

Entropy can be created
– by chemical reactions (for example, burning);
– in a wire with an electric current;
– by mechanical friction.

Actually, all of these processes can be looked at as kinds of friction. Whenever something flows through a conductor which poses a resistance to the current, there is 'friction'. In mechanical friction, momentum flows from one body to another one through a connection which is a bad conductor. Electric heating appliances have electricity flowing through a wire that resists the electric current. A kind of friction which is called reaction resistance needs to be overcome in chemical reactions.

We have discussed the question of where we get the entropy to heat a room or an object. Now we deal with the opposite problem of cooling an object. We already know one method. We pump entropy out of the object with a heat pump.

A second method works when the object is warmer than its environment (when its temperature is higher). What does one do when tea is too hot? One simply waits. The entropy flows off by itself.

In both cases, meaning with and without a heat pump, the entropy that disappears from the object being cooled appears again at another place. Would it be possible to somehow make the entropy disappear altogether? Can it be made to disappear so that it does not reappear somewhere else? Can it be *destroyed*? After all, we saw before that it can be *created* out of nothing.

Many inventors and many scientists have tried unsuccessfully to do this. Today, we firmly believe that entropy cannot be destroyed.

Entropy can be created but not destroyed.

At this point we will remind ourselves of two other quantities: energy and momentum. We have taken for granted the fact that these quantities can neither be created nor destroyed. When the amount of energy increases at a location, it decreases at another place, and if it decreases somewhere, it increases somewhere else. This is true of momentum as well.

Energy can be neither created nor destroyed.
Momentum can be neither created nor destroyed.

The fact that entropy can be produced poses interesting questions and has curious consequences.

Here is a first problem. Entropy can be created and is newly produced in countless processes that take place on Earth. An especially productive source of entropy is burning. Remember, burning not only takes place in ovens, heating boilers and car engines. It happens on a much larger scale in nature. In every form of life, from microbes to mammals, constant oxidation (burning) processes are going on, and thereby entropy is created.

Wouldn't the Earth's amount of entropy constantly increase because of this and wouldn't the Earth gradually become warmer? Actually, except for very small fluctuations, the temperature of the Earth has remained constant for millions of years. For an explanation it is not enough to consider only the Earth. First, it perpetually receives entropy with the light from the Sun. (In this case, entropy flows from a place of higher to a place of lower temperature: The sun has a surface temperature of about 6000 K, and the Earth's surface temperature is around 300 K.) Second, the Earth constantly gives entropy to outer space. (Again, entropy flows from a higher to a lower temperature. Space has a temperature of about 3 K.) The entropy given off by the Earth is carried by light, but in this case it is invisible infrared light. This infrared light carries off exactly the amount of entropy needed to keep the temperature of the Earth constant. The question remains of what happens to space if its entropy constantly increases. This question remains unanswered. It is actually a small problem compared to the unanswered questions concerning structure and development of the cosmos.

There is another odd consequence of the fact that entropy can be created but not destroyed. If you are shown a silent film, but no one tells you if it is running backwards or forwards, can you tell what direction it is running in? If you observe it in the right direction, the 'film' in Fig. 10.14 shows a burning candle. Observed in the wrong direction, it shows something that is impossible in the real world: a candle that becomes larger by itself. The film shows an *irreversible process*. Why is this process irreversible? Because entropy can only be created. A reversal would mean that entropy is destroyed and that is impossible.

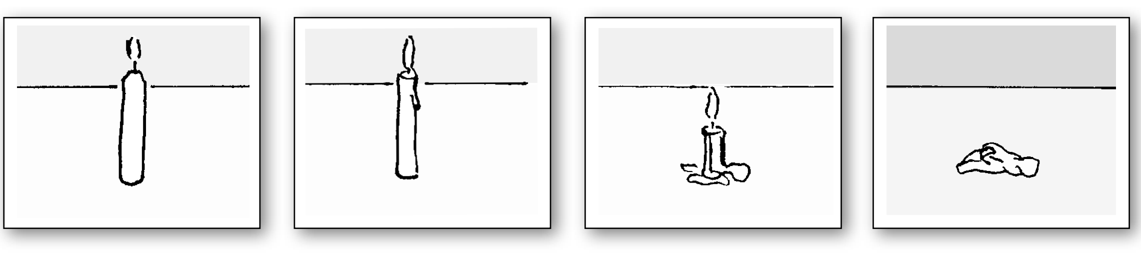


Fig. 10.14
The burning down of a candle is an irreversible process.

Fig. 10.15
Are the pictures here in the right order?

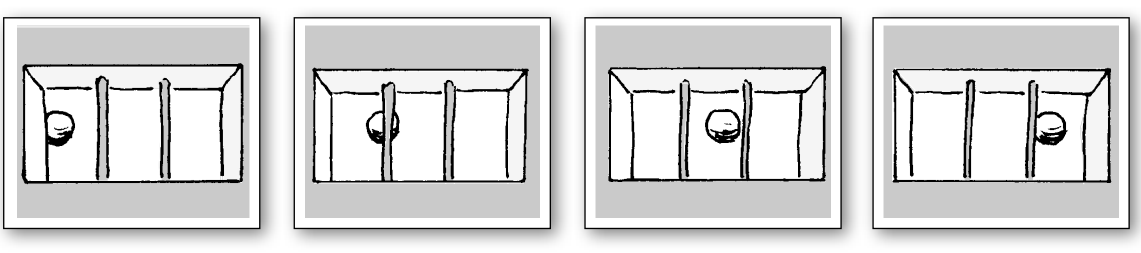


Fig. 10.16
The process of the ball flying by is reversible.

Another irreversible process is shown in the series of pictures in Fig. 10.15. A person slides down a rope. This process is also irreversible because entropy is created.

There are processes that run forwards as well as backwards. These are processes where entropy is not created. Fig. 10.16 shows a ball flying by a window. Does the ball move from left to right as shown in the film? Or is the film running backwards and the ball was actually moving from right to left?

Processes that produce entropy are irreversible.

Exercises

1. A lamp is connected to a battery. The lamp burns and the battery slowly empties. Describe the opposite process. (Assume that it is not forbidden to destroy entropy.)

2. Describe in detail which processes would take place if we reversed the process of a car driving along a street (allow entropy to be destroyed).

3. A person riding a bicycle brakes. What would happen, in detail, if the process ran backwards? (Assume that entropy can be destroyed.)

10.6 Entropy currents

The left end of the metal rod in Fig. 10.17 is heated and the right end is cooled. In other words, entropy flows into the rod from left to right, from a higher to a lower temperature. We say that an *entropy current* is flowing. The number of Carnot flowing per second through the rod gives us the *entropy current strength*:

$$\text{entropy current strength} = \frac{\text{entropy}}{\text{time interval}}$$

We use the symbol I_S for the entropy current. Using it we can write:

$$I_S = \frac{S}{t}$$

The unit for entropy currents is Carnot per second, abbreviated to Ct/s.

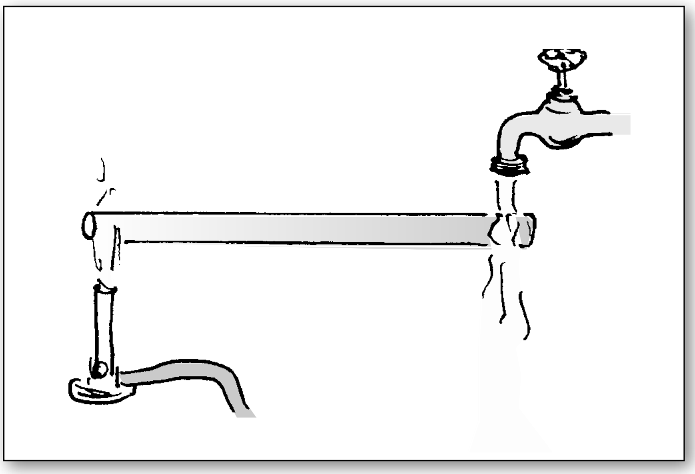


Fig. 10.17
An entropy current flows from the hot end to the cold end of the rod.

What does the strength of the entropy current between two points A and B depend upon? Let us take a look at Fig. 10.18. In the arrangement above, the temperature difference between A and B is greater than in the setup below. Otherwise everything is the same. The driving force for the entropy current is greater above than below, so the current is stronger there as well.

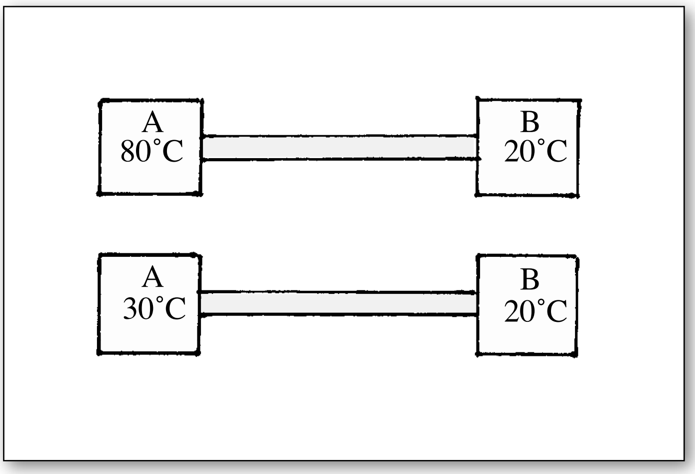


Fig. 10.18
In the upper arrangement, the temperature difference between bodies A and B is larger.

The greater the temperature difference between two points (the greater the driving force), the stronger the entropy current flowing from one to the other.

10.7 Thermal resistance

Entropy currents can be different when the temperature difference is the same. It does not depend only upon the difference of temperature, but also upon the type of connection, i.e., upon the *thermal resistance* of the connection, Fig. 10.19. What does the thermal resistance of a connection depend upon though?

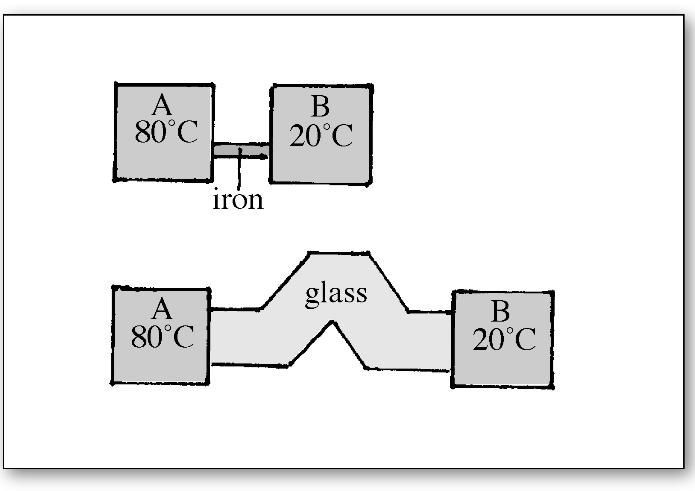


Fig. 10.19
Connections with different thermal resistances

Fig. 10.20 shows two entropy conductors, a and b. There is the same temperature difference of 60 K between the ends of each of them. The cross section of conductor b is twice the size of the cross section of conductor a. In each half of conductor b (i.e., in the upper half and the lower half) as much entropy is flowing as in conductor a. This means that in both halves of b together, twice the amount of entropy is flowing as in conductor a.

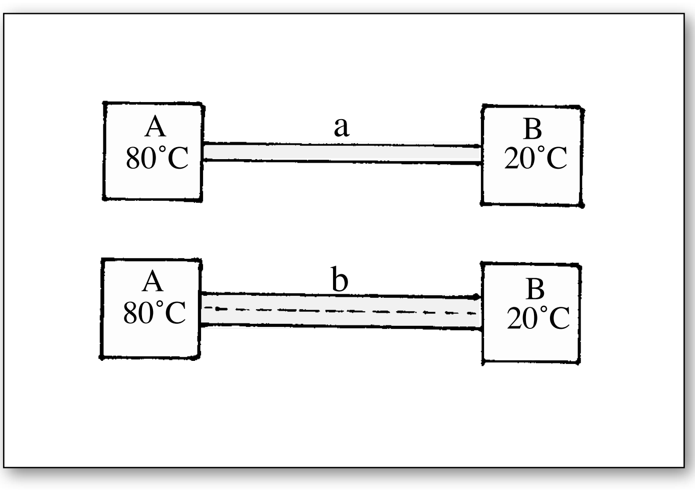


Fig. 10.20
A stronger entropy current flows through the thicker conductor.

Again, two conductors a and b are represented in Fig. 10.21. In this case, conductor b is twice as long as conductor a. We compare one half of b, say, the left side, to conductor a. They are both built identically, but there is a greater temperature difference in a than in the half we have chosen of b. Therefore, a weaker entropy current is flowing through b than through a. The other half of b has a weaker entropy current flowing through it as well.

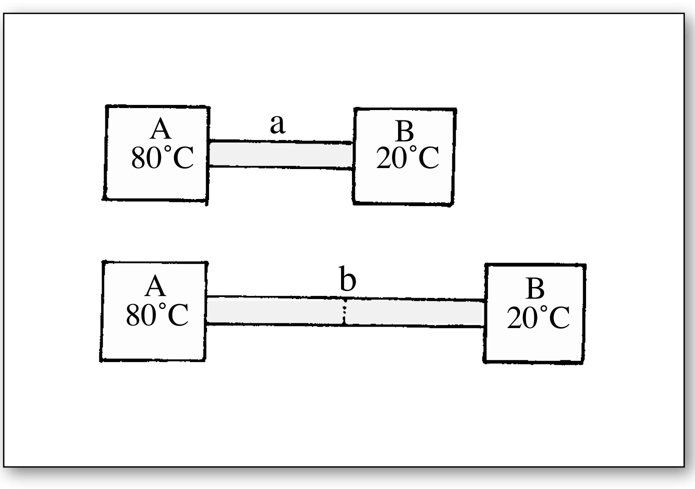


Fig. 10.21
A stronger entropy current flows through the shorter conductor.

Fig. 10.22 shows two conductors that are of the same length and have the same cross sectional areas. The temperature difference between their ends is also the same. In spite of this, a smaller entropy current flows through b than through a. This is because b is made of wood and a is made of copper.

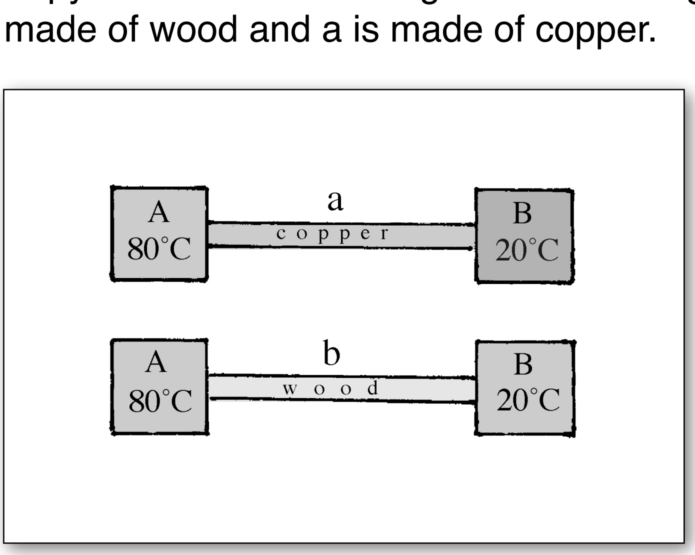


Fig. 10.22
A stronger entropy current flows through the conductor made of copper than through the one made of wood.

Every conductor has resistance to the entropy current flowing through it. The smaller the cross section of the conductor, and the longer it is, the greater the thermal resistance. The resistance also depends upon the material the conductor is made of.

Fig. 10.23 is a summary of what entropy currents and thermal resistance depend upon.

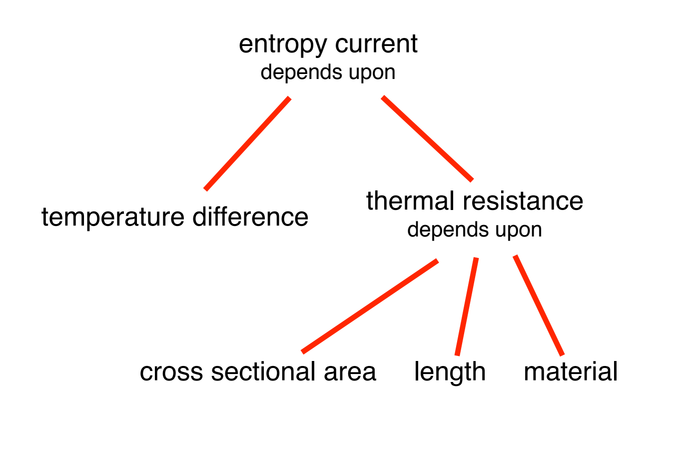


Fig. 10.23
Relation between current, temperature difference, and characteristics of the conductor.

We will now investigate some materials to see whether they have a low or a high thermal resistance, i.e., if they are good or bad thermal conductors. We hold a small rod made up of a certain material in our fingers. We hold the other end in a flame, Fig. 10.24. Depending upon the thermal resistance of the material the rod is made up of, we will feel it getting hot more quickly or more slowly.

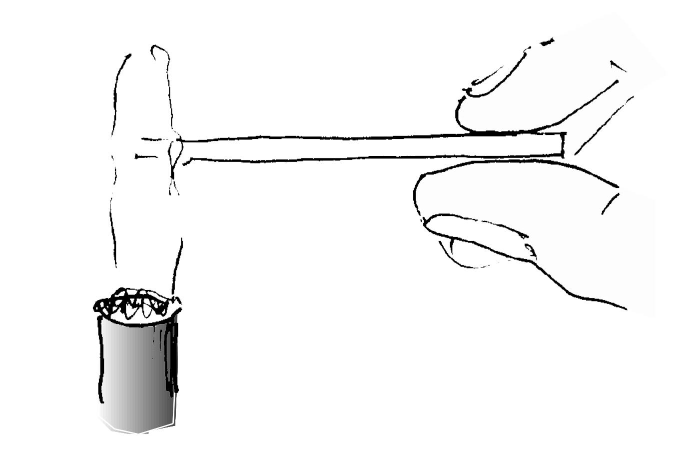


Fig. 10.24
The right end of the little rod will become hot more or less quickly depending upon its thermal resistance.

We notice that wood, glass and plastic have a rather high thermal resistance. Metals, on the other hand, have a low thermal resistance. They are good thermal conductors. Air and other gases have a very high thermal resistance. This is why materials with a high air content, such as bricks with cavities, gas-aerated concrete, frothed plastic and fibrous insulation material, are used for insulating buildings. A wool pullover keeps us warm because wool contains so many cavities filled with air.

We can now find out why a metal object feels colder than a wooden one.

First we want to state that this observation only holds for low temperatures. We put a piece of wood and a piece of metal into boiling water, so that they both reach a temperature of 100°C. We take both of them out of the water and touch them with our fingers. The metal one feels hotter than the wooden one. How can this be explained?

Touch a piece of wood and then a piece of metal, both at a temperature of 10°C, with your 25°C fingers. At first, entropy flows from your fingers to the object, Fig. 10.25. The wood becomes warm quickly at the place you touch it. It takes the temperature of your finger because it cannot conduct the entropy away. On the other hand, in the metal the entropy flows away into it from the point being touched, and the place being touched warms up only very little.

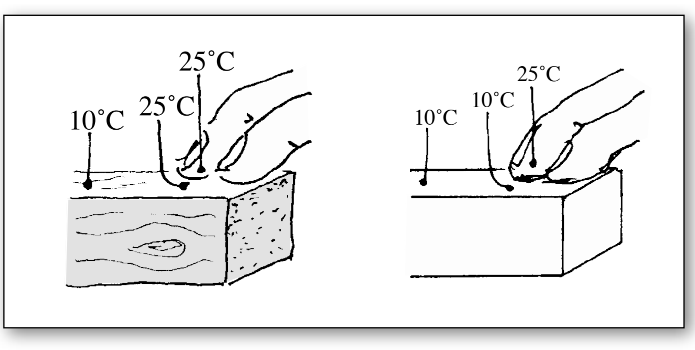


Fig. 10.25
The two objects have the same temperature before they are touched, but not afterwards.

If you touch an object that is a bad conductor, you do not feel the temperature it had before you touched it. You feel the temperature it takes from your fingers touching it.

Exercises

1. How should a house be built so that the heat loss (entropy loss) is kept as small as possible?

2. In a radiator of a central heating unit, the entropy should flow as easily as possible from the water inside the radiator to the outside. How can this be achieved? Name some other objects where good heat conduction is of interest.

10.8 Transport of entropy by convection

A temperature difference is the driving force for an entropy current. If entropy should flow from A to B, it is enough to make sure that A has a higher temperature than B. This kind of entropy transport is called *conduction of heat*. It is, so to speak, the normal way for entropy to get from A to B.

If we observe our environment carefully, we can see that most entropy transports, and especially transports of entropy over large distances, are not accomplished this way. There is another way of transporting entropy called *convective entropy transfer* or simply *convection*.

A liquid or a gas is heated and transported from A to B by means of a pump. The entropy is carried along with the substance being transported. In this case, no temperature difference is necessary as the driving force, but a driving force for the liquid or gas currents is needed.

An example of convective transport of entropy is a central heating unit, Fig. 10.26. Water is heated, possibly by burning oil, in the boiler which is usually in the cellar of the house. The heated water is pumped through pipes to the radiators in the various rooms. It gives off a part of its entropy in the radiators and then flows back through pipes and to the boiler again.

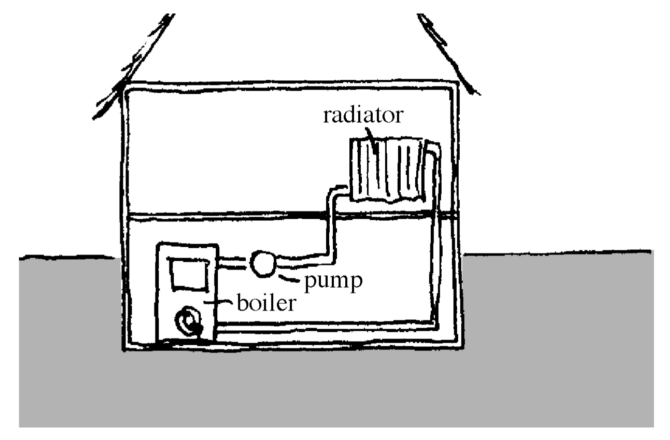


Fig. 10.26
Central heating unit. The entropy reaches the radiators convectively from the boiler.

It is much easier to realize convective entropy transports than the usual kind driven by temperature differences. The reason for this is that there are no really good thermal conductors. Even copper, which is generally considered to be a relatively good conductor, is actually a very bad thermal conductor. For example, it would be impossible to transport the entropy with copper rods from the boiler in the cellar into the various rooms of the house. By contrast, it is absolutely no problem to transport air or water together with their entropy over large distances.

> Convective entropy transport: Entropy is taken along by a flowing liquid or a flowing gas. No temperature difference is needed to transport entropy convectively.

There are many examples of convective entropy currents in nature and in technology.

Entropy should distribute over a room being heated by a radiator or space heater. How can this happen? Air is actually a very bad thermal conductor. Here, entropy is transported convectively with the air. In this case, the air moves without needing a pump. Heated air rises from a heater because warm air has a lower density than cold air, Fig. 10.27.

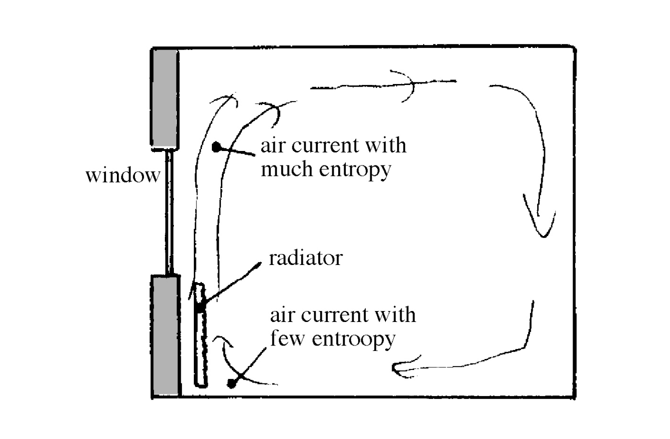


Fig. 10.27
Entropy is distributed in the room convectively.

Every automobile engine needs to be cooled, meaning that entropy must be taken away from it, Fig. 10.28. Most car engines are water cooled. The entropy is transported with water from the motor to the radiator, similar to the process in central heating units. The water is circulated by a cooling water pump. The entropy is given off in the radiator to the air flowing by.

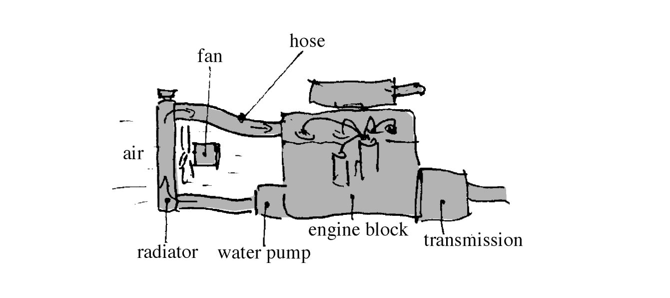


Fig. 10.28
Cooling of a car engine. The entropy goes convectively from the motor to the "radiator".

All of the large transports of entropy in nature, those that determine our weather, are convective transports. Entropy is moved over very large distances in the atmosphere by winds, i.e., by moving air.

The gulf stream is another good example of convective entropy transfer. It brings entropy from the Caribbean to Europe, Fig. 10.29. The result of this is a milder climate in Europe than would be expected by considering its geographic latitude.

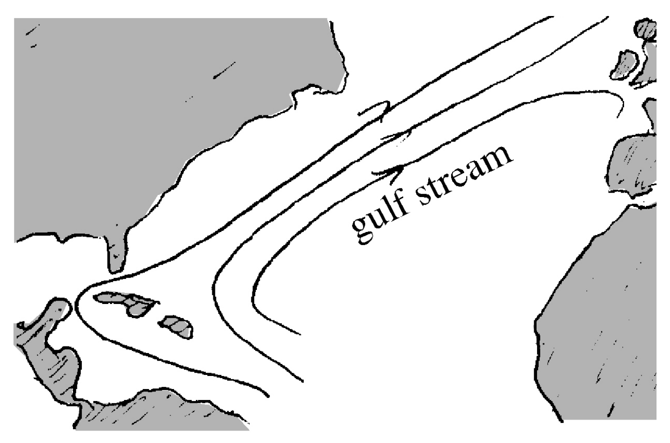


Fig. 10.29
The gulf stream. Entropy is brought from the Caribbean to Europe by a flow of water.

We will once more contrast transport of entropy by conduction to convective transport by following the path of the entropy in a house with central heating. The entropy produced in the flames reaches the outer wall of the boiler by convection. It moves through this wall in the normal way, driven by a temperature difference. It then flows with the water convectively to the radiators. It must, once again, flow through the radiator wall in the normal way. It flows further, convectively, from the surface of the radiator with the air to distribute over the room. We see that on its long way from the flames of the boiler into the room being heated, only tiny distances of a few millimeters are bridged by normal thermal conduction.

> All transport of entropy over large distances is done convectively.

Exercises

1. Describe the paths along which a house loses heat. Which losses are due to thermal conduction and which are due to convection?
2. Describe the path of entropy from inside a car engine until it reaches the air in the environment. Where on its path does the entropy flow because of a temperature difference, where does it flow convectively?
3. How does the heating in a car work? Describe the path of the entropy.

11

Entropy and energy

11.1 Entropy as energy carrier

We will consider the laws of balance for an electric heating unit. An electric heater is nothing more than a wire through which electricity flows, making it warm. As you know, this type of heating has many applications: hot plates, irons, light bulbs...

We know that a heater produces entropy. While it is running, the heater gives up entropy. We also know that a heater "uses" energy, meaning that the energy flows into the heater through the electric cable. The carrier for the energy flowing in is electricity.

The energy flowing continuously into the device with electricity must come out again. We ask the question we have asked so often before: What is the carrier of this energy?

The answer is self-evident. Along with the energy, entropy flows out of the heater. This entropy is the carrier we are seeking. We can make a general statement about this: Wherever and whenever an entropy current flows, an energy current is flowing.

Entropy is an energy carrier.

Electric heaters belong to the category of devices that we earlier called energy transfer devices or energy exchangers. The energy goes into the appliance with the carrier electricity. Entropy is produced in the appliance, and the energy leaves it with this entropy. The energy is transferred from electricity to entropy. Fig. 11.1 shows a schematic of our heater.

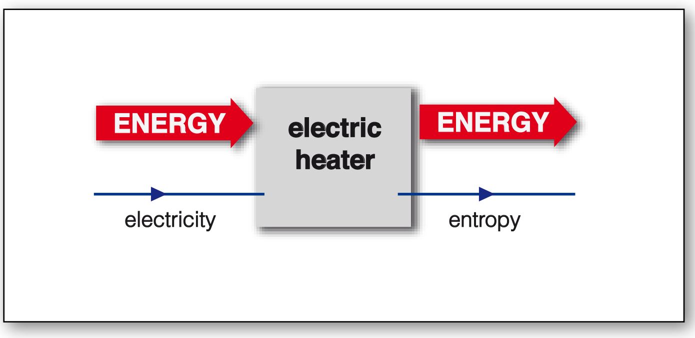


Fig. 11.1
Energy flow diagram of an electric heater

In one aspect, the flow diagram is incomplete. The carrier of the energy flowing in (the electricity) must also come out of the heater, because electricity can be neither produced nor destroyed. For this reason, in Fig. 11.2, the electricity has an outlet as well as an inlet. Notice that energy and electricity both have an inlet and an outlet, while entropy has only an outlet. Another way of saying this would be: In an electric heater, energy is transferred to the newly produced entropy.

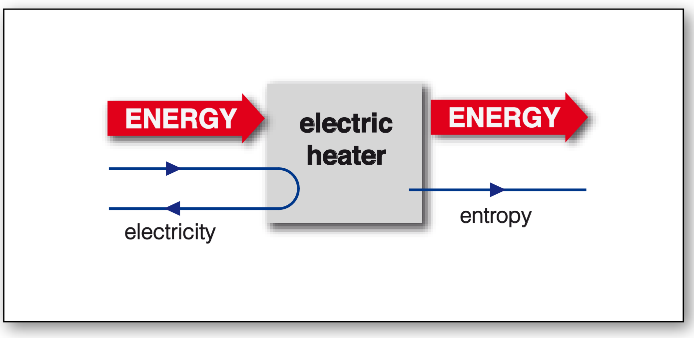


Fig. 11.2
A more complete energy flow diagram of an electric heater

The results of these considerations can be carried over to other processes where entropy is produced. Fig. 11.3 shows the flow diagram of an oil heater. Energy flows into the heater with the carrier "heating oil plus oxygen". When the energy is released, the oil and oxygen transform into exhaust gases (water vapor and carbon dioxide). Entropy is produced during combustion, and the energy leaves the heater with this entropy.

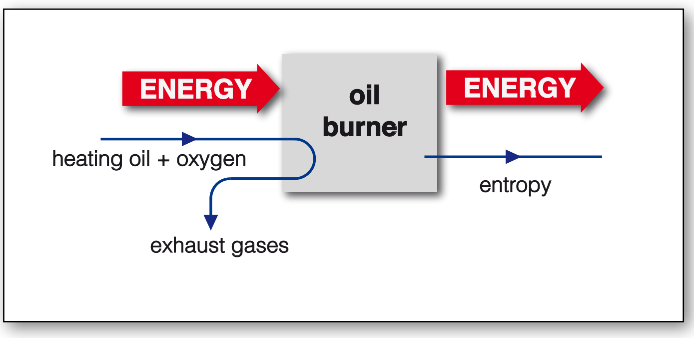


Fig. 11.3
Energy flow diagram of an oil burner

Exercises

1. Sketch the energy flow diagram for the process of friction in Fig. 11.4. Hint: The "energy exchanger" is the bottom of the crate rubbing against the floor.

2. A tower of building blocks collapses. During which part of this process is entropy produced? Where does the energy needed for this come from?

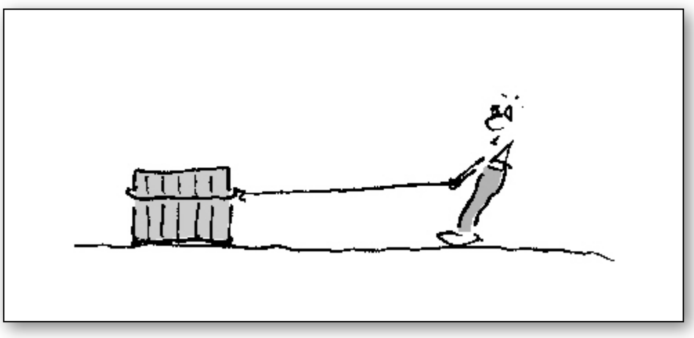


Fig. 11.4
For Exercise **1**. Entropy is produced on the underside of the crate.

11.2 The relationship between energy currents and entropy currents

Every entropy current is accompanied by an energy current. How do the strengths of these currents relate to each other? A partial answer to this question is easy: A strong entropy current is related to a strong energy current. This can be expressed more precisely. Two entropy currents of the same strength carry twice the amount of energy that only one of them does. In mathematical terms:

$$P \sim I_S \,. \qquad (1)$$

Of course, this is not the complete relation between P and I_S. In order to find the still missing part we will again consider laws of balance. However, this time we will not do this for an electric heater, but for an electric heat pump. It is better suited to what we are doing for the moment.

Fig. 11.5 shows a flow diagram of this energy exchanger. This time, for every current flowing out of it there is one of the same strength flowing in. This holds for the entropy current as well. Energy flows with the carrier electricity into the appliance. The electricity leaves the appliance again after it has released its energy. The energy arriving with the electricity is transferred to the entropy flowing into the heat pump. This energy leaves the heat pump with the entropy flowing out.

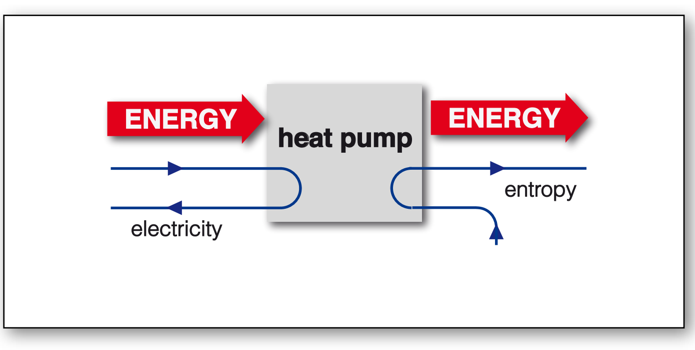


Fig. 11.5
Energy flow diagram of a heat pump

We will take a closer look at the right side of the flow diagram. The energy arrow on the right represents only the energy taken over from the electricity. The right side of the diagram could be depicted in more detail, as in Fig. 11.6. The entropy flowing *into* the heat pump also carries energy. However, the entropy flowing out carries more energy than the entropy flowing in because it carries the energy taken over from the electricity as well. Fig. 11.5 shows only the "net energy current".

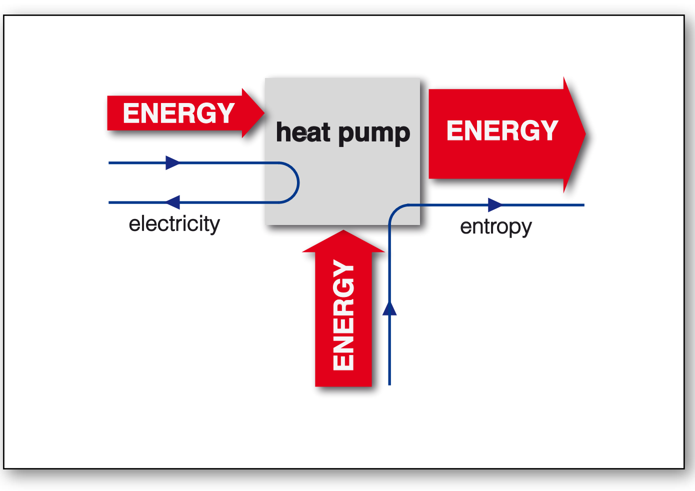


Fig. 11.6
A heat pump. The energy currents flowing with the entropy are individually represented.

We see in Fig. 11.6 that two entropy currents of the same strength can carry different amounts of energy. The one flowing in carries little energy, and the one flowing out carries a lot of energy. This means that the energy current depends upon more than just the strength of the entropy current.

What is the difference between the entropy inlet and the entropy outlet? It is the temperature. Therefore, the energy current depends upon the temperature of the conductor through which the entropy is flowing. We could also say that the factor of proportionality which makes relation (1) into an equation, depends upon the temperature.

We have expressed the facts in an unnecessarily complicated way. Actually, the factor of proportionality is just the absolute temperature itself:

$$P = T \cdot I_S \qquad (2)$$

A coincidence? Absolutely not. The temperature scale that everyone uses and that we have already used a lot is defined by equation (2).

An entropy current I_S carries an energy current $T \cdot I_S$.

Equation (2) shows that the temperature can be interpreted in the following way:

The temperature specifies how much an entropy current is charged with energy.

We can now establish an exact, quantitative energy balance for the heat pump. We call the high temperature at which the entropy is flowing out of the machine T_A, and the low temperature at which the entropy is flowing in, T_B. An energy current of

$P_B = T_B \cdot I_S$

flows in along with the entropy at low temperature. At the exit (high temperature) the energy current is

$P_A = T_A \cdot I_S$.

The resulting net energy current is

$P = P_A - P_B = T_A \cdot I_S - T_B \cdot I_S$.

Or otherwise expressed

$$P = (T_A - T_B) \cdot I_S \,. \qquad (3)$$

This net current must be equal to the energy current flowing over the electric cord and into the heat pump. Equation (3) gives the energy used by the heat pump. We interpret equation (3) as follows:

The more entropy the heat pump must transfer, the more energy it uses.

The greater the temperature difference to be overcome, the more energy is used.

Example: A heat pump heating a house transfers 30 Ct per second from the outside to the inside of the house. The temperature outside is 10°C. The temperature inside the house is 22°C. What is the energy consumption (the "power") of the pump?

In this case, we do not need to convert the Celsius values to absolute ones, because the differences are the same on both scales. We have $T_A - T_B$ = 12 K. From this we obtain

$P = (T_A - T_B) \cdot I_S$ = 12 K · 30 Ct/s = 360 W.

We will now assume that the same house is heated with a standard electric heater. This means that entropy is not pumped in from outside, but is produced in the house. Again, the temperature in the house should be 22°C, and of course we need 30 Ct/s again in the house because exactly this amount is lost through its walls. The energy current coming out of the electric heater is calculated with equation (2), where T = (273 + 22) K = 295 K and I_S = 30 Ct/s:

$P = T \cdot I_S$ = 295 K · 30 Ct/s = 8850 W.

According to our calculations, the energy consumption by a normal electric heater is much greater than that of a heat pump. In reality, the difference is not as great as it appears though, because in every heat pump there is some entropy production due to friction and electrical resistance.

Exercises

1. A house heated by an oil heater to a temperature of 20°C, loses heat at 35 Ct/s. What is the energy consumption of the heater?

2. A car's radiator has a temperature of 90°C. It emits 60 Carnot per second into the air. What is the energy current flowing out of the radiator into the air?

3. The temperature on the sole of a 1000-W iron is 300°C. How much entropy per second is coming out of the iron?

4. A swimming pool is heated by a heat pump. The heat pump takes entropy out of a stream flowing nearby. The temperature of the water in the stream is 15°C, and the water in the swimming pool is 25°C. The water in the swimming pool continuously loses entropy at a rate of 500 Ct per second into the environment. In order to keep this temperature, the heat pump must continuously replace the lost entropy. What is the energy consumption of the heat pump?

5. (a) A house is heated with a heat pump. The temperature outside is 0°C and the temperature inside is 25°C. The heat pump transfers 30 Ct/s. What is the energy consumption?

(b) The same house is heated by a standard electric heater. This means that 30 Ct/s are produced inside the house, rather than pumped in from the outside. What is the energy consumption?

11.3 Entropy production by entropy currents

An entropy current flows through a rod made of a well conducting material, Fig. 11.7. The current is sustained by a temperature difference. The sides of the rod are insulated so that they do not lose entropy there. At the beginning of the experiment, the temperature will change at points along the rod. After a while these changes stop and the so-called *steady-state* is attained.

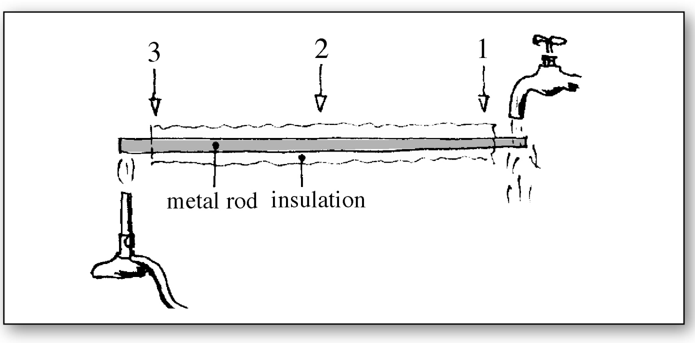


Fig. 11.7
More entropy comes out at the right end of the rod than flows in at the left.

The equation relating the entropy and energy current to each other leads us to a surprising statement here.

We observe three different locations on the rod: The cold right end, the middle, and the hot left end. The values related to these three locations are indicated by a "1", a "2", and a "3". An energy current having a value P_3 flows into the rod from the left. Because steady-state has been attained, energy does not build up anywhere in the rod. The energy current must have the same value everywhere:

$$P_3 = P_2 = P_1 \,. \qquad (4)$$

Now we know that the energy current P is related to the entropy current I_S by

$$P = T \cdot I_S \qquad (5)$$

We replace the energy currents in equation (4) with help from equation (5) and obtain

$$T_3 \cdot I_{S3} = T_2 \cdot I_{S2} = T_1 \cdot I_{S1} \,. \qquad (6)$$

We know that the temperature T_3 is greater than T_2, and T_2 is greater than T_1:

$$T_3 > T_2 > T_1 \,.$$

In order for equation (6) to be valid, we must have

$$I_{S3} < I_{S2} < I_{S1} \,.$$

Thus, the entropy current increases to the right. On the right, where the water cools the rod, more entropy is flowing out than is flowing in at the left, where the flame is. This must mean that entropy is being produced in the rod. How is this possible?

Basically, this result isn't as surprising as it may appear at first. We learned earlier that entropy is always created when some sort of frictional process occurs, when a current meets some resistance. Exactly this is going on here. In this case though, no gas or liquid is flowing, nor are momentum or electricity, but the entropy itself. When entropy flows through something having a thermal resistance, entropy is created.

In our minds, we can divide the entropy at the exit of the rod (at the right end), into two parts. We have the part that flowed in from the left, and the part that was newly created on the path from left to right. Therefore

$$I_{S1} = I_{S3} + I_{S\,\text{produced}}$$

The amount of entropy produced per second in the rod is represented by $I_{S\,\text{produced}}$.

If entropy flows through a thermal resistor, additional entropy is produced.

Example: The heating wire of a 700-W immersion heater, Fig. 11.8, has a temperature of 1000 K (727°C). The entropy current coming out of the wire has a value of

$$I_S = \frac{P}{T} = \frac{700\ \text{W}}{1000\ \text{K}} = 0{,}7\ \text{Ct/s}\,.$$

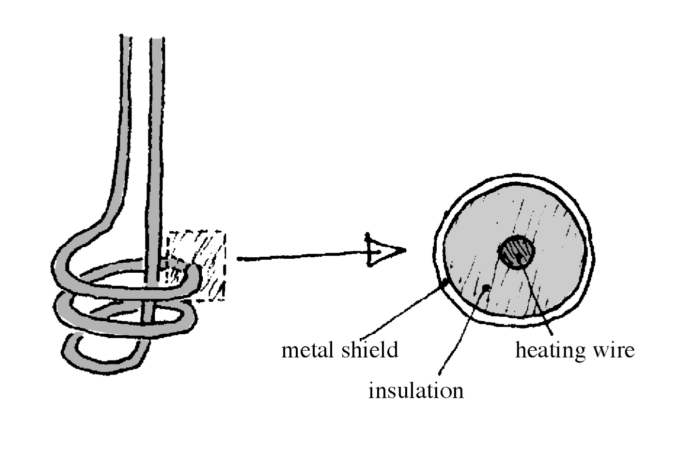


Fig. 11.8
An immersion heater. A cross section on the right (simplified and enlarged).

The surface of the immersion heater has the same temperature as the water. We assume that the water temperature is 350 K (77°C). The entropy current at the surface of the heater is therefore

$$I_S = \frac{P}{T} = \frac{700\ \text{W}}{350\ \text{K}} = 2\ \text{Ct/s}\,.$$

On the short path from the heating wire to the surface of the immersion heater

$$(2 - 0.7)\ \text{Ct/s} = 1.3\ \text{Ct/s}$$

are created. 0.7 Ct/s are created in the wire by the electric current. More is produced by the entropy on its way to the outside than is created by the electric current.

Exercises

1. A house is heated with 20 kW. The temperature inside is 20°C, the temperature outside is –5°C.

(a) What is the entropy current flowing out of the house at its inner wall?

(b) What is it at the outer wall?

(c) How much new entropy is produced per second as a result of the entropy flowing out?

2. The heating wire of a 1000-W hot plate has a temperature of 1000 K.

(a) How much entropy is produced per second in the wire?

(b) A pot of water with a temperature of 373 K is on the hot plate. How much entropy per second flows into the water?

(c) How much entropy is produced on the way from the heating wire to the water?

11.4 Heat engines

An energy flow diagram best explains what a heat engine is, Fig 11.9: It is an energy exchanger which receives energy with the carrier entropy and gives it away with the carrier angular momentum. The fact that the energy carrier at the exit of the machine is angular momentum means that the energy comes out through a rotating shaft. The machine is there to drive something.

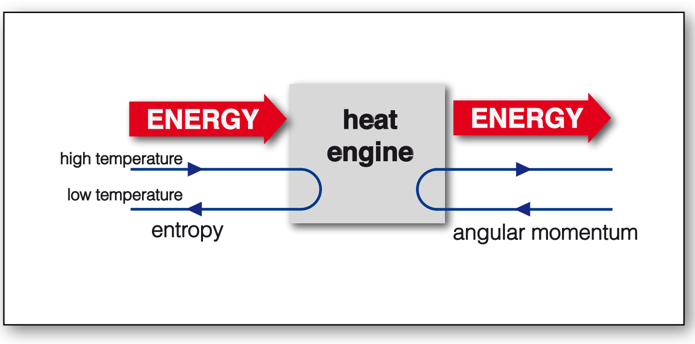


Fig. 11.9
Energy flow diagram of a heat engine

The following belong to the category called heat engines:

– steam turbines;
– reciprocating steam engines;
– all combustion engines (Otto and Diesel engines);
– jet engines;
– other, less used engines.

Later we will see how these engines work in detail. For the moment, we will consider only what all heat engines have in common. We will begin with a little detour.

Fig. 11.10 shows an energy flow diagram of a water turbine. This isn't a heat engine. Water at high pressure flows into the water turbine, and flows out again at a low pressure. The water at high pressure carries a lot of energy, the water at low pressure carries little. While the water in the turbine is "falling" from high to low pressure, it releases energy. This energy leaves the turbine through the shaft with angular momentum as the energy carrier.

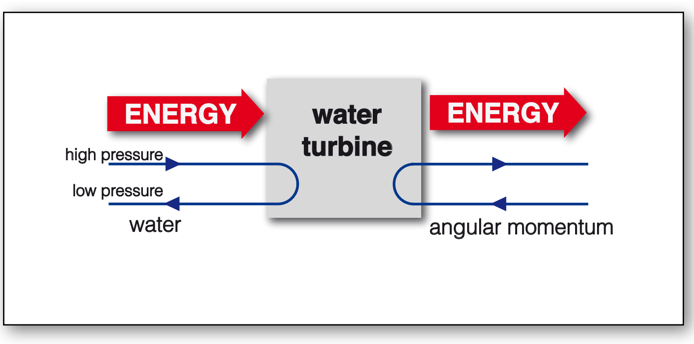


Fig. 11.10
Energy flow diagram of a water turbine

A comparison of Fig. 11.10 and Fig. 11.9 shows that the heat engine and the water turbine both have something in common. Entropy flows at a high temperature into the heat engine. This is entropy carrying a lot of energy. The same entropy flows out of the device at a low temperature, meaning it has little energy. While the entropy in the machine "falls" from a higher temperature to a lower temperature, energy is released. This energy also leaves through a rotating shaft. This means that it comes out with the carrier angular momentum.

> In a heat engine, energy is transferred from the energy carrier entropy to the energy carrier angular momentum.

We will calculate the energy emitted per second by a heat engine. At the entrance for the entropy, the machine receives an energy current of $T_A \cdot I_S$ at the high temperature T_A. At the entropy exit, it then emits an energy current of $T_B \cdot I_S$ at the low temperature T_B. The difference is the current of the energy transferred to the angular momentum. An energy current having a value of

$$P = T_A \cdot I_S - T_B \cdot I_S = (T_A - T_B) \cdot I_S$$

leaves the engine with the angular momentum.

> The stronger the entropy current flowing through the engine, and the greater the temperature difference through which the entropy current flows down in the engine, the more energy will be given up by the heat engine together with the angular momentum.

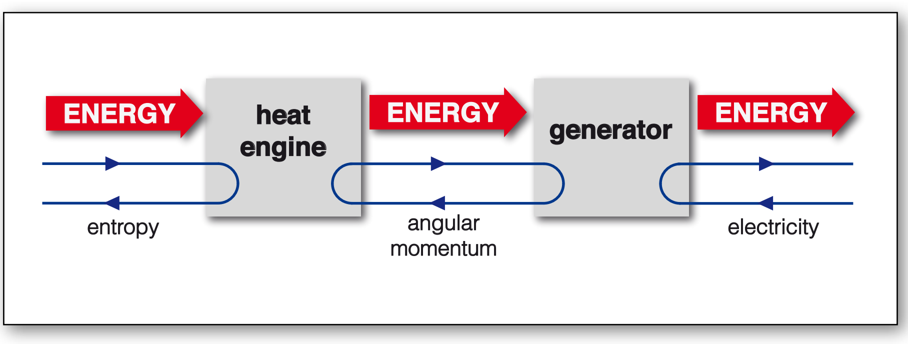


Fig. 11.11
Energy flow diagram of a thermal power plant

In most electric power plants, the generator is driven by a heat engine. The flow diagram of the devices connected to each other is shown in Fig. 11.11. The two energy exchangers can be represented by just one box, Fig. 11.12. Compare this flow diagram to the one of the electric heat pump, once again shown in Fig. 11.13. (It is the same as the one in Fig. 11.5.) The only difference in the two flow diagrams is the direction of the arrows.

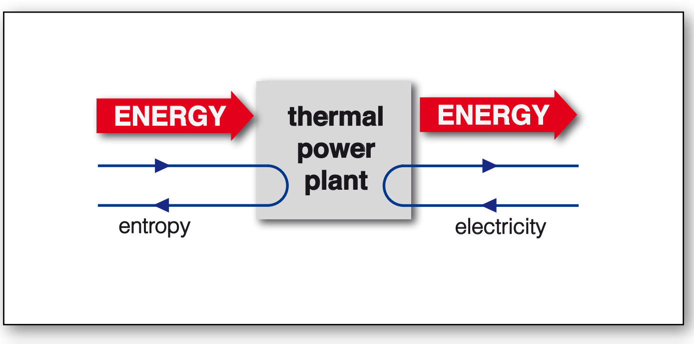


Fig. 11.12
Energy flow diagram of a thermal power plant. Turbine and generator are represented by a single symbol.

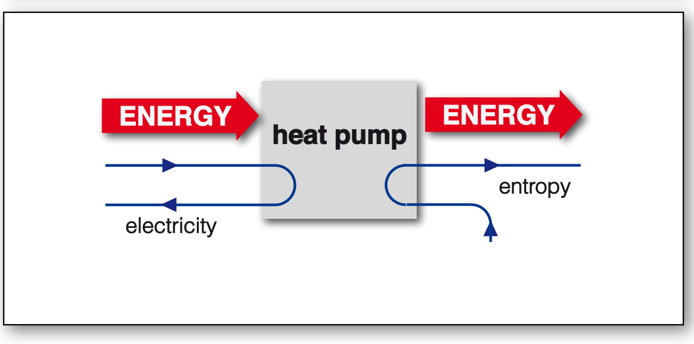


Fig. 11.13
Energy flow diagram of a heat pump

The power plant does exactly the opposite of what a heat pump does. While the electric heat pump transfers energy from the carrier electricity to the carrier entropy, in the power plant in Fig. 11.12, the energy is transferred from entropy to electricity.

A thermal power plant transfers energy from entropy to electricity. Such a power plant is a large and complicated construction. There are some devices that do exactly the same thing, namely, transfer energy from entropy to electricity, but they are small, handy, and robust at the same time. They are called *Peltier devices*.

A Peltier device can even run in reverse, as a heat pump. It is a heat pump which is uncomplicated, rather inexpensive, and compact all at the same time.

Unfortunately, Peltier devices lose a lot of energy. For this reason, they are only appropriate for applications where loss is not considered important.

11.5 Entropy sources for heat engines

There are always two problems to solve when a heat engine is to be operated:

1) A source of entropy at a high temperature is needed.

2) It must be possible to get rid of the entropy at a lower temperature. There must be a so-called "trash dump" for the entropy.

These problems can be solved in different ways.

Natural sources of entropy

This is the solution least damaging to our environment: Natural sources of entropy at high temperatures are exploited.

There are places on Earth where hot steam is contained in layers of rock at depths that are not too great. This steam is allowed to flow to the Earth's surface through drilled holes and can be used to drive power plants. Unfortunately, there are not many such sources of this *geothermal* energy.

The huge amounts of entropy at very high temperatures received by the Earth with sunlight provide another possibility. This entropy is being exploited in *solar power plants*. Although this entropy source is inexhaustible, it presents us with some problems that are not easy to solve. One of these is that sunlight is thinly distributed. This means that the entropy, and the energy with it, is strongly diluted. It must therefore be "collected" on large surfaces that the Sun shines upon. This collecting can be accomplished by erecting mirrors so that the light is concentrated on a boiler. Another problem related to solar energy is that the Sun does not always shine. It does not shine at all in the night, and in winter, when energy is most needed, it shines only weakly.

Artificial entropy sources

Most of the entropy used to drive heat engines today is acquired in a less elegant manner: It is produced by burning fuels or by nuclear fission.

Heat engines are widely used, and present not only the problem of how to acquire entropy but also the problem of "thermal trash". We will see how these problems are solved for the most important heat engines.

Thermal power plants

Most power plants operate with steam turbines. In coal-fired power stations, entropy is produced in a steam boiler by burning coal. Entropy is produced in nuclear power plants by splitting the atomic nuclei of uranium and plutonium.

When entropy leaves the power plant, its temperature is only slightly higher than the temperature of the surrounding environment. The entropy is mostly released into the water of a large river. If no river is available, or if a river's water does not suffice, the entropy is given up to the air in cooling towers.

Combustion engines

Entropy is created by burning gasoline or diesel oil inside an engine. Most of it leaves the engine with the exhaust fumes. The flow diagram in Fig.11.9 doesn't actually describe a combustion engine because the entropy is not introduced to the engine from outside.

Piston steam engines

These were the most important engines before electric and combustion engines were invented. They were used in steam locomotives, steam ships, steam rollers, and steam plows. They also drove threshing machines. Piston steam engines were used to drive the machines in many factories as well.

Entropy was produced for these engines in the boiler by burning coal. The steam driving the engine was commonly allowed to escape into the air after it had done its job. The entropy escaped to the air with the steam.

Jet engines

These are used to drive almost all large airliners. Jet engines do not exactly satisfy our definition of a heat engine. They don't give up their energy through a shaft with angular momentum, but together with momentum, Fig. 11.14. These engines „pump" momentum out of the air and into the airplane.

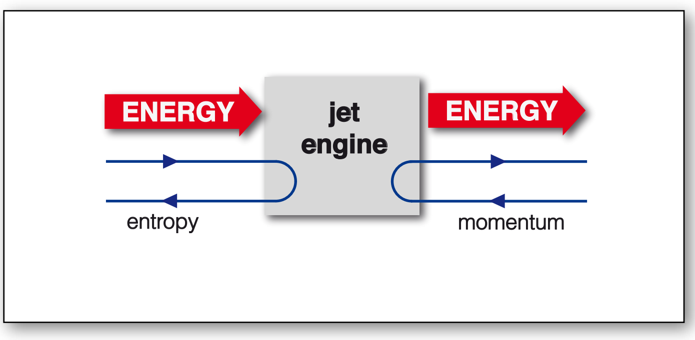


Fig. 11.14
Energy flow diagram of a jet engine

As with combustion engines, the entropy is produced in the engine by burning fuel, and it leaves the jet engine with the exhaust gases.

Exercises

1. An entropy current of 100 Ct/s flows through a heat engine. At the entrance, the temperature is 150°C, at the exit it is 50°C. How much energy per second does the engine emit with the energy carrier angular momentum?

2. A power plant gives off an energy current of 1000 MW with electricity. The temperature of the steam at the entrance to the turbine is 750 K, at the exit it is 310 K. What is the entropy current that flows away with the cooling water? What is the energy current carried by this entropy current?

3. Think of some possibilities for applications of entropy with high temperature found in nature. Also discuss possibilities you might consider unrealistic.

11.6 Energy loss

On the way from the faucet to the nozzle, Fig. 11.15, water is lost. 2 liters per second come out of the faucet but only 1.8 liters per second get to the nozzle. The difference, i.e., 0.2 liters per second, flows out of the hole in the hose. We have a *loss* of 0.2 l/s. Usually, loss is expressed as a percent of the original quantity. The loss, which we express as a percentage, is symbolized by *V*. Therefore, in our case,

$$V = \frac{0{,}2\ \text{l/s}}{2\ \text{l/s}} \cdot 100\% = 10\%\,.$$

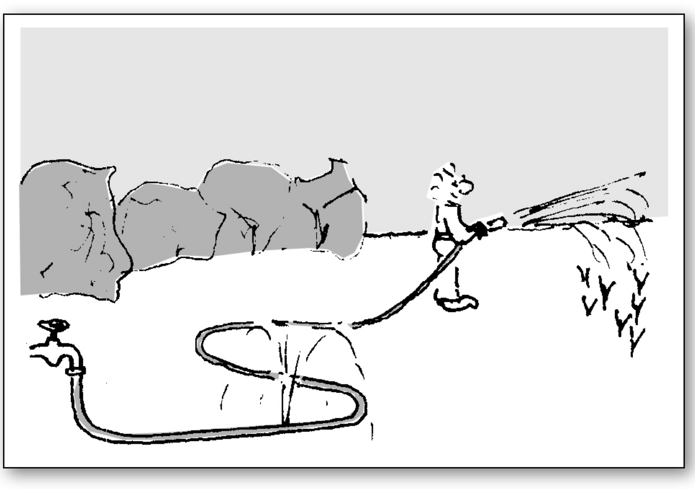


Fig. 11.15
Water gets lost through the hole in the hose.

In most devices, where energy is transferred from one carrier to another, and in most conductors for transferring energy, energy gets lost. What does this mean? Energy cannot be destroyed! It is similar to the water in Fig. 11.15. A part of the energy does not get where it should go. It seeps out, so to say.

Energy loss is almost always connected to creation of entropy. We consider a water turbine. So far, we have sketched the flow diagram of a water turbine like the one shown in Fig. 11.16 (also see Fig. 11.10). This is actually a perfect, idealized turbine as we would never see it in the real world because in every real turbine, entropy is produced unintentionally. This happens at various locations: by friction created by the water rubbing against the walls of the pipe, by water rubbing against itself ("fluid friction") and by friction in the bearings of the turbine shaft. The entropy produced also leaves the turbine by various paths: partly in the water flowing away and partly into the surrounding air.

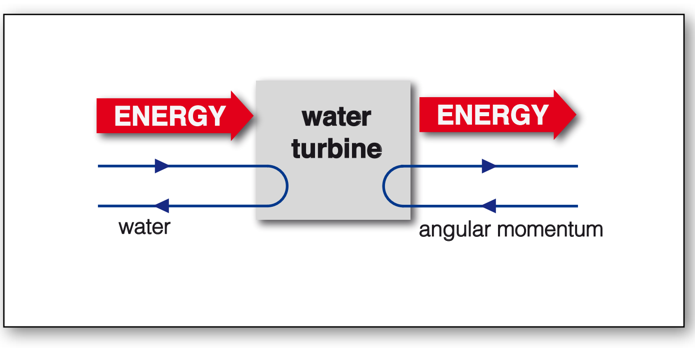


Fig. 11.16
Energy flow diagram of an ideal water turbine

Energy flows away with this entropy. Fig. 11.17 shows the energy flow diagram of a real turbine. The strengths of the currents are indicated by the thicknesses of the energy arrows.

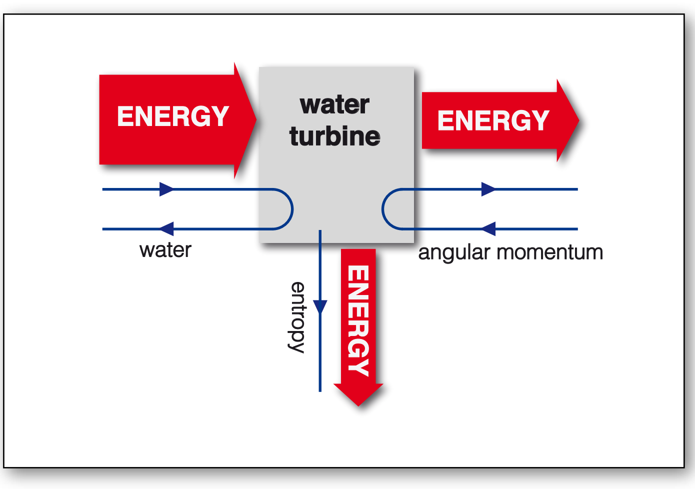


Fig. 11.17
Energy flow diagram of a non-idealized water turbine

The value of the energy current due to loss is called P_V. The relation between the produced entropy and the lost energy is then

$$P_V = T \cdot I_{S\,\text{produced}}$$

and the device's percentage loss is

$$V = \frac{P_V}{P_{in}} \cdot 100\%\ . \qquad (7)$$

The energy current flowing into the engine is indicated by P_{in}.

Fig. 11.18 shows the energy flow diagram of a real (not idealized) electric motor. Here as well, entropy is created unintentionally. A part of this entropy is created in the wires (entropy is always produced when an electric current flows through a wire). Another part is created in the bearings.

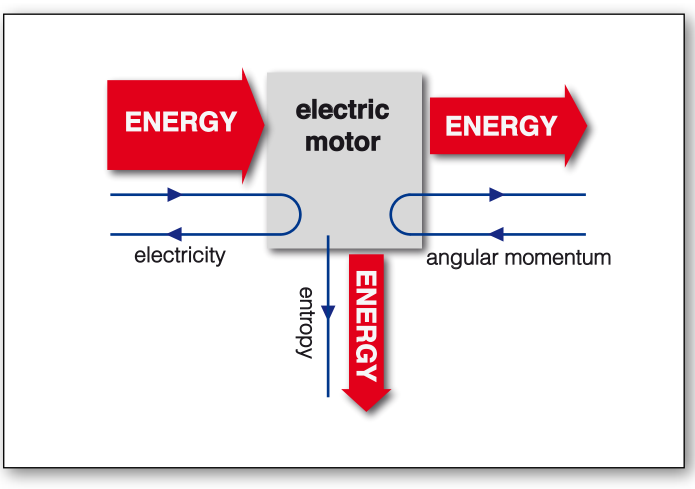


Fig. 11.18
Energy flow diagram of a non-idealized electric motor

The energy loss in a simple electric cable is also calculated with Equation (7).

We have seen that energy loss depends upon entropy production. Of course one would wish to avoid this loss. Therefore remember:

> Avoid creating entropy.

The loss in some energy exchangers is very large. Table 11.1 shows some typical values.

	loss
Large steam turbine	10 %
Large electric motor	10 %
Toy electric motor	40 %
Solar cell	90 %
Coal fired power plant	57 %
Nuclear power plant	67 %

Table 11.1
Typical values of energy loss

You probably wonder about the high losses in power plants. This is due only to a small extent to the losses in steam turbines and the generator. They occur mostly because of the entropy produced in the burner or in the reactor. How can we define loss in this case? Don't we need to create this entropy in order for the power plant to run? Not necessarily.

It is possible to transfer the energy of coal (or maybe uranium) directly to electricity without taking the detour through entropy and angular momentum. The devices that do this are called *fuel cells*. A fuel cell functions similarly to a battery. It actually represents a kind of battery where the substances being used up are constantly renewed. Up until now, fuel cells have worked only with fairly pure liquid and gaseous fuels, and not with coal. Moreover, the life span of these fuel cells is not yet long enough for them to compete with the power plants in use today.

Exercises

1. An automobile engine emits 20 kW through its shaft. Only 18 kW reach the wheels because entropy is produced (by friction) in the bearings and the gearbox. What is the percentage loss?

2. An electric motor with a loss of 40% uses 10 W. How much energy does it emit with angular momentum? How much entropy is produced per second? (The ambient temperature is 300K.)

3. A generator with a loss of 8% gives up an energy current of 46 kW with the electricity. What is the energy current that flows over the engine shaft into the generator? What is the energy current due to loss? What is the current of the produced entropy? (Ambient temperature is 300 K.)

11.7 The relationship between entropy content and temperature

If entropy is added to a body, its temperature rises. At least, this was the case of the objects we have dealt with so far. Later on we will get to know cases where things are different.

What factors determine how much the temperature of a body will rise if it is given a certain amount of entropy?

The first factor is naturally the size of the body, or rather, its mass. Its influence can be expressed in this way: Two bodies A and B are made of the same material. A has twice the mass of B. At the same temperature, A has twice the entropy of B.

Second, the entropy content also depends upon the material the object is made of. Fig. 11.19 shows how the temperature increases with the entropy content for a body of copper and one of aluminum. They both have a mass of 1 kg. We see in the figure that less entropy is needed to bring copper up to a certain temperature than is needed for aluminum. For example, at a temperature of 300 K, copper contains 500 Ct, but aluminum contains 1000 Ct, i.e., twice that of copper.

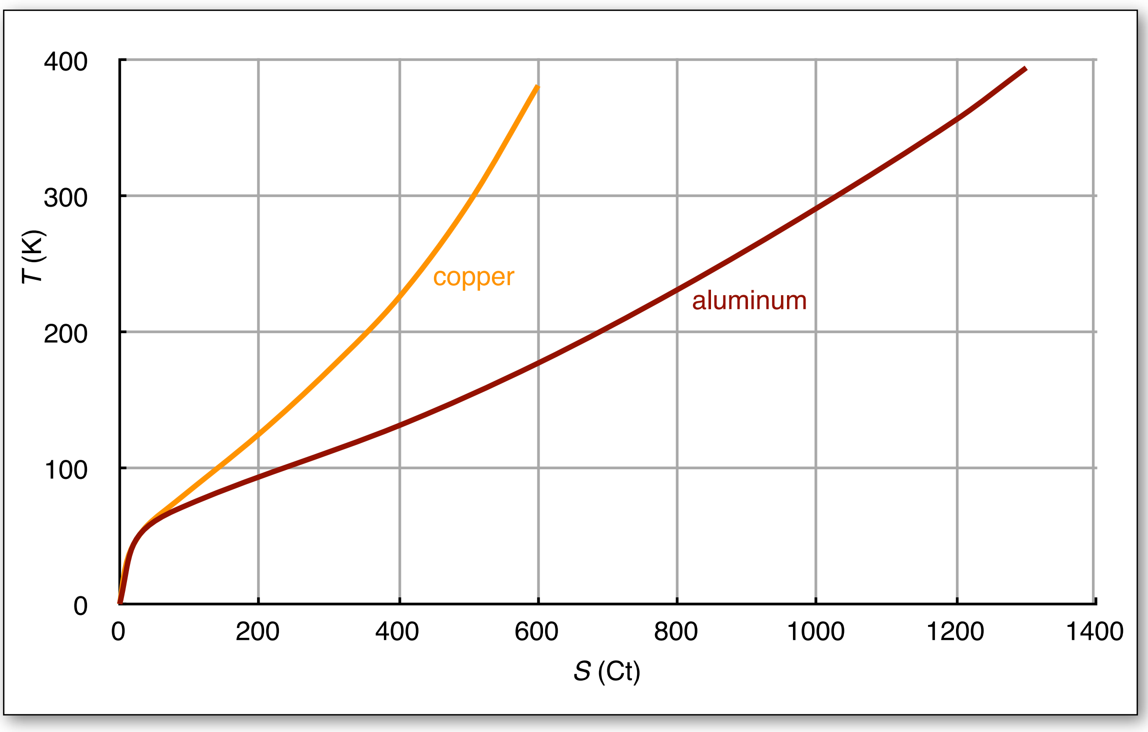


Fig. 11.19
Temperature as a function of the entropy content for 1 kg of copper and 1 kg of aluminum

The diagram shows that, with a given amount of entropy, copper warms up more than aluminum. With 500 Ct, copper reaches a temperature of 300 K, but aluminum reaches only about 150 K.

If we are only interested in what happens in the vicinity of normal ambient temperature, it is more useful to use a graph where the axis does not start at zero, i.e., an enlarged section of the original figure.

Fig. 11.20 shows such a section for 1 kg of various materials: copper, iron, aluminum, heating oil, and water. The steeper the curve, the less entropy is needed to produce a given temperature change.

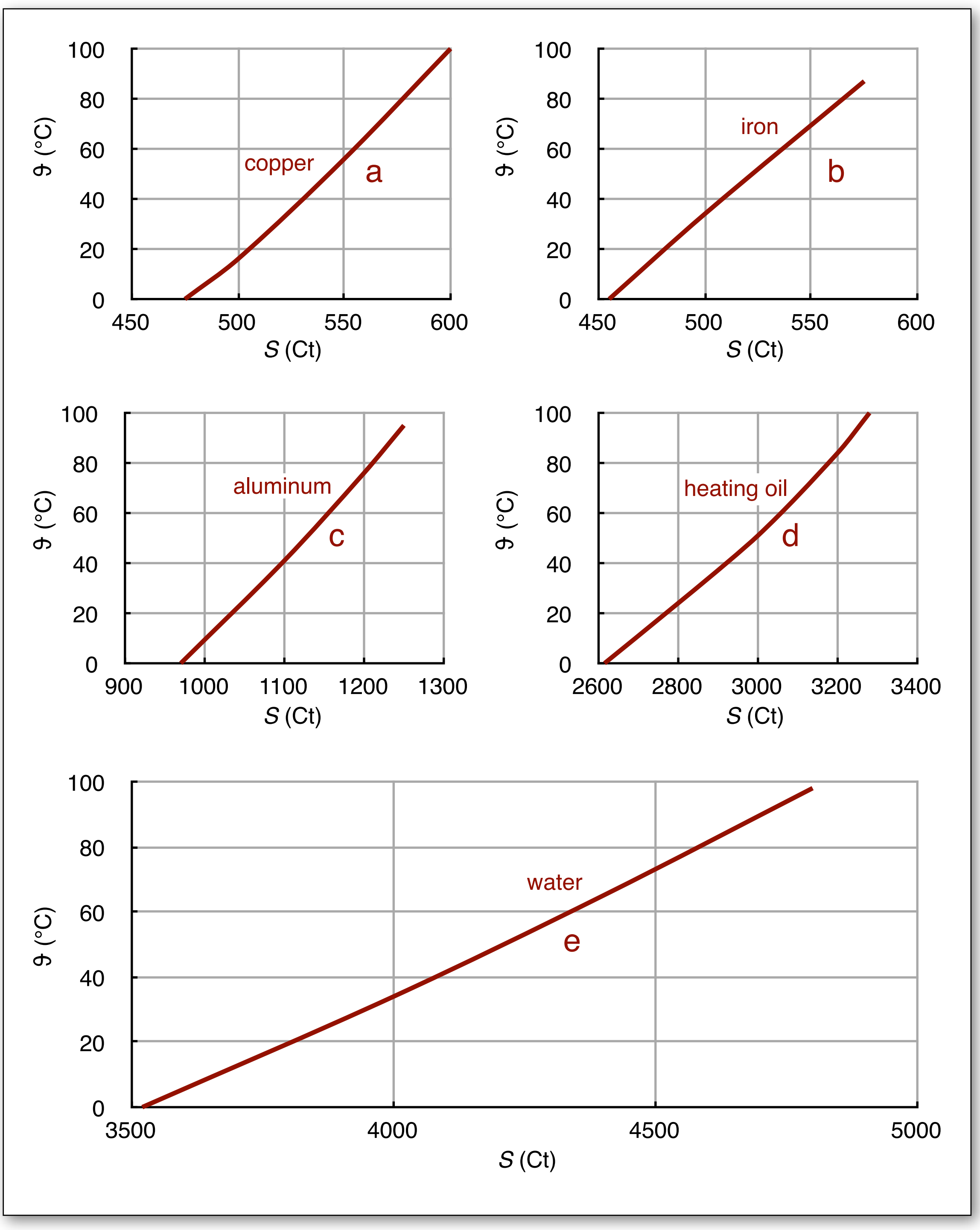


Fig. 11.20
Temperature as a function of the entropy content for 1 kg of: (a) copper, (b) iron, (c) aluminum, (d) heating oil, and (e) water. The entropy scales do not begin at $S = 0$ Ct. The temperatures do not begin at absolute zero, but at zero on the Celsius scale.

Exercises

1. A kilogram of copper and a kilogram of aluminum with initial temperatures of 25°C each receive 80 Ct. Which material heats up more? By what factor do the temperature changes differ?

2. How much entropy is needed to heat 100 l of water from 20°C to 100°C? (1 l of water has a mass of 1 kg)

11.8 The relation between adding energy and change of temperature

If water is to be heated, entropy must be added. Energy also goes into the water with the entropy. This fact is probably known by most people: It costs money to heat water, and one pays for the energy.

We will now set up a formula that gives us information about energy use in heating water. We call the amount of energy added to the water in the heating process ΔE. This should not be mistaken for the total amount of energy contained in the water. In order to heat 1 kg of water from 20°C to 100°C, a certain amount of energy is necessary. We need double this amount of energy to heat 2 kg of water from 20°C to 100°C. Therefore:

$\Delta E \sim m.$

The energy needed for heating is proportional to the mass of the water.

Moreover, the energy ΔE also depends upon by how much we wish to increase the temperature. If the temperature is to be raised by 20°C, more energy is needed than would be if it should only increase by 10°C. We put an immersion heater into a given amount of water and measure the increase of temperature ΔT as a function of the added energy ΔE. We find that ΔT is proportional to ΔE :

$\Delta E \sim \Delta T.$

This relation is no longer valid at very high or very low temperatures although it is satisfied between 0°C and 100°C. Together with the previous proportionality, we have:

$\Delta E \sim m \cdot \Delta T$.

In order to make an equation out of this proportional relation, we insert a factor of proportionality c :

$$\Delta E = c \cdot m \cdot \Delta T .$$

c is called the *specific heat*.

In order for the left and right sides of the equation to both have the same unit, c must be measured in J/(kg · K).

The value of c still depends upon the material of the body being heated or cooled. For water it is

$c = 4180$ J/(kg · K).

Exercises

1. A half liter of water is to be heated by a 500-W immersion heater from 25°C to 100°C. How much time is needed for this? (1 l of water has a mass of 1 kg.)

2. How much energy is used in a five minute shower?

First calculate how many l liters of warm water are used during the five minutes. Assume that 0.1 l per second of water comes out of the shower head. Also assume that the water flows into the water heater at 15°C and flows out again at 45°C.

12

Phase transitions

12.1 Phase transitions

We put an immersion heater in a glass of water, turn it on and measure the temperature of the water, Fig. 12.1. While the immersion heater is delivering entropy to the water, the water's temperature increases, at least at first. When the temperature reaches 100°C, the water begins to boil. The temperature does not increase anymore although the immersion heater continues to emit entropy. Why is that?

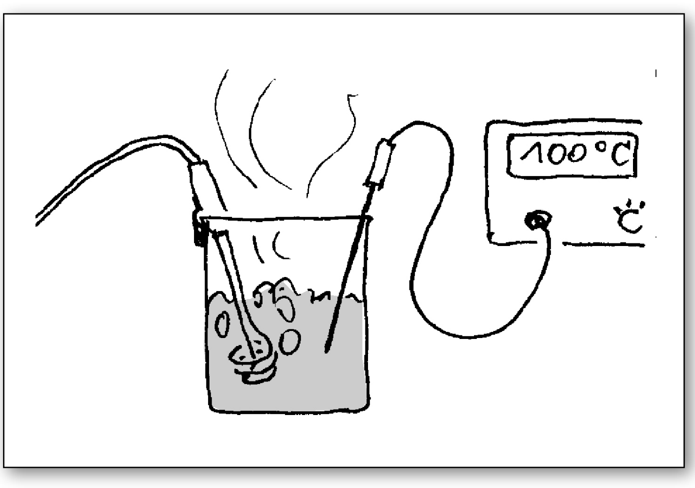


Fig. 12.1
Despite continued supply of entropy, the temperature stops increasing at 100°C.

When water boils, liquid water transforms into gaseous water, or *steam*. The steam has the same temperature as the boiling liquid water, that is 100°C. The entropy added to the water is apparently used to vaporize the water. We conclude that steam contains more entropy than liquid water.

Steam can be further heated. To do so, it is conducted through a pipe which is heated from outside, Fig. 12.2.

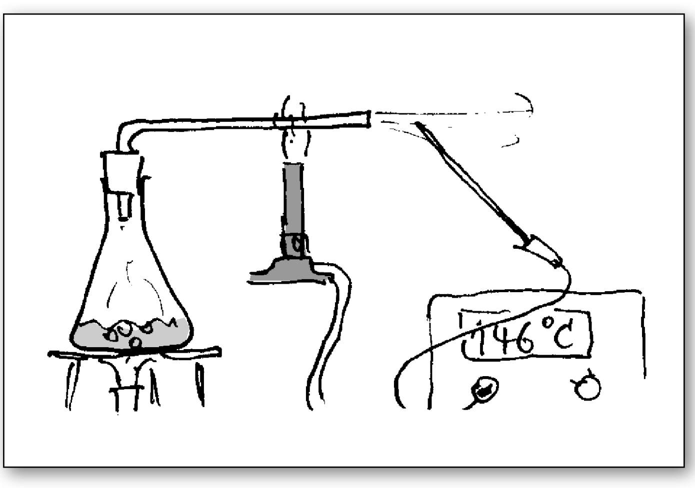


Fig. 12.2
The water vapor (steam) that has an initial temperature of 100°C, is further heated.

In Fig. 12.3, the temperature of 1 kg of water is plotted as a function of the entropy content of the water. This is over a larger span of temperatures than in 11.20(e). From the curve, we see that 1 kg of steam contains about 6000 Ct more than 1 kg of liquid water.

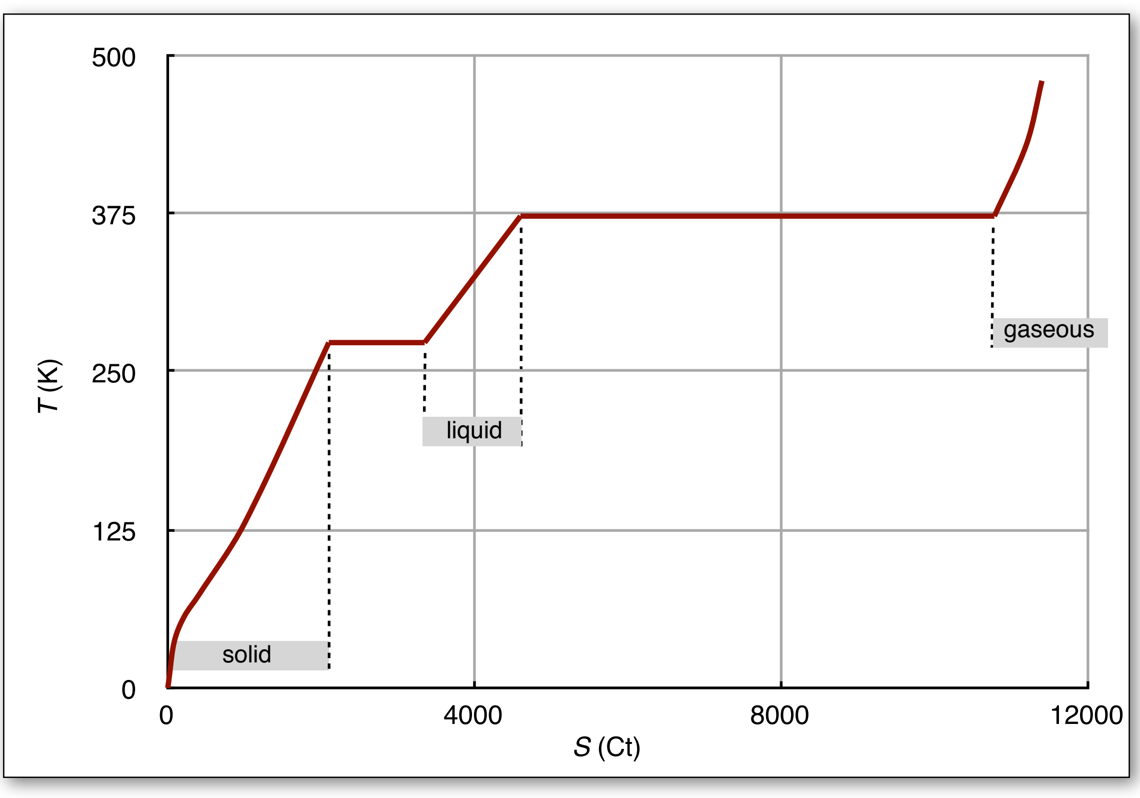


Fig. 12.3
Temperature as a function of entropy content for 1 kg of water at p = 1 bar

> 1 kg of steam contains about 6000 Ct more entropy than 1 kg of liquid water.

The diagram also shows that a similar phenomenon occurs during the change from the solid to the liquid phase. Liquid water contains around 1200 Ct more than solid water (ice). It is necessary to add 1200 Ct of entropy to transform 1 kg of ice at 0°C into 1 kg of liquid water at 0°C (in other words, to melt the 1 kg of ice). The opposite is also true: In order to transform 1 kg of water into 1 kg of ice, 1200 Ct must be removed.

> 1 kg of liquid water contains about 1200 Ct more entropy than 1 kg of ice.

A comment about wording: It is said that a substance appears in various *phases*. Water has a solid, a liquid and a gaseous phase. The gaseous phase is called steam, or vapor. Steam is water in its gaseous form. There are also special words for the transitions between phases:

solid → liquid: melting;
liquid → solid: solidifying or freezing;
liquid → gas: vaporizing;
gas → liquid: condensing.

Water is not the only substance that has various phases. Other substances have too. You are already aware that metals can melt. They can even be vaporized. All substances normally appearing in gaseous form can be liquefied and brought into the solid phase. Table 12.1 shows the melting temperatures and boiling temperatures of some substances.

substance	melting temperature (°C)	boiling temperature (°C)
Aluminum	660,0	2450,0
Copper	1083,0	2590,0
Iron	1535,0	2880,0
Water	0,0	100,0
Ethanol	-114,5	78,3
Oxygen	-218,8	-183,0
Nitrogen	-210,0	-195,8
Hydrogen	-259,2	-252,2

Table 12.1
Some melting and some boiling temperatures

There are many more phases than just "solid", "liquid", and "gaseous". Substances usually have several solid phases that differ from each other in many of their characteristics. Some substances have many liquid phases with very distinct characteristics.

Exercises

1. Using Fig. 12.3, find out how much entropy is in 1 kg of steam at 100°C and how much entropy is in 1 kg of liquid water at 100°C. By what factor is the value for the steam greater than the one for the liquid?

2. How much entropy is needed to transform 10 l of liquid water at 90°C into steam at 100°C?

3. 6000 Ct is necessary to melt an ice block. What is the mass of the ice block?

4. A quarter of a liter of carbonated water is cooled from 20 °C to 0 °C by using ice cubes. How much of the ice melts in the process? (Carbonated water is mainly water.)

5. The steam jet of an espresso machine is used to heat a glass of milk (0.2 l) from 15°C to 60°C. How many grams of steam are used? (Milk is essentially water.)

12.2 Boiling and evaporating

We have seen that water begins to boil at 100°C. It transforms to the gaseous state at lower temperatures as well, but more slowly. This process is called evaporation. Here are the different expressions again: The change from liquid to gas is always called “vaporization”. If the vaporization occurs at boiling temperature, meaning that it happens quickly, it is called boiling. If it happens below boiling temperature, meaning more slowly, it is called “evaporation”.

Why does vaporization by boiling go quickly and vaporization at lower temperatures more slowly? How do the processes differ? We consider a water surface at various temperatures, Fig. 12.4.

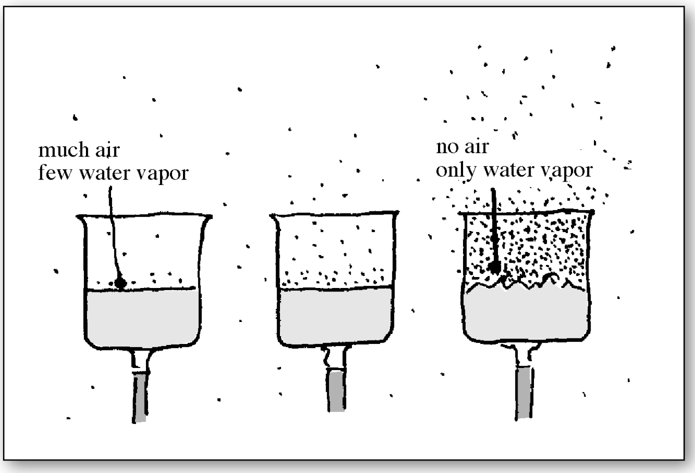


Fig. 12.4
When water boils, the steam pushes the air above its surface away.

At a temperature of 20°C, there is a slight amount of water vapor just above the water’s surface. In order for the process of vaporization to take place, this water vapor must disappear upward. It must move up to where the air contains less water vapor. The process by which a gas (in this case water vapor) “pushes through” another one (in this case air) is called *diffusion*. The second gas represents a great resistance to the motion of the first one. In our case this means that the water vapor can hardly move away from the water’s surface.

When the temperature is higher, there is more water vapor above the surface of the water. The driving force for the process of diffusion is greater and the water vapor moves away more quickly. Therefore, the liquid water can replace it more quickly and evaporation goes faster.

Finally, at 100°C there is only pure steam directly above the surface of the water. In order for this steam to move away from the surface of the water, it does not need to push through the air any longer. It does not need to diffuse, but can freely flow like water in a pipe or air moved by the wind. The vapor moves as quickly now as it can be replaced by the liquid water, and the water does this as quickly as it receives the necessary entropy for vaporization from the heater.

We can now understand an interesting phenomenon. If air pressure is less than 1 bar (normal pressure), water boils at a temperature lower than 100°C. If air pressure is lower, the steam coming off of the water’s surface can completely displace the air sooner, i.e., at a lower temperature.

This phenomenon can be observed in the mountains. On a high mountain where air pressure is lower, the boiling temperature of water is lower than 100°C. Air pressure is about 0.5 bar at an altitude of 5400 m. The boiling temperature of water at this altitude is 83°C.

12.3 Phase transitions in nature and engineering

During a phase transition, a substance absorbs entropy at a constant temperature, or it emits it, depending upon the direction of the transition. This fact is often applied in technology. It is also the explanation for some interesting natural phenomena.

Latent heat and evaporative cold

When you climb out of a swimming pool, and especially when the surrounding air is moving, you are chilly. The water on your skin is evaporating. It needs entropy to do this and this is taken from your body. The evaporation goes more quickly when the water which evaporates is carried away by the moving air around you.

Hot steam is more dangerous than hot water

It isn't really a problem when you get a little bit of 100°C water on your finger. It is worse if you get 100°C steam on it. In both cases, entropy is transported to the finger causing possible burns. Steam is more dangerous because the steam condenses on the finger giving up an additional amount of entropy to the finger.

Freezing mixtures

Salt water freezes at lower temperature than standard, pure water. We put a small piece of crushed ice (or snow) in a glass. We measure the temperature and find (as expected) 0°C. We now put a large amount of salt into it and stir. The temperature sinks to below –10°C.

With the addition of the salt, the melting temperature lowers. A part of the ice melts. Entropy is needed for this. Because we do not supply entropy from outside, the ice-water mixture cools down. More ice melts and the temperature decreases further. This process continues until the temperature reaches the new melting temperature and then comes to a stop.

Entropy storage

It is possible to store entropy by heating an object. When the entropy is released from the object, it cools down. This method is used in so-called night storage heaters, Fig. 12.5. A night storage heater is composed mostly of ceramic stones. At night, when energy is cheaper, the stones are charged with entropy. They heat up to over 600°C. During the day, the entropy is retrieved by blowing air past the hot stones.

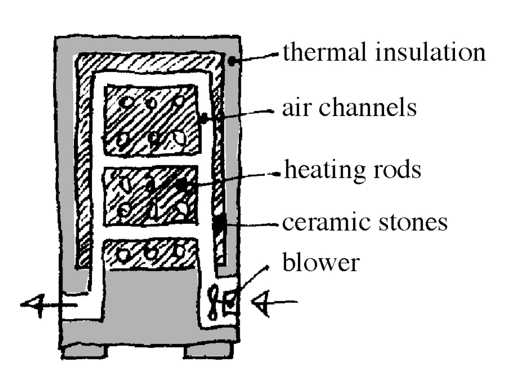


Fig. 12.5
Night storage heater

It would be desirable to store the entropy, which is available abundantly in summer for use in winter. The night storage heater method is not suitable for this because the stones cannot store much entropy.

A method with more potential makes use of a phase transition. A substance is chosen whose phase transition from solid to liquid takes place at an acceptable temperature. About 50°C would be convenient. (A phase transition of liquid to gaseous cannot be used because gases take up too much room.) A large amount of the substance is melted during the summer months by solar entropy (and energy). In winter, the entropy is retrieved to heat a house.

When the price for energy strongly increases in the future, this method for using solar energy could become a competitive alternative.

Cooling drinks with ice

A soda can be cooled in a refrigerator. The entropy of the soda is pumped out by the refrigerator's heat pump. Sometimes we wish to cool a soda, or at least keep it cool, while it is standing on a table. You know how it is done: You throw an ice cube in. Why don't you just pour some cold water into the soda? This would be much less effective. The ice in the soda melts. In order to melt, it needs entropy and it gets it from the soda. The melting process takes as long as needed for the soda to reach 0°C (assuming there is enough ice in the glass).

Liquid nitrogen

If something must be cooled to a much lower temperature, but there is no appropriate refrigeration machine available, liquid nitrogen can be used. It is also inexpensive.

The boiling temperature of nitrogen is 77 K (–196°C). How can liquid nitrogen exist when the ambient temperature is much higher? It is stored in well insulated containers. The slight amount of entropy that escapes through the insulation results in a constant very slow boiling of the nitrogen. The temperature of the liquid nitrogen remaining in the container always stays at 77 K, exactly in the same way that boiling water remains at a temperature of 100°C. Liquid nitrogen can be stored for days like this.

Transport of entropy with phase changes

Earlier we saw that transport of entropy by convection is more effective than by conduction. There is, however, a means of transport that works even better than normal convection, Fig. 12.6. The substance in the pipes is vaporized on the left by the entropy source. In the process it absorbs a lot of entropy. It then flows to the right through the upper pipe. It condenses in the coil on the right and gives up the entropy it had absorbed. Earlier central heating units functioned with this principle and were called steam heaters. They had some disadvantages though: They were hard to control and unpleasant noises occurred in the radiators during condensation of the steam.

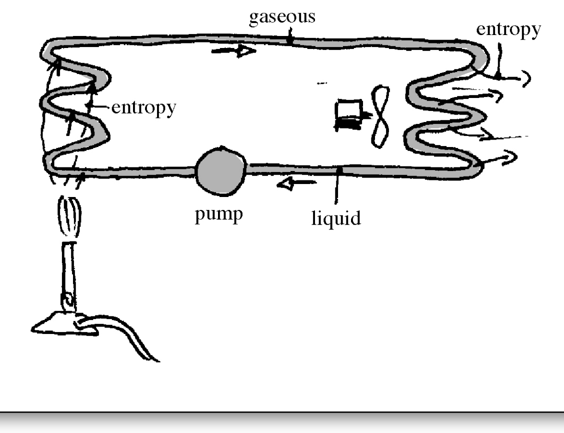


Fig. 12.6
On the left, a substance is vaporized. In the process, it absorbs a lot of entropy which it emits during condensation on the right.

Today, this method is mostly used by heat pumps, as they are found in refrigerators. The coolant vaporizes in the coiled pipe inside the refrigerator, absorbing entropy in the process. In the coil on the outside it condenses and emits the entropy. (In order for condensation to occur where it is hotter, and evaporation where it is colder, a compressor is used to keep the pressure higher at the warm side, and lower at the cold side.)

Nature also uses this way of transporting entropy. Constant evaporation and condensation processes take place in the atmosphere. When water evaporates, the temperature decreases. The vapor is carried away by winds to another place where it condenses making that place warmer.

13

Gases

13.1 Gases and condensed substances

Substances can be solid, liquid or gaseous.

The liquid and the gaseous phases have something in common: Liquid and gaseous substances can both flow. When the wind blows, when a fan or hairdryer is running, air flows. Water flows in rivers and streams as well as in the oceans. It flows when we turn on the water faucet. Because liquid and gas flows have a lot in common, they have been combined into one class of substances called *fluids*. Fluids are considered the opposite of solids.

However, solid substances have some characteristics in common with liquids. These are characteristics which differentiate them from gases. As we have learned earlier, solid and liquid substances have much higher densities than gases. For this reason, solid and liquid substances are often classified together and are called *condensed substances*. Condensed substances are the opposite of gases, Fig. 13.1.

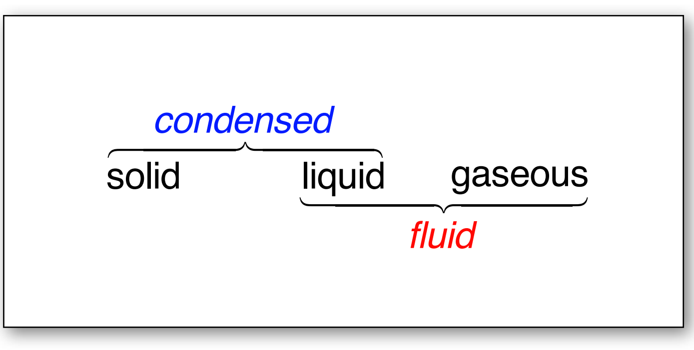


Fig. 13.1
Two types of classification of substances

In the following, we will be interested in further characteristics which differentiate gases and condensed substances.

The tendency to disperse

We pump the air out of a glass container and drip a little water into it, Fig. 13.2. The water falls downward just as it would in a non-evacuated container. We repeat the experiment but let air instead of water into the container. In order to see where the air goes, we let it flow through a cigarette first (finally, something a cigarette is good for). These experiments show:

> Gases fill the entire space available to them, condensed substances do not.

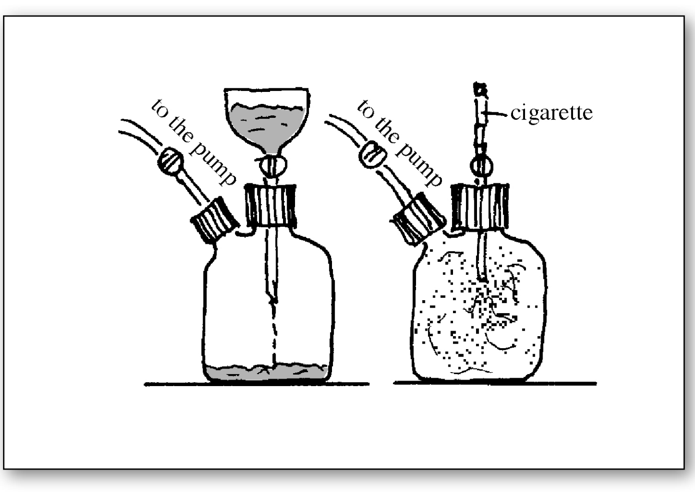


Fig. 13.2
Gases take up all the room available to them. Liquids do not.

Simplification is often necessary when something should be expressed briefly. The sentence above is just such a simplification. The statement is valid most of the time, but not always. For instance, it is invalid for the air above the surface of the Earth. In theory, this air could disperse into all of outer space but it does not leave the Earth. Why not?

Compressibility

There is air in a cylindrical container with a movable piston. When the piston is pushed in, the air is compressed, Fig. 13.3a. If there is water in the container instead of air, Fig. 13.3b, the piston cannot be pushed in. Water cannot be compressed. However, if you look very carefully, you can make out a tiny amount of compression that is so small it can be ignored for most purposes.

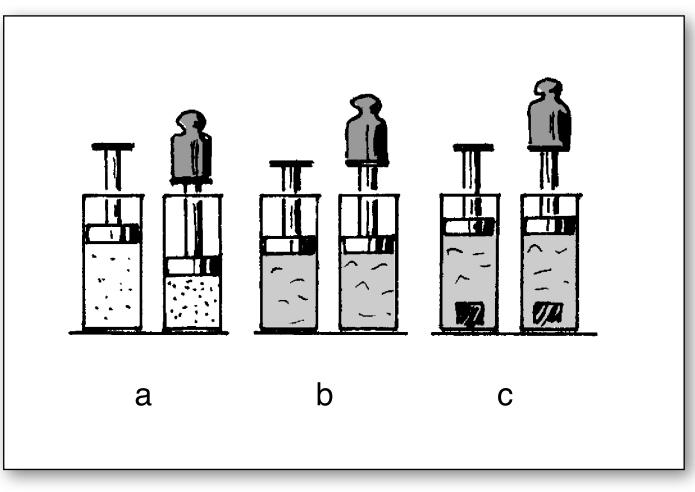


Fig. 13.3
Gases (a) are compressible, liquids (b) and solid substances (c) are not.

Even when a solid object is also put in the water, Fig. 13.3c, the piston cannot be pushed in because solid objects are (almost) incompressible. Some solid bodies give the impression of being slightly compressible. Foam is an example of this. but it is the air in the pores of the foam and not the solid substance itself that is compressed.

We summarize our observations as follows:

> Gases can be compressed, condensed substances hardly at all.

"Compression" means that the volume of a given amount of substance is reduced while retaining the same mass. The formula $\rho = m/V$ shows that the density of a substance increases when it is compressed. It is possible to increase the density of a compressible substance by raising the pressure. If a substance is incompressible, raising the pressure will not result in any change of density. We can summarize this as follows:

> The density of gases increases with increasing pressure, the density of condensed substances does (almost) not.

This fact has interesting consequences. For example, the density of water in a lake does not increase as one goes downward although the pressure does. The density of the water is just about the same no matter what the depth is. This is about 1000 kg/m^3. It is a different story, however, with the air above the Earth's surface. Air pressure decreases with altitude, and as a result, so does the density. For this reason, breathing becomes more and more difficult for people climbing high mountains.

Thermal expansion

Gases and condensed substances react differently when entropy is added to them.

If a solid body is heated, its volume hardly changes. The same is true for liquids. It is different with gases, though. If air is heated in a container open at the top, Fig. 13.4a, it expands strongly and "overflows". Because air is invisible, this overflowing cannot be seen. However, a trick can be used to make it visible, Fig. 13.4b.

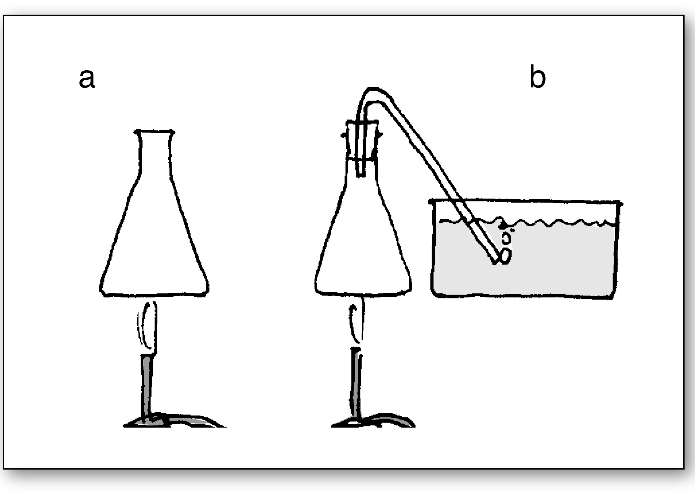


Fig. 13.4
Gases expand when entropy is added to them. In the experiment on the right, overflowing of air from a container is shown visually.

> Gases expand when entropy is added to them, condensed substances hardly at all.

Exercises

1. Why do bicycles have tires filled with air? Why aren't the tires filled with water?

2. Fig. 13.5 shows a hot air balloon. The balloon is open at the bottom. The air in it is heated by a gas flame. Why does the balloon rise upward?

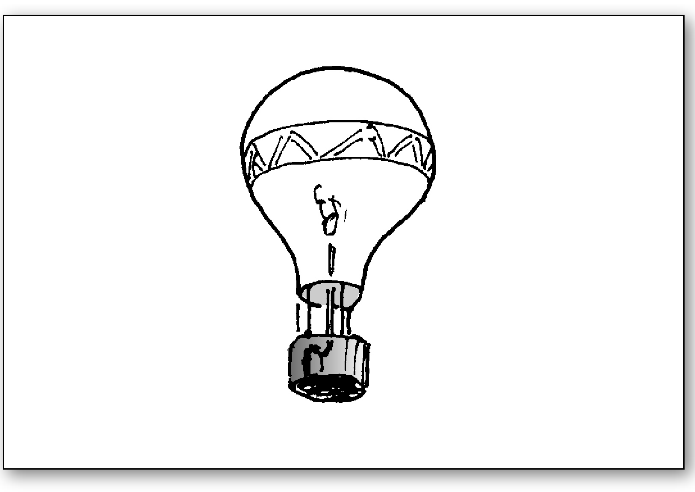


Fig. 13.5
Hot air balloon (for Exercise **2**)

13.2 Thermal properties of gases

In the previous section we have compared gases to condensed substances. From now on we will deal only with gases. They are more interesting than condensed substances with regard to thermal properties.

First we will again add entropy to a gas. This time we hinder it from expanding by keeping it in a container with a fixed volume, Fig. 13.6. The manometer shows that the pressure increases while entropy is being added. We can summarize this observation and the last one from the previous section as follows:

> If entropy is added to a gas at constant pressure, the volume increases.
>
> If entropy is added to a gas with constant volume, the pressure increases.

The temperature of the gas increases in both cases.

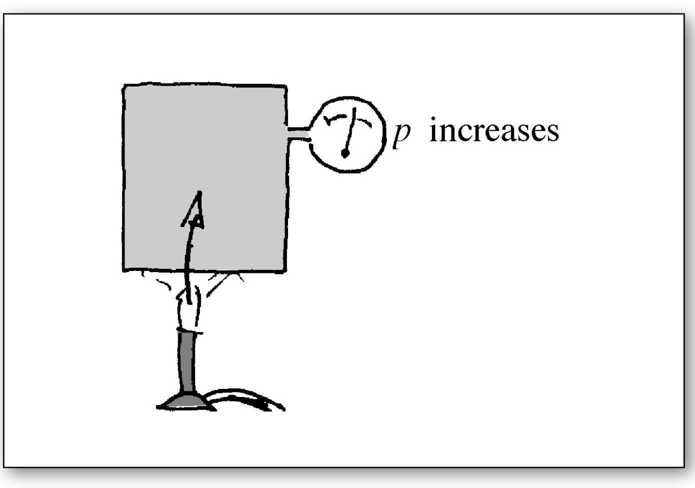


Fig. 13.6
If entropy is added to a gas at constant volume, the pressure increases.

These processes can be symbolically described by indicating whether the four quantities, entropy, temperature, volume, and pressure, stay constant, decrease, or increase:

$S\uparrow \quad T\uparrow \quad V\uparrow \quad p = \text{const}$ (1)

$S\uparrow \quad T\uparrow \quad V = \text{const} \quad p\uparrow$ (2)

Once again, we compress the air in a cylinder. This time we measure the temperature, Fig. 13.7. We find that the temperature increases as the air is compressed. If the air is allowed to expand, the temperature goes down again.

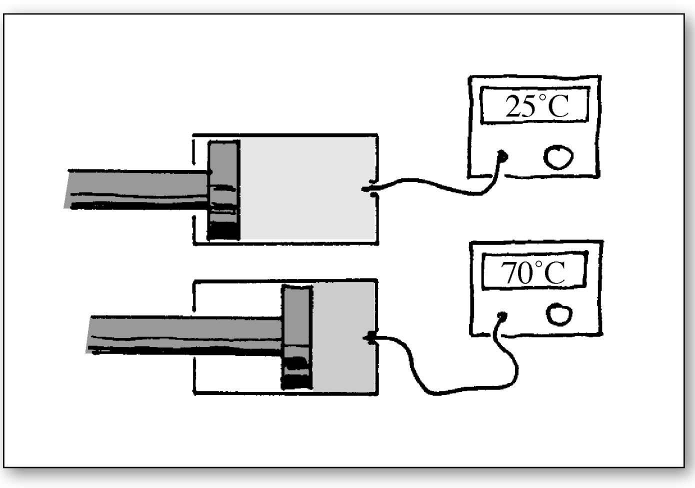


Fig. 13.7
When a gas is compressed, its temperature increases.

This behavior is plausible because the entropy in the air is compressed into a smaller space as well. A lot of entropy in a small space means a high temperature.

> The temperature of a gas increases if its volume is reduced.

Expressed in symbols, the result is

$S = \text{const} \quad T\uparrow \quad V\downarrow \quad p\uparrow$ (3)

The expressions (1) to (3) describe various processes that can be carried out with gases. Of course, the reverses of these three expressions are also true. The reverse of (1) is:

$S\downarrow \quad T\downarrow \quad V\downarrow \quad p = \text{const}$

In each of the processes (1) to (3), a different quantity is kept constant. In (1), it is the pressure, in (2) the volume, and in (3) the entropy. The only process missing is the one where the temperature stays constant. This process is also easy to perform. It is enough to compress the gas of Fig. 13.7 very slowly, Fig. 13.8. Actually, the compression would cause a rise in the temperature but if we push only very slowly, the temperature can constantly adjust to the temperature of the environment. In the process, entropy flows from the gas into the environment. Therefore, there is less entropy in the air at the end than before. Expressed in symbols, we obtain:

$S\downarrow \quad T = \text{const} \quad V\downarrow \quad p\uparrow$ (4)

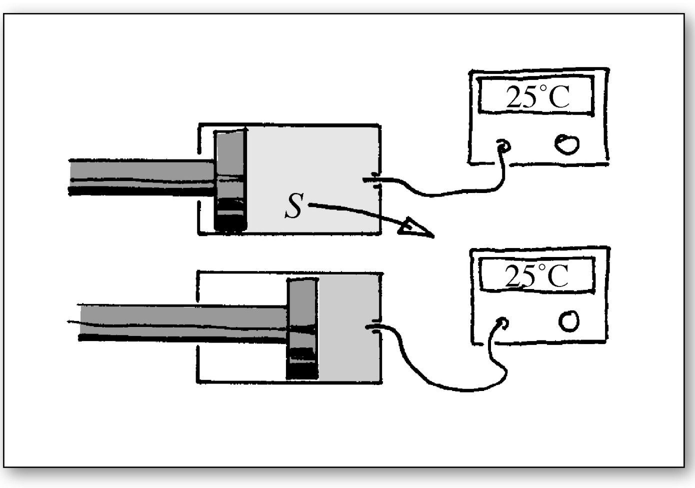


Fig. 13.8
When the piston is pushed very slowly into the cylinder, entropy escapes from the gas.

This statement is also interesting. It fits together with something we experienced earlier: The larger the volume of a portion of a substance is (at constant mass and temperature), the more entropy it contains. We encountered this behavior with the phase transition from liquid to gas. At the same temperature, the gas (large volume) contains more entropy than the liquid (small volume).

In Fig. 13.9, the four processes (1) to (4) are once again summarized along with their reverse processes.

$S\uparrow$	$T\uparrow$	$V\uparrow$	p = const	1a
$S\downarrow$	$T\downarrow$	$V\downarrow$	p = const	1b
$S\uparrow$	$T\uparrow$	V = const	$p\uparrow$	2a
$S\downarrow$	$T\downarrow$	V = const	$p\downarrow$	2b
S = const	$T\uparrow$	$V\downarrow$	$p\uparrow$	3a
S = const	$T\downarrow$	$V\uparrow$	$p\downarrow$	3b
$S\downarrow$	T = const	$V\downarrow$	$p\uparrow$	4a
$S\uparrow$	T = const	$V\uparrow$	$p\downarrow$	4b

Fig. 13.9
Symbolic representation of four processes. In each of them, one of the four quantities S, T, V, and p is kept constant.

Exercises

1. We need a bottle that can be well sealed and a bowl with hot water and a bowl with cold water (the two sections of a kitchen sink will do).

a) The air in the open bottle is cooled by the cold water. Then the bottle is sealed and pushed down into the hot water. The cap on the bottle is slightly loosened so that it no longer sealed. What happens? What is your explanation?

b) The air in the open bottle is heated by the hot water. Then the bottle is sealed and pushed down into the cold water. The cap on the bottle is slightly loosened so that it is no longer sealed. What happens? What is your explanation?

2. There are equal amounts of the same gas at the same temperature in two containers. The same amount of entropy is added to both the gases. In one case the volume is kept constant, and in the other one the pressure is kept constant. Is the change of temperature in both gases the same? If not, which container has the greater change of temperature? Does the temperature go up or down? Give reasons!

3. How can the temperature of a gas be lowered even when entropy is being added to it?

13.3 The operating mode of heat engines

In section 11.4 we saw that in a heat engine, entropy goes from a higher to a lower temperature and in the process 'drives' something. This is similar to how water in a water turbine goes from high to low pressure and drives something.

How can entropy be brought from a higher to a lower temperature in order to set something in motion?

It is no problem to bring entropy from a higher to a lower temperature without driving anything. This tends to happen by itself. One lets entropy simply 'slide down' a heat conductor from a higher to a lower temperature (see Section 11.3). The energy that one would like to transfer to a useful energy carrier, angular momentum, for example, goes away along with the newly produced entropy. It is wasted.

How can we bring entropy from a high to a low temperature without producing more entropy? Since we know the thermal characteristics of gases, this is no longer a problem for us. Fig. 13.10 shows how it is done.

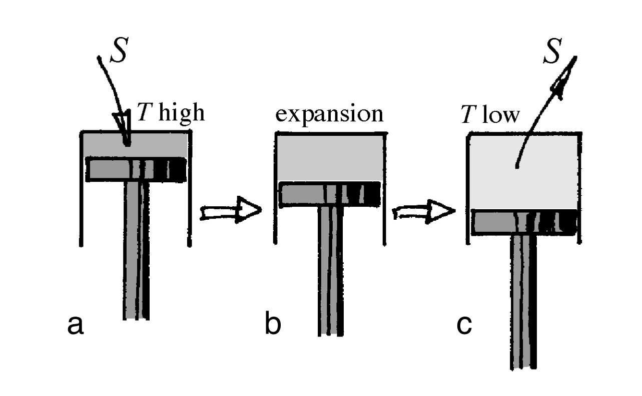


Fig. 13.10
Operation mode of heat engines. (a) Entropy is introduced into a compressed gas. (b) The gas relaxes, its temperature goes down and it supplies energy. (c) The entropy is given away at a lower temperature.

The entropy is put into a compressed gas, and then the gas is allowed to expand. According to line (3b) in Fig. 13.9, the temperature falls and at the same time, the piston is pushed outward. The energy released by the entropy exits through the rod of the piston and to a crank that sets a drive shaft in motion.

> A gas is allowed to expand in a heat engine. In the process, the pressure and temperature of the gas go down and the gas supplies energy.

This is the basic idea behind all heat engines. There are a number of various technical implementations of this idea: steam engines, steam turbines, gasoline engines, diesel motors, jet engines, and more.

We will take a closer look at two of these machines. First, the steam engine because it has played a very important role in the past, and second, the gasoline motor because most cars are driven by one.

The reciprocating steam engine

The biggest problem to be solved in the realization of a machine built upon the principle of Fig. 13.10, is getting the entropy into and out of the machine *quickly*. In no way does it function as suggested in Fig. 13.10, namely that entropy is let flow by normal heat conduction into the cylinders. That would go much too slowly. We already know a trick for getting entropy quickly from one place to another which is convection. It is done just this way in steam engines.

The gas is heated up outside the cylinder and then allowed to flow along with its entropy into the cylinder. It expands there, at the same time supplying energy to the piston. Afterwards, it is released from the cylinder together with its entropy.

Fig. 13.11 shows the details of this process for a steam engine. The gas used is steam. The steam is produced in a boiler and then *‚overheated'*. The slide valve controls the steam inlet and outlet of the cylinder. At first, the piston is all the way on the left (Fig. 13.11a). New hot steam flows from the left into the left-hand part of the cylinder. After the piston moves a little to the right (Fig. 13.11b), the steam inlet is closed. The steam pushes the piston further to the right expanding as it does so. Pressure and temperature both go down. The piston reaches the right-hand turnaround point (Fig. 13.11c), and begins to move back. In the meantime, the outlet has been opened. The expanded, cooled steam is pushed out along with its entropy.

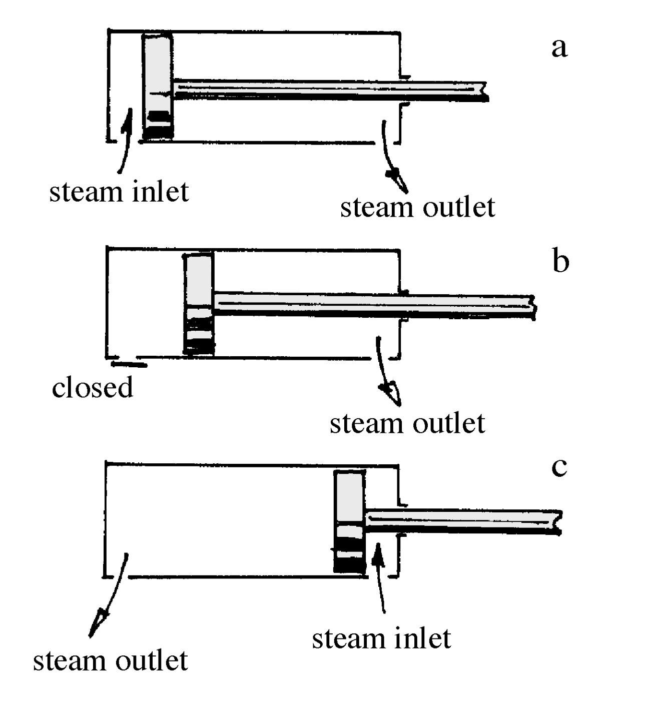


Fig. 13.11
A reciprocating steam engine at three different points in time.

Corresponding processes take place on the right-hand side of the piston. The steam on the right side pushes the piston to the left.

The parts of such a steam machine are easily recognizable in a steam locomotive, Fig. 13.12.

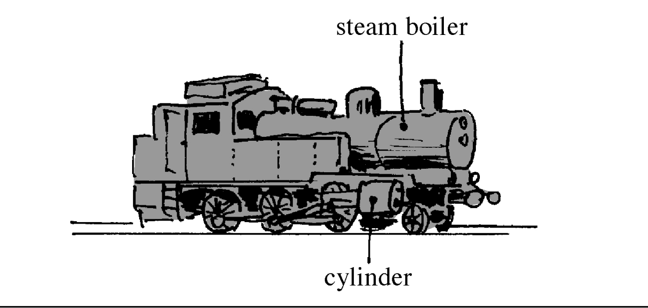


Fig. 13.12
A steam locomotive

The Otto engine

In this case, the trick of getting entropy quickly into the cylinder is to produce it there by burning a mixture of gaseous gasoline and air. This combustion takes place explosively or in other words, very fast.

The cylinder must first be filled with the combustible gasoline and air mixture. This can be done by initially letting the engine work as a pump for one revolution.

Each half revolution of the drive shaft is called a *stroke*. Charging the engine or pumping, takes two strokes. During the aspiration stroke, the gasoline and air mixture is sucked into the cylinder, Fig. 13.13a. It is then compressed during the compression stroke, Fig. 13.13b. At this point, the piston is at the upper dead center position and it is ready to work, Fig. 13.13c. The gasoline and air mixture is ignited by an electric spark produced by the spark plug. It burns instantly. Entropy is produced in the process and temperature and pressure strongly increase. Now the hot gases push the piston downward whereby temperature and pressure decrease. This stroke is called the *working stroke*, Fig. 13.13d. Afterwards, during the *exhaust stroke*, the exhaust gases are pushed out the exhaust pipe along with their entropy, Fig. 13.13e.

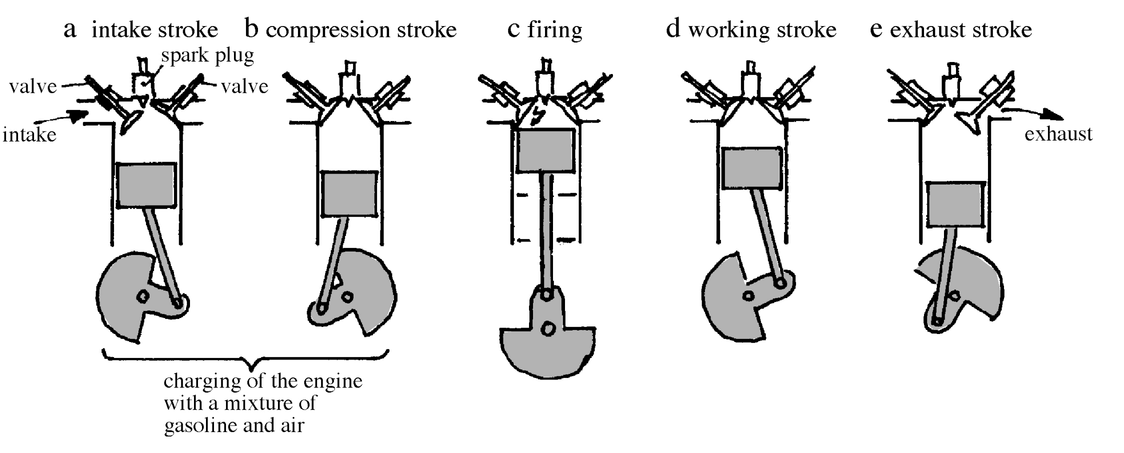


Fig. 13.13
An Otto engine at five points of its working cycle

Such a single cylinder engine works only one quarter of the time, as you see, i.e., during the working stroke. The three other strokes run on momentum. An Otto engine works more smoothly when it has several cylinders that work alternately. Most car engines have four cylinders. When this kind of engine is running, at any given moment one of the cylinders has its working stroke.

An Otto engine needs a series of auxiliary devices:

– A carburetor where the gasoline is evaporated and mixed with air;
– the fuel pump that transports the gasoline from the tank to the carburetor;
– spark coils and interrupters for producing the high electric voltage needed for the spark;
– the ignition distributor which sets the high voltage for the correct spark plug.

Exercises

1. Imagine the "working substance" in the heat engine of Fig. 13.10 is not a gas but a liquid. Would the engine work? Give reasons!

2. A diesel engine is built very similarly to an Otto engine. The difference: It has no spark plugs. The diesel fuel/air mixture ignites by itself. How is this possible?

3. Instead of closing the steam inlet of the cylinder of a reciprocating steam engine after it has moved a little to the right, it could be left open until the piston has moved all the way to the right. The engine would be stronger and would give off more energy. This mode of operation is possible in steam locomotives. It is used to start motion and to move uphill. What is the disadvantage of this mode of operation?

13.4 Why the air above the Earth's surface gets cooler with altitude

It is colder on top of a high mountain than down in the valley. The higher one climbs, the lower the temperature. For every 100 meters of altitude gained, the temperature sinks about 0.6 °C. In airplanes, the captain often announces impressively low outside temperatures. For an airplane at 10,000 m altitude, it is about –55°C.

How can these low temperatures be explained? Shouldn't the temperature differences above and below balance out? As we know, entropy flows from places of higher temperature to places of lower temperature. There is an obstacle in this case, though. Entropy flows only when the resistance is not too great, and air is a highly insulating material. A few millimeters of air between the layers of glass in a double glazed window is already very effective. There is a layer of air between the upper and lower parts of the Earth's atmosphere which is several kilometers thick. Temperature equalization by heat conduction is practically impossible because of this.

How does this temperature difference come about? We must apply our knowledge of the thermal characteristics of gases. The air in the Earth's atmosphere is in constant motion. In the next section we will see what causes this. For the moment, let us just imagine that someone is constantly stirring it up.

We consider a certain portion of the air that is moving downward. It is compressed because pressure increases in the downward direction. Because the amount of entropy in the portion of air remains the same, the temperature must rise according to line (3a) of Fig. 13.9.

Exactly the opposite would happen with a portion of air that is moving upward: the temperature goes down.

A certain amount of air with a certain amount of entropy changes temperature by moving upward or downward. Further up, it is cooler, and further down, it is warmer. To every altitude belongs a certain temperature.

13.5 Thermal convection

Warm air rises, as everyone knows. Why is this? The explanation is easy now that we have become experts on the thermal properties of gases. We consider the radiator of a central heating unit. The air near the radiator is heated and expands (see section 13.2). In the process, its density becomes less than that of the unheated air around it. The heated air tends to move upward (see section 4.8). That is basically all there is to it.

Now, after the air has moved upward, something more happens with it. It gradually gives its entropy into the surrounding unheated air and objects in the room, cooling down while doing so. Its density increases again, and it is displaced by the freshly heated air rising upward. It flows downward again and replaces the warm, rising air. In short: It circulates, Fig. 13.14. This kind of constant flow process is called *thermal convection*.

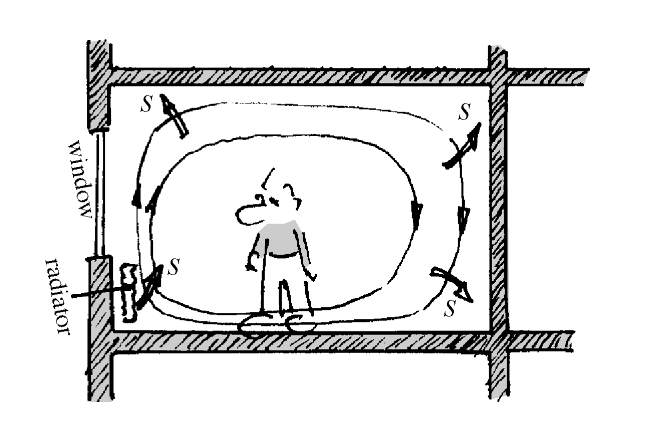


Fig. 13.14
Thermal convection in a heated room

Thermal convection is responsible for many entropy transports in nature and technology. We have just looked at an example of this. Thermal convection makes sure that the entropy emitted by the radiator is distributed over the entire room.

Thermal convection plays an important role in the creation of winds. Some wind systems are produced in very complicated ways, but in other cases, simple thermal convection is responsible.

An example is a *sea breeze*. This is a wind that blows from the ocean to land during the day. Solar radiation greatly increases the temperature on land, but the temperature of the water only rises a little. This is because the entropy distributes over a much greater depth in water than on land. The air over land expands, becoming less dense and rising Fig. 13.15. Air flows from the ocean (where it is not expanding), in the direction of land. The air from land flows back to the ocean at a few hundred meters elevation where it sinks again. The Earth's surface which is heated by the Sun corresponds to the radiator in our last example.

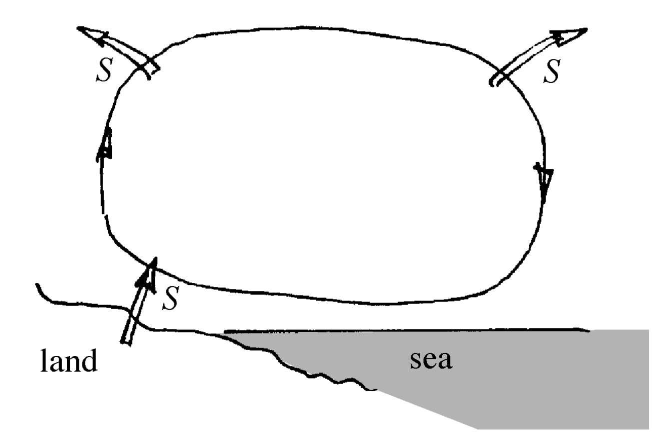


Fig. 13.15
The Sun heats up the land mass greatly, but the ocean only a little. Convective currents are created.

Temperature differences that lead to different heating of air are not only to be found between land and ocean, but on many other places on the surface of the Earth. Wherever there is a place where the Earth is warmer than its surroundings, an updraft occurs. If the place is cooler than its surroundings, there is a downdraft.

The updrafts that occur in warm places (so called thermals) are often used by birds and gliders for rising through the air.

Trade winds are an example of thermal convective flow, Fig. 13.16. Air is strongly heated near the equator. It rises and flows at a high elevation to the north and to the south, in other words, to areas that are cooler. Near the 30th parallel (north and south), it sinks again and flows back to the equator. This back-flow in the direction of the equator constitutes the trade winds.

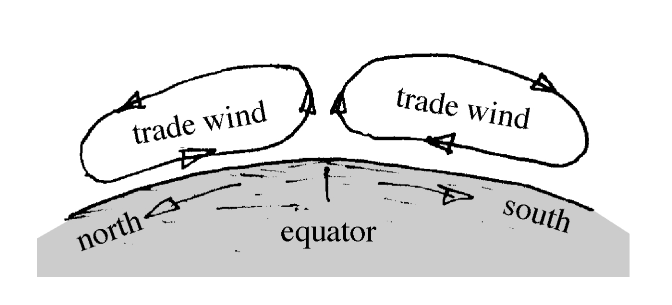


Fig. 13.16
How the trade winds are created

Now we want to look at thermal convection from another viewpoint. Air at a low elevation absorbs entropy and then rises. The temperature of the rising air decreases upwardly because its density decreases. It then gradually releases its entropy where it is at a higher temperature than its surroundings. It now emits its entropy at a lower temperature than the one at which it absorbed it.

The same thing happens with the air as with the working fluid in a heat engine. Entropy is absorbed at a high temperature and released at a lower one. Every thermal convective flow can be considered a heat engine where no drive shaft is set in motion but air is moved.

Finally, energy is often taken out of moving air by windmills, wind turbines, and sailboats. The energy of convective flow in a room could be used to turn a pin wheel.

Exercises

1. Liquids expand only very slightly when entropy is added to them. However, this slight expansion is enough to set thermal convective flows in motion. Give an example. Where is entropy added to the liquid, and where is it removed?

2. Why does the flame of a candle point upward from the wick and not downward?

14

Light

14.1 Transport of entropy through empty space

A hot object normally cools down all by itself. Its entropy flows into the surroundings: into the air and whatever it is standing upon. We now wish to prevent this cooling. One might think this is very easy. We only need to put the object into a vacuum, Fig. 14.1. Entropy cannot leak out through the air, because there is no air. We hang the object (we will call it G) from long thin threads leading only to a very slight heat leak.

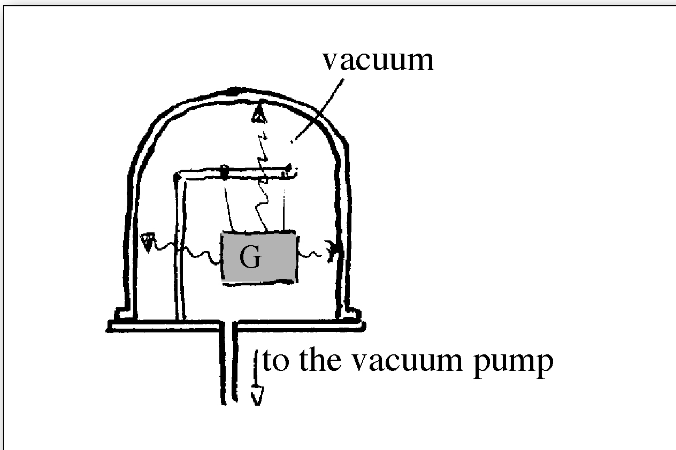


Fig. 14.1
Object G cools although it is in a vacuum.

We now observe something remarkable. First, the bell jar becomes noticeably warmer and second, object G cools down (which we can confirm if we take G out of the bell jar). In other words: The entropy has left the hot object although no heat conductor was available.

Actually, the experiment could be done by placing G into empty outer space. It would cool down there too.

The entropy is apparently able to move through empty space through some invisible connection or with an invisible carrier. It is easy to discover how this occurs and what the carrier is, if G is heated until it glows. When it glows, it emits something we all know, light. Light passes through empty space particularly easily. For example, it travels the 150 million kilometers between the Sun and the Earth with almost no loss. The light from the object carries the entropy. Therefore, the radiating object constantly emits entropy.

However, our problem is not quite solved yet. Object G wasn't glowing. It wasn't radiating any light. Or was it?

We must now learn a few things about light.

14.2 Types of light

We send a thin ray of sunlight (or the light of a strong light bulb or an arc lamp) through a glass prism; then we let it fall on a white screen behind the prism. What we see there is not a white spot, as might be expected, but a colorful stripe, a *spectrum*, Fig. 14.2.

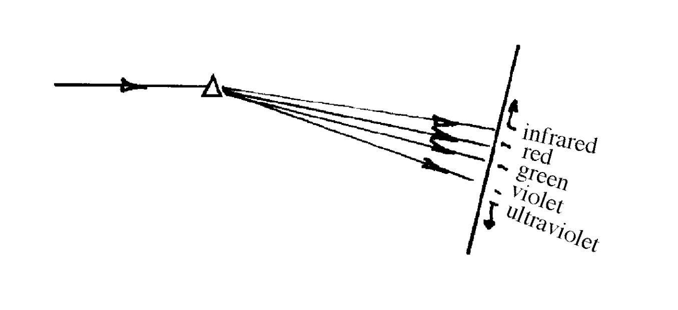


Fig. 14.2
White light is divided into its components by means of a glass prism.

The light of the Sun and the light of the lamp are made up of various kinds of light. These various kinds of light create different color perceptions in our eyes. When all types of light mix in our eyes, we see “white”.

The prism diverts these different types of light more or less strongly thus splitting the light. Red light is the least diverted followed by orange, yellow, green, and then blue. Violet light is the most strongly diverted.

The light that we can make out with our eyes is only a very small portion of all the kinds of light that occur in nature and that people can create with technology. There are many other kinds of light than visible light. We just have no sensory organ to detect them. All of these kinds of radiation, visible and invisible, are called “electromagnetic radiation”.

Even sunlight and lamplight contain invisible radiation. It is also diverted by the prism. It can be detected with special devices. One finds that there is “light” that is more strongly diverted than violet. This is called *ultraviolet radiation*. There is also “light” which is less diverted than red. It is called *infrared radiation.*

The temperature of a body determines what types of light it radiates and how much it radiates.

The hotter an object is (the higher its temperature) the more light it radiates. Only at a temperature of 0 K does it cease to radiate.

Moreover, the composition of the radiation shifts when the temperature of the radiating body is changed. The Sun’s surface has a temperature of around 5800 K. The light it radiates is mostly visible light. The filament of a light bulb has a temperature of about 3000 K. The fraction of infrared light in its radiation is larger than the visible one. If a body has a temperature of 1100 K (about 800°C), it is red hot. Only red remains of visible light, most of the light is infrared. Below 900 K (about 600°C), the object emits only infrared light.

The higher the temperature of a body, the more electromagnetic radiation it emits.

At the temperature found on the Sun’s surface (5800 K), most of the radiation is made up of visible light. The lower the temperature of the radiating body, the lower the portion of visible light and the greater the fraction of infrared light. Below 900 K it emits only infrared light.

14.3 Transport of entropy and energy with light

We return again to our body G which is cooling in a vacuum. We have stated that G emits visible or invisible light that is able to transverse the vacuum. The entropy emitted by G in cooling, must be carried away with the light.

We already know that entropy is an energy carrier. Whenever and wherever entropy flows, energy is also flowing. The cooling body emits entropy as well as energy with the light.

Light (visible and invisible) carries entropy and energy.

One might draw a wrong conclusion from the observations we just made. If every body radiates entropy as long as its temperature is greater than 0 K, it should continuously cool down until reaching 0 K (if it is in a vacuum). This does not happen. In fact, just the opposite does. If the temperature of body G is brought to a temperature less than the temperature of its surroundings and is then put into a vacuum (as in Fig. 14.1), it does not cool down but warms up.

It heats up although it is emitting entropy. How can this be explained? We have forgotten to take something into account. It is not only our object that radiates, but so do the objects in the surroundings. G emits entropy along with the radiation but it also absorbs entropy with the radiation emitted by the objects surrounding it. If G's temperature is higher than that of its surroundings, it emits more entropy into the surroundings than it receives, Fig. 14.3a. If its temperature is lower than that of the objects nearby, it receives more entropy than it emits, Fig. 14.3b. In both cases, the final state is the same: The temperatures equalize, reaching thermal equilibrium.

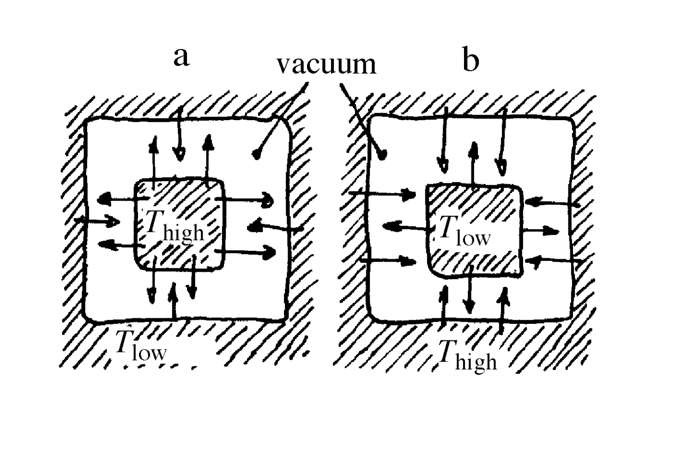


Fig. 14.3
Even in a vacuum, the bodies adjust to thermal equilibrium.

Even when transport of entropy occurs with electromagnetic radiation, the (net) entropy current flows from places of higher to places of lower temperature.

Exercise

A body K is put between two parallel walls A and B. These walls have different temperatures T_A and T_B , see Fig. 14.4. T_A is higher than T_B.

a) What can be said about the temperature that K reaches?

b) What can be said about the energy currents between the walls themselves and between the walls and K?

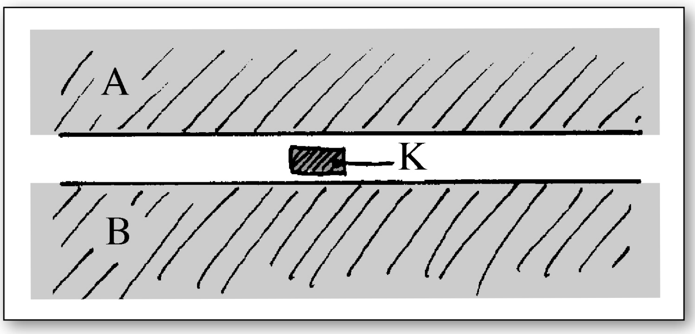


Fig. 14.4
For the exercise

14.4 The temperature of light

The light emitted by a body has the same temperature as the body itself. This means that the light coming from the surface of the Sun has the same temperature as the surface of the Sun, about 6000 K. This statement appears implausible at first because if sunlight has this temperature, wouldn't it immediately burn everything exposed to it? If it has this temperature, it should be measurable by putting a thermometer in the sunlight.

In order to solve this problem, we must look more closely into how to use a thermometer correctly. The object or substance we wish to measure the temperature of must be put in contact with the thermometer. If we put a thermometer in the solar radiation, the thermometer is in contact with the sunlight. The sunlight "touches" the thermometer, but the thermometer is also touched by other things.

First, there is the air. The air, as well, touches the thermometer. Whose temperature does the thermometer show? That of the air or that of the sunlight? The thermometer makes a compromise and shows a temperature that is neither that of the air nor that of the sunlight.

One can try to help the situation by putting the thermometer in a transparent vacuum container. The temperature it now shows is still far from the expected 6000 K. This is not surprising because we have forgotten something else. The thermometer again makes a compromise. The thermometer is in contact with not only the sunlight, but also with the infrared radiation of the environment. This radiation has the temperature of the surrounding area, about 300 K. While the Sun's radiation falling upon the thermometer comes from only a narrow direction, the 300 K light comes from almost all directions, Fig. 14.5. It is normal that in this case as well, the measurement greatly favors the ambient temperature.

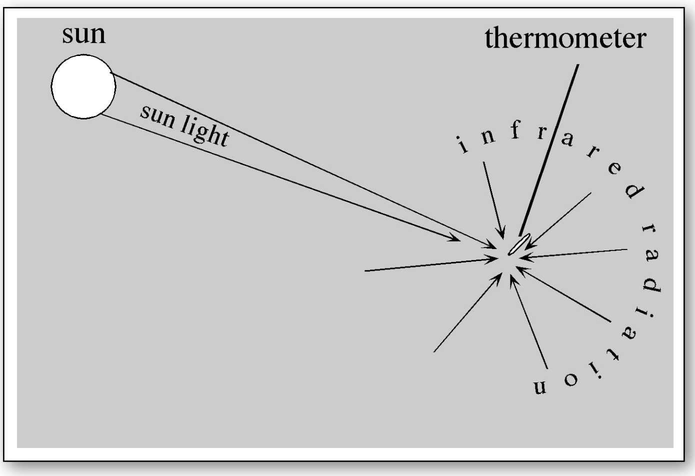


Fig. 14.5
The sunlight comes only from a narrow domain of directions. From all the other directions infrared light arrives.

How then can we measure the temperature of sunlight? One must make sure that the sunlight falling upon the thermometer does not only come from one narrow direction, but from every direction. This can be achieved with the help of lenses or mirrors, Fig. 14.6. If the Sun can be seen in every direction from the thermometer, it will show the temperature of the Sun. Naturally, our standard thermometers are useless for this.

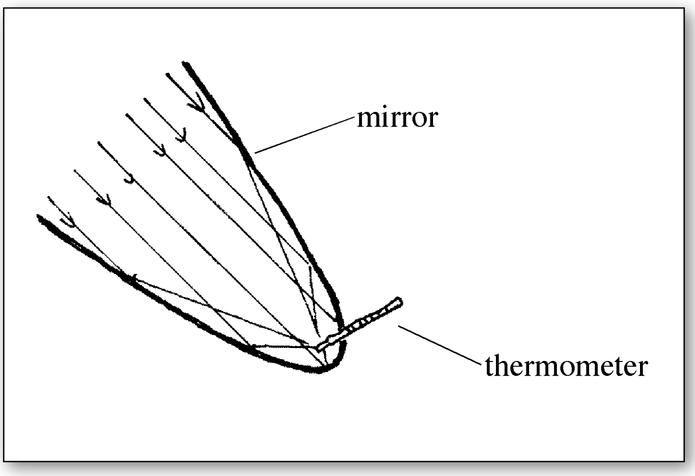


Fig. 14.6
The parabolic reflector makes sure that sunlight of all directions falls on the thermometer.

You know that very high temperatures can be generated with a lens or so-called burning glass by concentrating sunlight onto a small spot, possibly on a piece of wood. The process can be described as follows: One attempts to expose all sides of the wood to the light so that the wood takes upon the temperature of the light. Actually, the light coming from a burning glass does not reach the wood from all sides. The wood reaches a very high temperature, but not anywhere near that of the light.

14.5 Entropy and energy balances of the Earth

The Earth constantly receives entropy and energy with the light from the Sun.

The intensity of the energy current coming from the sun and falling upon one square meter of the Earth is an important number which is easy to remember. It is just about 1 kW. The *solar constant* is said to be 1 kW/m². The 1 square meter surface being considered must be perpendicular to the direction of the sunlight, Fig. 14.7. If this surface is at an angle, it will naturally receive less than 1 kW. This solar constant value only holds for a cloudless sky.

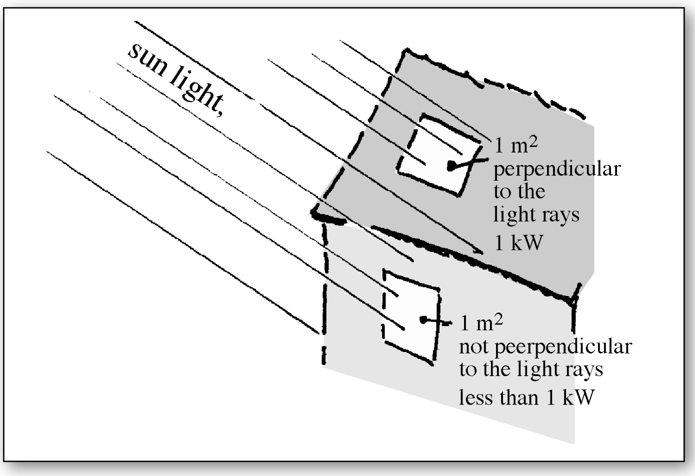


Fig. 14.7
An energy current of 1 kW hits a one square meter surface perpendicular to the direction of the sunlight.

Solar constant: 1 kW/m²

If the Earth did not emit any entropy or energy, it would continuously warm up, but it does not warm up. It is easy to see how the Earth keeps its temperature constant. The Earth's temperature is not 0 K, so it emits infrared light that carries entropy and energy away with it.

Sunlight reaches Earth from just one side, while the Earth itself radiates in all directions, Fig. 14.8.

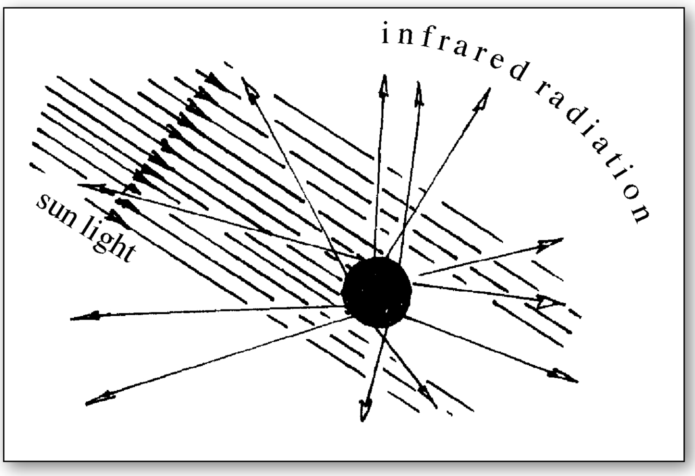


Fig. 14.8
The Earth gets sunlight from a very narrow domain of directions, and it radiates in all directions.

The Earth does not heat up or cool down so the energy current flowing out must be exactly as strong as the one flowing in:

$P_{out} = P_{in}$

It isn't as simple with entropy, however. A lot of entropy is created on Earth. This means that the light radiating from the Earth must carry more entropy than the sunlight falling upon it does. The infrared light being radiated into outer space must carry away not only the entropy coming from the Sun but the entropy created on Earth as well. In this way the amount of entropy on Earth remains constant:

$I_{S\,out} = I_{S\,in} + I_{S\,produced}$

The energy and entropy balances of Earth are described by the same equations that were used for the balances of the rod in Fig. 11.7 in section 11.3.

The Earth can also be compared to a heated house. The heater constantly delivers a certain energy current and a certain entropy current. The entire energy current flows out of the house again through leaks. Both the entropy created by the heater and the entropy created in the house and in the walls flow out with the energy current.

The fact that the entropy current flowing away from the Earth or house adjusts to a value which is constant over time, means that we have a steady-state.

14.6 The greenhouse effect

As we know, the atmosphere allows visible light to pass through. (If this was not the case, it would be dark both night and day). Infrared radiation has a much harder time getting through the atmosphere. Carbon dioxide (present in small amounts) is at fault for this. Carbon dioxide, in chemical symbols CO_2, is a kind of insulating material for infrared radiation. We will now investigate what would happen if the amount of CO_2 in the atmosphere were to rise for some reason.

The sunlight coming in would be untouched by this. The Earth would be heated by the Sun as before. However, heat loss would be lessened because the radiation cannot flow out as well as before, and the temperature would increase. Higher temperatures mean more radiation. The radiation would gradually increase for as long as needed to reach the old value, i.e., until the energy currents flowing in and out are again the same. The new "steady-state" would differ from the old one (before the CO_2 content was raised) in its temperature. The temperature would now be higher.

The higher the CO_2 content of the atmosphere, the higher the average temperature the Earth adjusts to.

To make this clear, we will once again compare the Earth to a heated house. If the insulation of the house is made better, but the house is heated like before, the house will reach a higher temperature. At this higher temperature, the energy current flowing out of the house through the heat leaks is exactly as strong as the energy current emitted by the radiators.

The CO_2 content of the atmosphere is about 0.03% (0.03% of air molecules are CO_2 molecules). Presently, the fraction of CO_2 is increasing greatly. This is a result of burning coal at power plants, from heating oil in central heating units of houses and from fuels (gasoline and diesel oil) for automobiles. Carbon dioxide is used up by plants. In the process they produce oxygen. This decomposition process by plants is currently decreasing because the tropical rain forests are being continually deforested. For these reasons, it can be expected that the Earth's temperature will increase in the next decades. Even when this increase is only a few degrees, it can have devastating consequences. It could happen that parts of the polar ice caps melt resulting in a higher sea level. The ocean would then flood large areas of land.

When the atmosphere lets sunlight through unhindered, but does not let the infrared radiation from the Earth through, we have what is called the *greenhouse effect*. The same thing happens in greenhouses. In this case, though, glass plays the role of the atmosphere. Glass also allows visible light to pass through, but not infrared radiation. It holds back the infrared radiation produced in the greenhouse, creating a higher temperature than the one it would have if the infrared radiation could pass through it.

Exercise

If the CO_2 content of the Earth changes, the Earth adjusts to a new temperature. We had a very similar situation in Chapter 4. The air resistance of a body was changed. As a result, the velocity of the body adjusted to a new velocity. What process was taking place? Compare both phenomena.