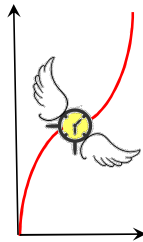


Synchronisierte Uhren, Selbstorganisation und Emergenz:

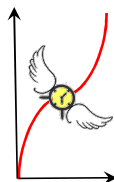


**Experimente und Modelle
universeller Phänomene**

**Harmon. Oszill. & Resonanz:
curricularer Standard
Synchronisationsphänomene: ??**



Manfred Euler
euler@ipn.uni-kiel.de



**We are all of us clocks whose
faces tell the passing years.**

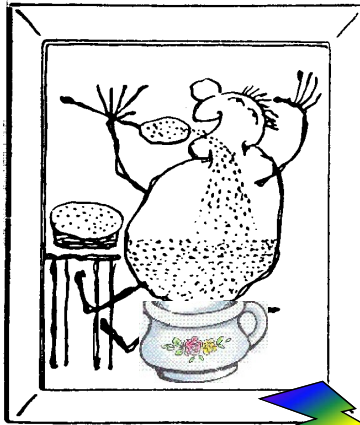
Eddington.



Experimente zur Uhrensynchronisation und universelle Modelle

- Selbstorganisation am Beispiel selbsterregter Oszillatoren
- Universelle Synchronisation: eine subjektive Auswahl von Experimenten
- Das Adler-(Kuramoto) Modell: Komplexitätsreduktion auf Phasenmodulation
- Synopse numerischer, analytischer, geometrischer und intuitiver Zugänge
- Synchronisation von de Broglies Uhren:
quasi-relativistische Dynamik in einem klassischen „Spielzeugmodell“
- Ausblick (Phil.-Kog.Psych.-Päd.) – Neues erschließen:
Kreativität und die Interaktion komplementärer Wissensformen
„Toy models as engines of intuition“

Das innere Walten der Natur



Wilhelm Busch
Maler Klecksel

Wie entgeht Lebewesen dem Tod (biologisch) ?

Gleichgewicht (physikalisch)?

Wesentliche Ergänzung:

Offenes System

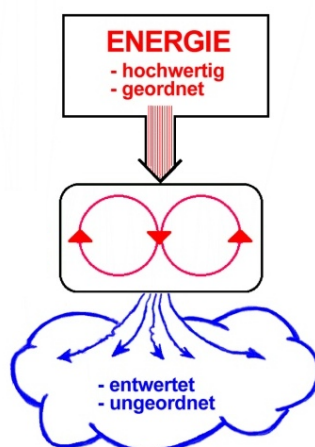
Im- & Export

(Materie, Energie, Entropie)

Dynamisches Gleichgewicht

Selbstorganisation

Komplexe Dynamik in offenen Systemen



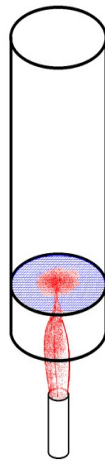
Emergenz neuer
Eigenschaften

Selbstorganisation
Leben
Wahrnehmen
Kognitive Prozesse



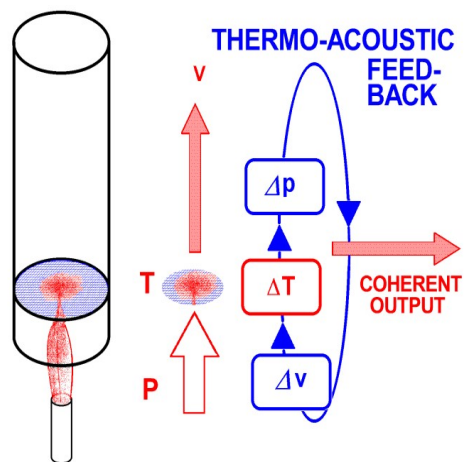
A thermo-acoustic model:

The sounds of self-organization

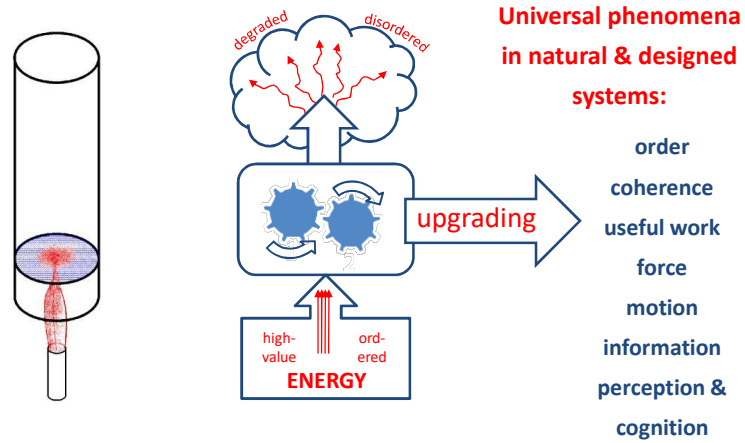


The sounds of self-organization

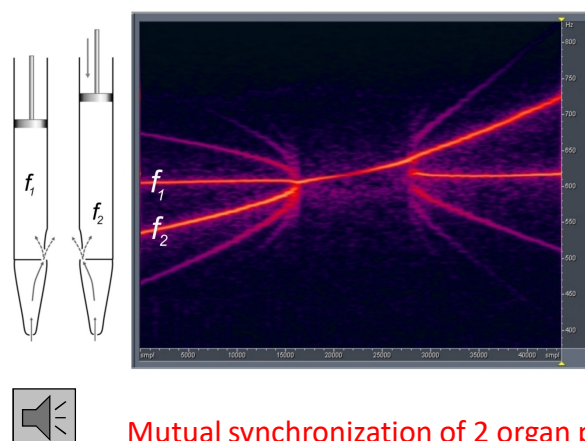
A thermo-acoustic laser analogue



Rijke-tube - a model for self-organization

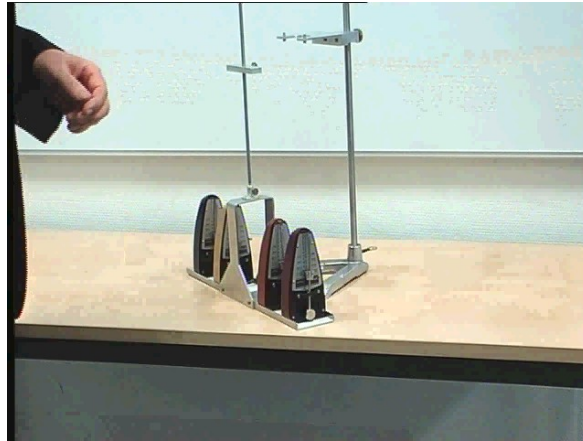


Coupled acoustic self-oscillations

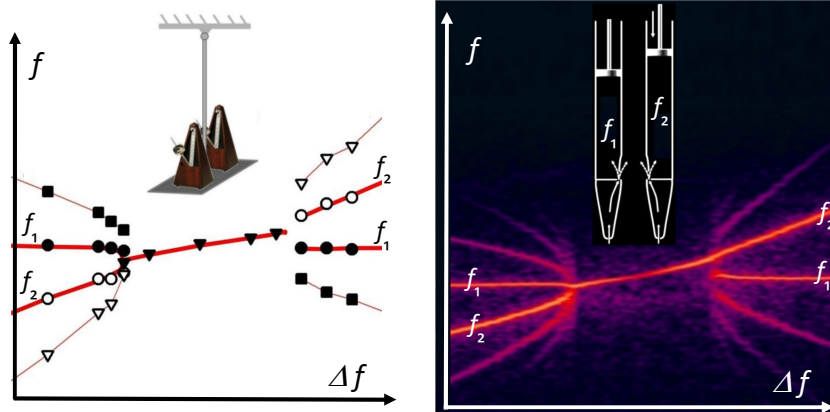


Mutual synchronization of 2 organ pipes

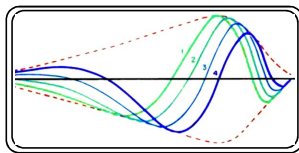
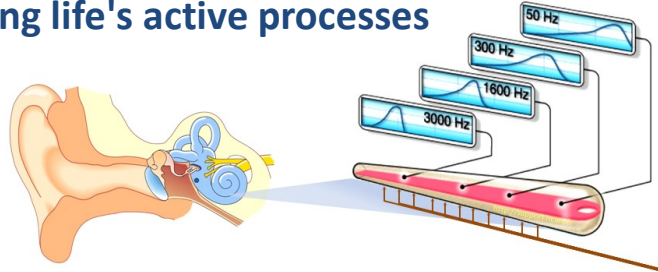
Synchronization of pendulum clocks (metronomes)



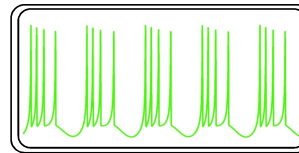
Universal bifurcation patterns (hard/soft) & coupling products (f_{comb})



Experiments in hearing: exploring life's active processes



Dual coding of sound signals



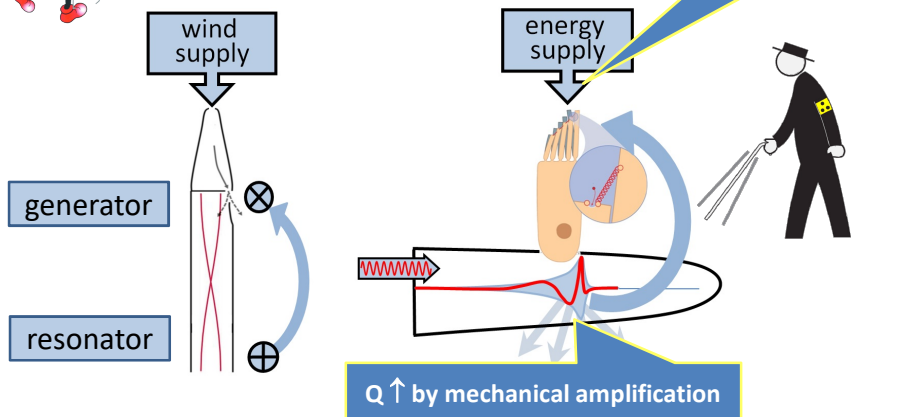
- Spectrum via travelling waves
- Amplification by feedback
- Place principle: $f \leftrightarrow x$

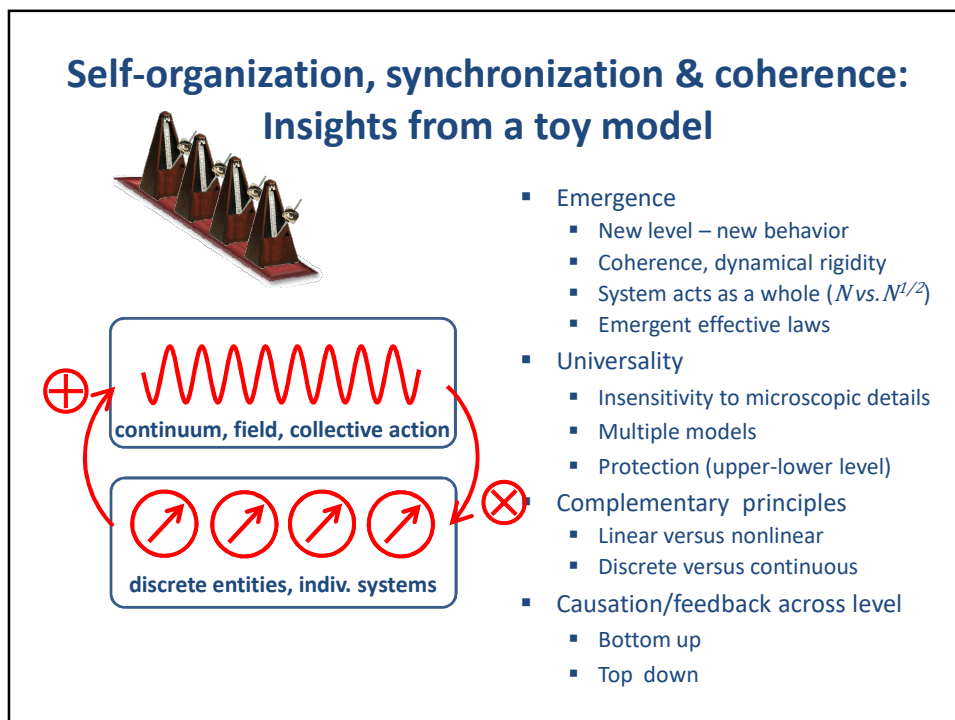
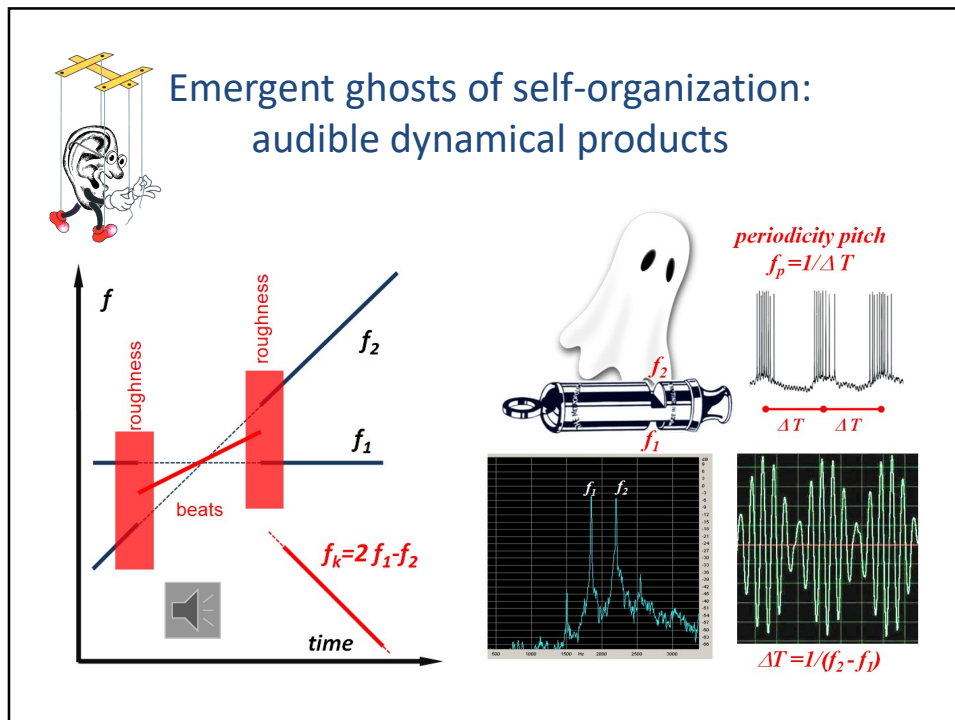
Both processes
include
synchronization

- Neural timing
- Rate & phase code
- Periodicity pitch: $f \leftrightarrow 1/\Delta t$



Self-organization & active hearing: organ pipe analogy



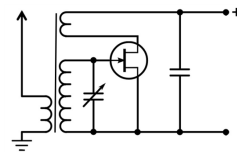


Theoretical approaches to Sync: Exercises in model order reduction

- Continuous systems:
nonlinear PDE



- Discrete systems:
nonlinear ODE (2nd order)



- Further approximations:
nonlinear ODE (1st order)
for angle



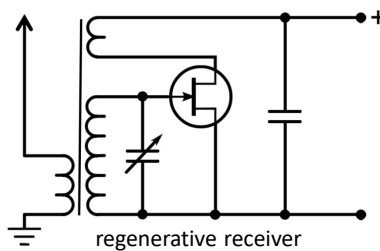
From resonance to sync: van der Pol oscillator as basic model



Overcoming friction: phase dependent energy input

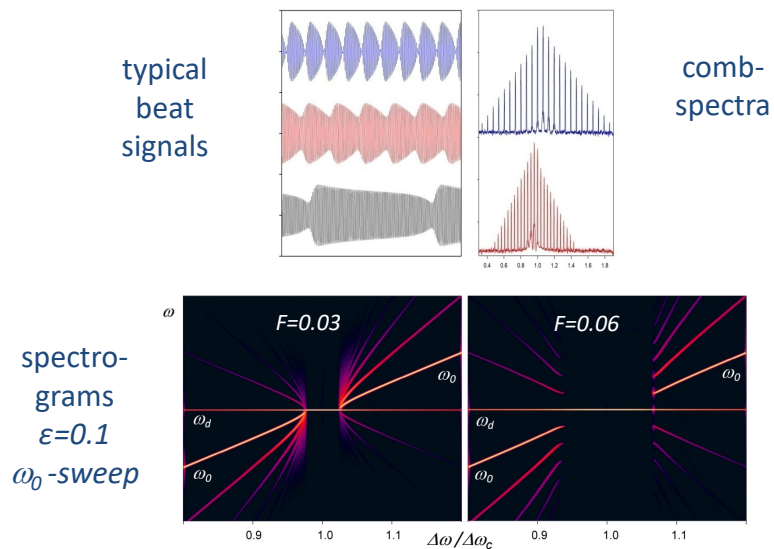
$$\ddot{x} + \varepsilon(x^2 - 1)\dot{x} + \omega_0^2 x = F \sin \omega_d t$$

Driven version first studied in radio engineering

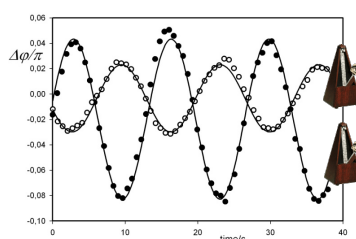


Technical:
regenerative audion receiver
Historical:
notorious propaganda machine

Driven VDP: numerical integration



Model order reduction: An intuitive guess of the Adler equation



$$\frac{d\phi}{dt} = \Delta\omega - B \sin \phi$$

- Symmetric coupling of 2 metronomes



- Faster clock is slowed down
- Slower clock is speeded up
- Periodic frequency pulling

- free running oscillators:
phase rate $d\phi/dt = \Delta\omega = \omega_0 - \omega$
- coupling adds periodic phase modulation term $\propto -\sin(\phi)$

Model order reduction: formal approaches

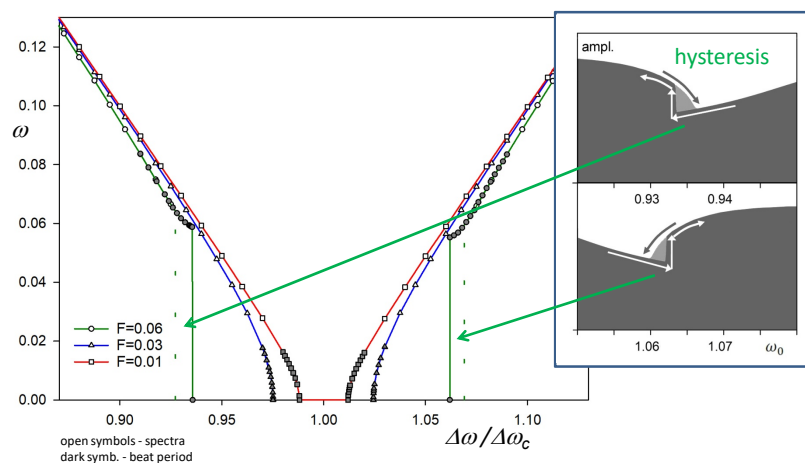
- Derivation for electronic oscillators: $\frac{d\varphi}{dt} = \Delta\omega - \Delta\omega_c \sin \varphi$
Adler (Proc. IRE 1946)
- Different time scales \Rightarrow averaging (Krylov, Bogoliubov)
discards fast oscillations keeps slow modulation terms:
$$x(t) = a(t) \sin(\omega t + \varphi(t))$$
- Reduced equations \Rightarrow coupled amplitude and phase modulation (ε & F small, $\omega_0 \cong \omega$, A =amplitude HO):

$$\frac{da}{dt} = \frac{\varepsilon A}{2} \left(1 - \frac{A^2}{4} \right) + \frac{F}{2\omega} \sin \omega t$$

$$\frac{d\varphi}{dt} = (\omega_0 - \omega) - \frac{F}{2A\omega} \cos \varphi$$

Complex behavior of driven VDP-oscillator: locking, suppression, hysteresis, chaos

Order in complexity: Adler equation fits in the weak driving limit **r** & **b**



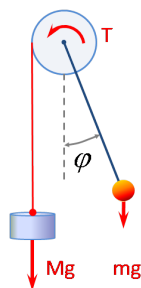
Making phase modulation tangible: A synopsis of different approaches/tools

- Numerical models
- Many possibilities and tools
- Example table calculation (Sigma Plot)
- Simple algorithm - straightforward iteration loop (VDP):

```

to=cell(2,1) ao=cell(2,2) vo=cell(2,3) so=cell(2,4)
tn=to+dt
an=-om0^2*sa+eps(1-sa^2)*va +F*cos(tn*omd)
vn=va+an*dt
sn=sa+va*dt
cell(2,1)=tn cell(2,2)=an cell(2,3)=vn cell(2,4)=sn
  
```

Making phase modulation tangible: Analogue mechanical models



- Asymmetric cylinder on inclined plane
- Pendulum driven by constant torque
- Both with strong viscous friction
- Neglect inertial effects

$$\cancel{I\ddot{\varphi}} + \beta\dot{\varphi} + mgl \sin \varphi = T$$



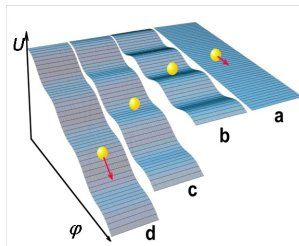
$$\dot{\varphi} = \Delta\omega - \Delta\omega_c \sin \varphi$$

Making phase modulation tangible: Energy landscape models



Washboard
model

Skiing moguls
(bumps)



- Motion of a 'phase particle' in potential
 $U(\varphi) = -\Delta\omega \cdot \varphi + \Delta\omega_c \cdot \sin(\varphi)$
- inclined plane with sinusoidal ripples
- Detuning $\Delta\omega \leftrightarrow$ inclination
- Coupling strength \leftrightarrow amplitude of ripples
- Phase rate in strong viscous friction:

$$\frac{d\varphi}{dt} = -\frac{dU}{d\varphi} = \Delta\omega - \Delta\omega_c \cos \varphi$$

Making phase modulation tangible: Analytical solution of Adler equation

- With $\gamma = \frac{\Delta\omega}{\Delta\omega_c}$ & $\Delta\omega_c = \omega_0$ & separation: $\int \frac{d\varphi}{\gamma - \cos \varphi} = \int \omega_0 dt$
- Integration somewhat tedious,
with Weierstrass substitution: $\tan \frac{\varphi}{2} = u, \quad d\varphi = \frac{2du}{(1+u^2)}, \quad \sin \varphi = \frac{2u}{1+u^2}, \quad \cos \varphi = \frac{1-u^2}{1+u^2}$

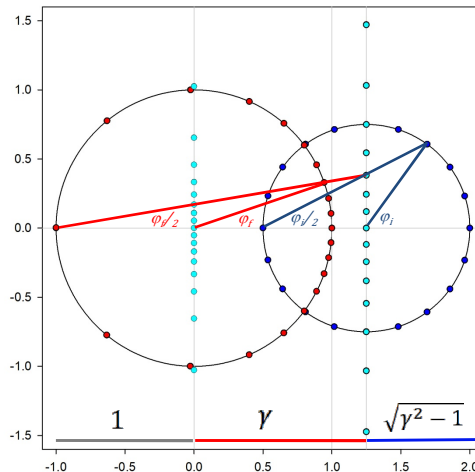
$$\tan \frac{\varphi}{2} = \frac{\gamma - 1}{\sqrt{\gamma^2 - 1}} \tan \left(\frac{\omega_0 \sqrt{\gamma^2 - 1}}{2} t \right)$$

$$\varphi = 2 \tan^{-1} \left(\frac{\gamma - 1}{\sqrt{\gamma^2 - 1}} \tan \left(\frac{\omega_0 \sqrt{\gamma^2 - 1}}{2} t \right) \right)$$

- $\tan(\varphi/2)$ substitution \Rightarrow stereographic projection of circle to a line
- The world's "sneakiest substitution" (Spivak) – helps to crack important integrals
- Sneaky math opens up rich physics from cartography to relativity
- Mapping of initial angle to final angle – re-parametrize time
- Dynamical geometry:
"tangible" visualization

$$\tan \left(\frac{\varphi_f}{2} \right) = \frac{\gamma - 1}{\sqrt{\gamma^2 - 1}} \tan \left(\frac{\varphi_i}{2} \right) = \frac{\sqrt{\gamma^2 - 1}}{\gamma + 1} \tan \left(\frac{\varphi_i}{2} \right)$$

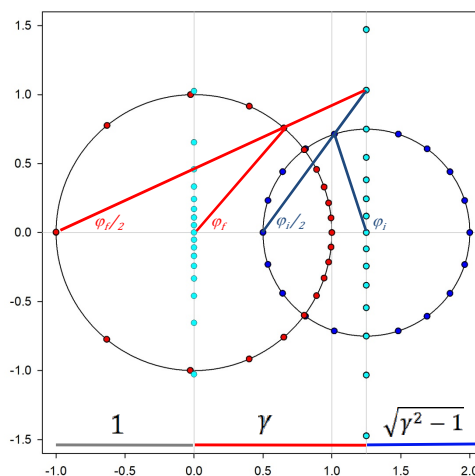
Making phase modulation tangible: A geometric representation of half angle trsf.



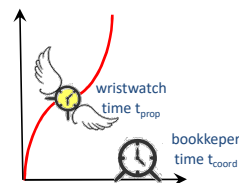
- the strange “cogs of sync”
- continuous gear, periodical
- reparametrize time – obtain phase rate via geometry
- 2 different phase rates

$$\tan\left(\frac{\varphi_f}{2}\right) = \frac{\sqrt{\gamma^2 - 1}}{\gamma + 1} \tan\left(\frac{\varphi_i}{2}\right)$$

Making phase modulation tangible: A geometric representation of half angle trsf.



- the strange “cogs of sync”
- continuous gear, periodical
- reparametrize time – obtain phase rate via geometry
- 2 different phase rates
- key to SRT models
- 2 clocks: $t_{prop} - t_{coord}$



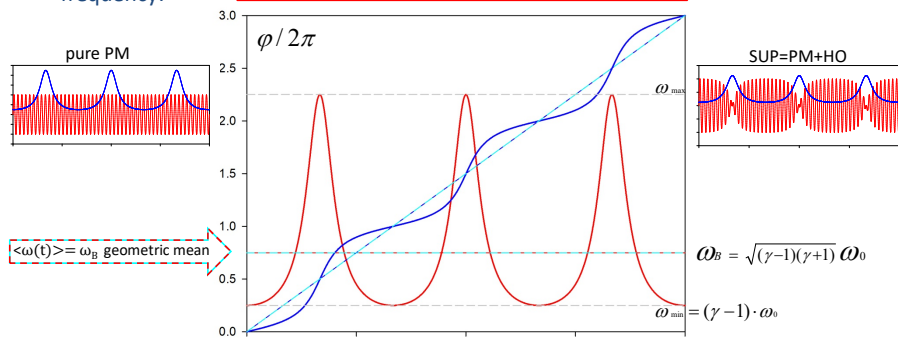
Phase modulation & instantaneous frequency

- Phase

$$\varphi = 2 \tan^{-1} \left(\frac{\gamma - 1}{\sqrt{\gamma^2 - 1}} \tan \left(\frac{\omega_0 \sqrt{\gamma^2 - 1}}{2} t \right) \right)$$

- Phase rate = instantaneous frequency:

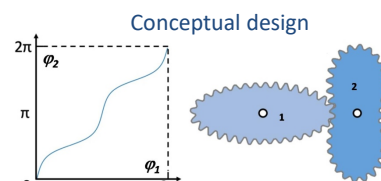
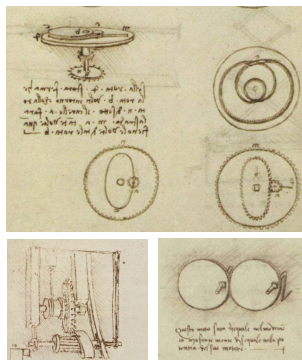
$$\dot{\varphi} = \frac{(\gamma^2 - 1)\omega_0}{\gamma + \cos(\sqrt{\gamma^2 - 1} \cdot \omega_0 t)} = \omega(t)$$



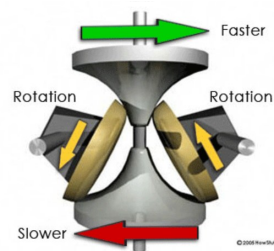
From Physics to Mechanical Technology:

A creative design exercise: CVT to simulate phase rate in sync

Basic ideas: Leonardo da Vinci

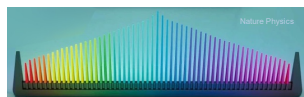
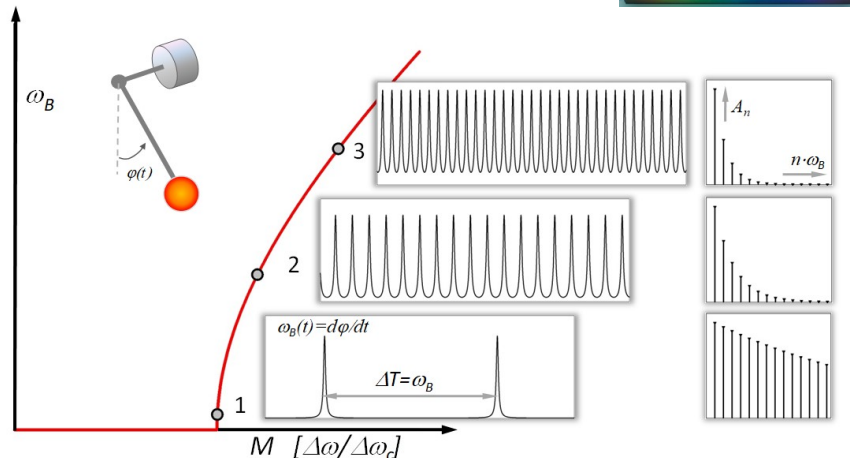


Applications



The creative leap from mechanical gears to gears & rulers of Quantum Technology

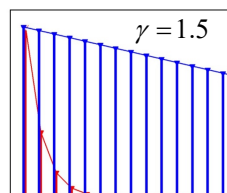
Beats & comb-spectra



Use 'relativistic' notation

$$\gamma = \frac{1}{\sqrt{1-\beta^2}}, \quad \beta\gamma = \sqrt{\gamma^2 - 1}$$

Fourier series expansion



$$R = \gamma - \beta\gamma$$

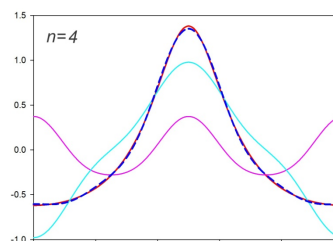
Comb spectra: a largely elementary analysis

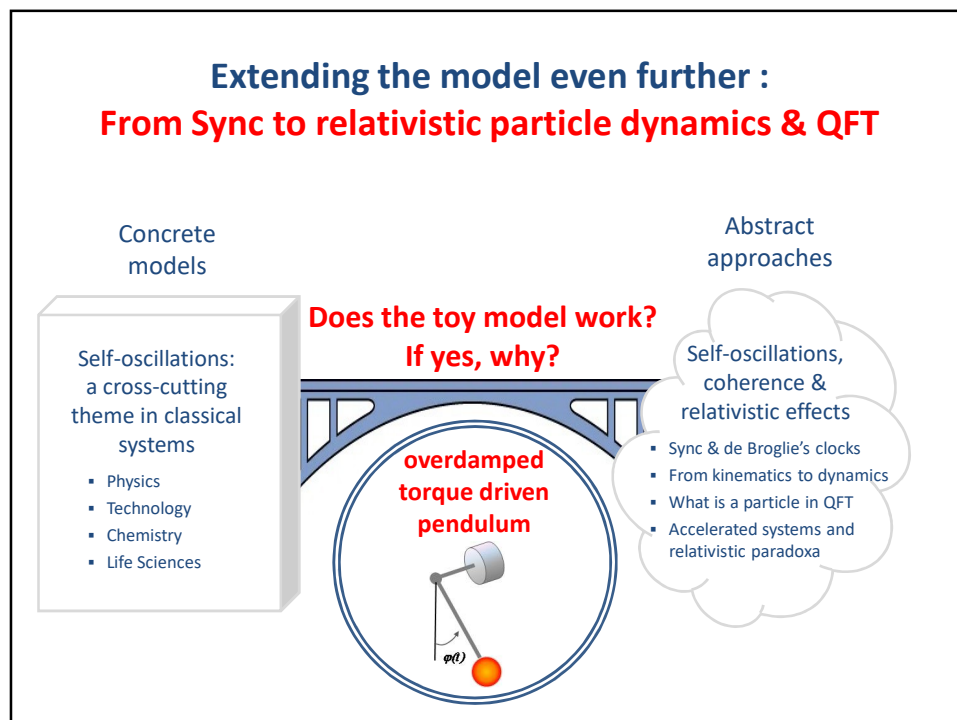
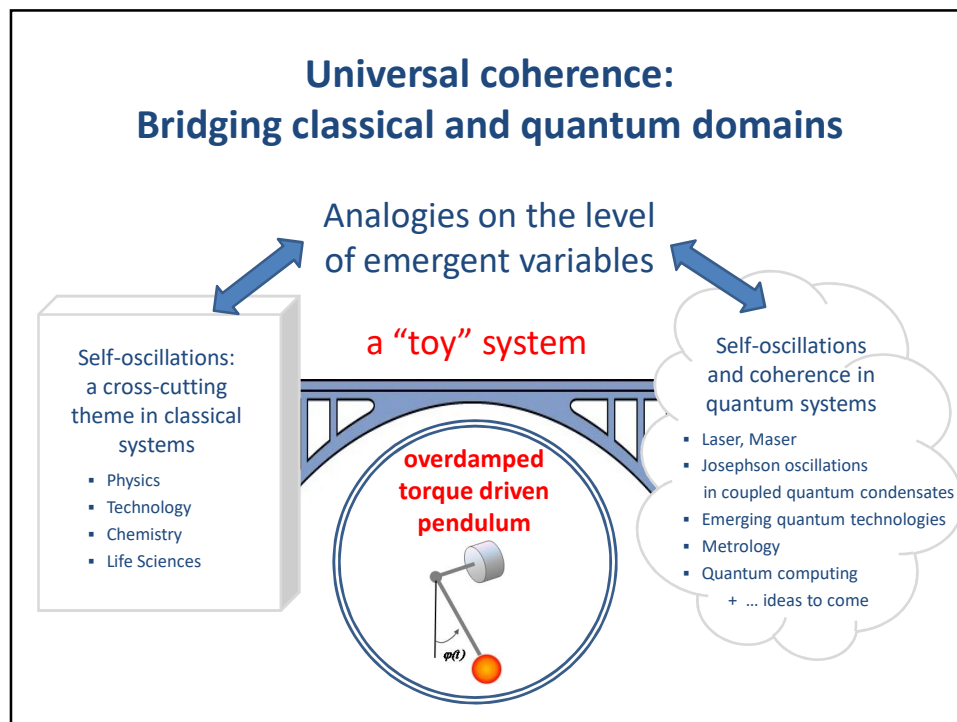
$$\dot{\phi} = \frac{(\beta\gamma)^2 \omega_0}{\gamma + \cos(\omega_B t)}, \quad \omega_B = \beta\gamma \omega_0$$

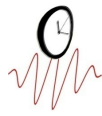
$$\dot{\phi} - \beta\gamma = \sum_n (-R)^n \cdot 2\beta\gamma \cdot \cos(n \cdot \omega_B t)$$

$$12) \int \frac{\cos x dx}{1+p \cos x} = \frac{\pi}{\sqrt{1-p^2}} \left\{ \frac{\sqrt{1-p^2}-1}{p} \right\}^x$$

NOUVELLES TABLES
D'INTEGRALES DEFINIES
D. BERENS DE HAAN





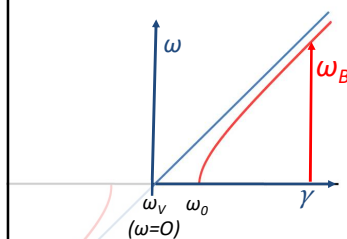


The cornerstone of QT: matter waves & moving clocks - de Broglie's basic ideas

- A (massive) particle is a clock: $E_0 = \hbar \omega_0$
- Beat of clock \Leftrightarrow particle mass: $\omega_0 = mc^2/\hbar$
- The puzzling result: different relativistic transforms of
 - Apparent “relativistic” mass increases $M = m \gamma$
 - Clock rate decreases, time dilation $\omega = \omega_0 / \gamma$
- Kinematic solution of puzzle:
matter waves with $\lambda_B = h/p$ in phase with particle clock (Diss. de Broglie)
- De Broglie’s “Law of phase harmony” \Leftrightarrow Lorentz invariance of phase
- Relativistic dispersion: $E = ((pc)^2 + (E_0)^2)^{1/2}$, $v_g = dE/dp$, $v_{ph} = p/E$, $v_g v_{ph} = c^2$
- Kinematics leaves open possible mechanisms
- Puzzle can be solved also by “real” dynamics: sync model

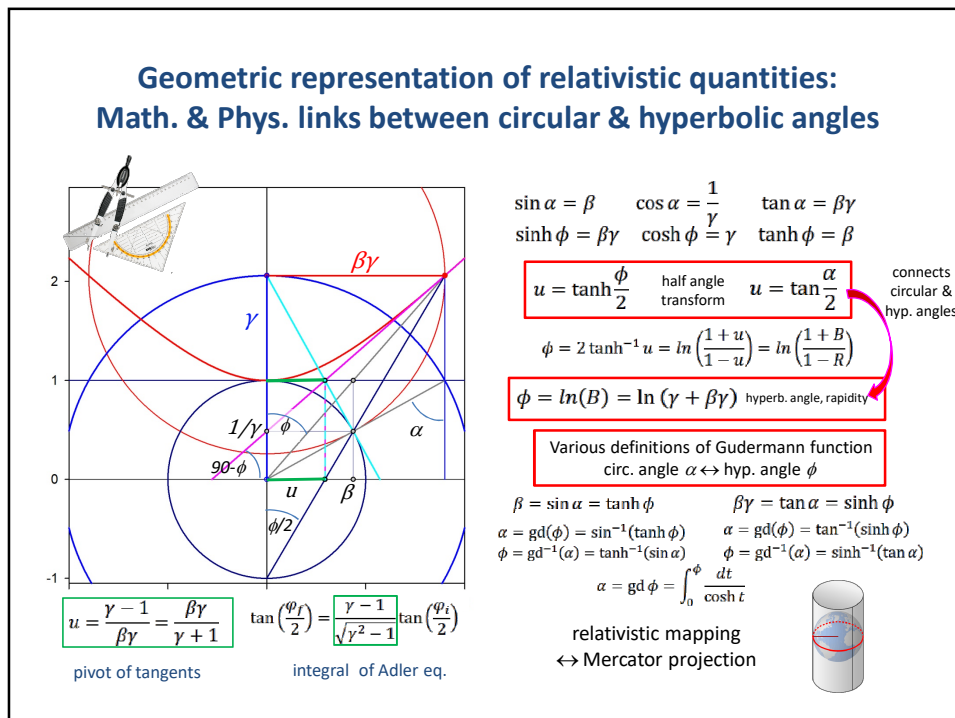
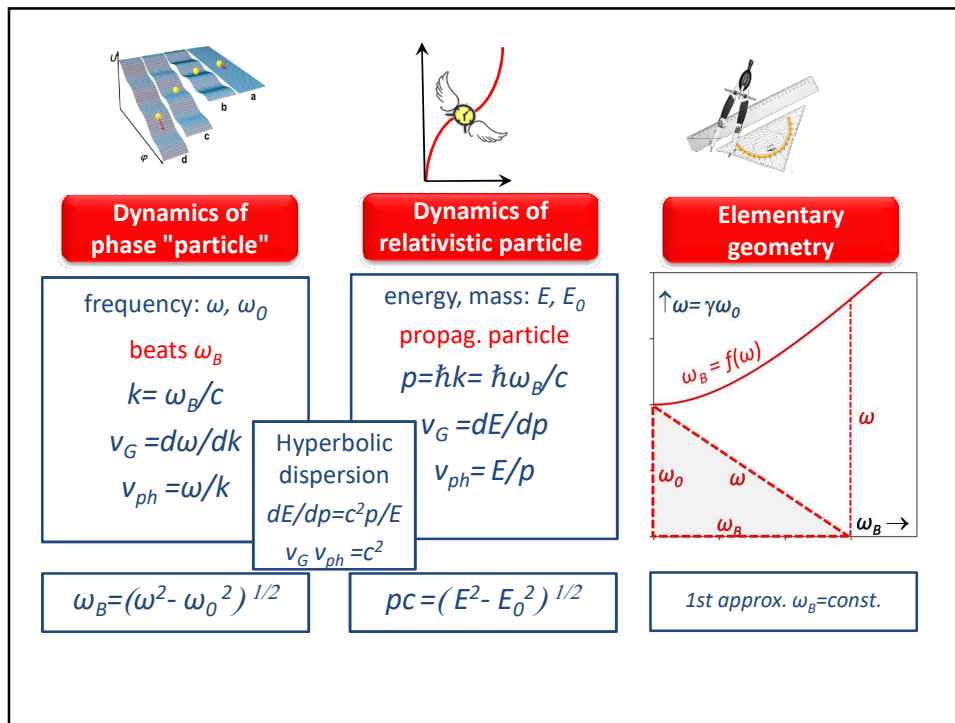


Synchronizing de Broglie's clocks - from kinematics to dynamics



Correspondence rules:
(de Broglie frequency –
not a common notion!)

- Sync provides a dynamical model
- Particle: clock running forever $\Rightarrow Q = \infty$
- Dynamical model: self sustaining oscillator (ω_0)
- $E = \hbar \omega_0$ energy threshold to create excitation
- Particle clock interacts with background field ω_v - resonates actively with vacuum
- Dynamics depends on boosted frequency ω
 - For $\omega < \omega_0$ ($\gamma < 1$) \Leftrightarrow no excited state
 - For $\omega > \omega_0$ ($\gamma > 1$) \Leftrightarrow excitation in motion, beats
 - Beat frequency: $\omega_B = (\omega^2 - \omega_0^2)^{1/2} = \beta \gamma \omega_0$
- Particle \Leftrightarrow excitation on a sea of energy (cf. Dirac)
- $E = \hbar \omega = \hbar \gamma \omega_0$
- $p = \hbar k = \gamma m v = \beta \gamma m c = \hbar \omega_B / c = h / \lambda_B$
- $\lambda_B / \lambda_0 = 1 / \beta \gamma$ $\omega_B / \omega_0 = \beta \gamma$



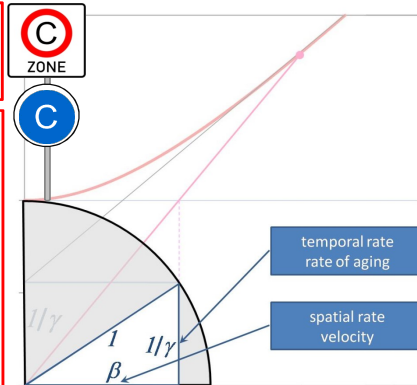
Sync - a model for dividing up total energy between:

- energy & momentum
- temporal periodicity & spatial periodicity
- rate of time (clock rate, rate of aging) & the rate of space (velocity)

Every object's spacetime rate is c :
 c is limiting rate
 as well as commanded rate

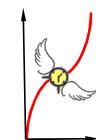
Beyond the simplicity of $\omega_b = \text{const}$:

- Sync & acceleration
- Opens up the Pandora box of relativistic paradoxa/puzzles
- Micro-version of twin travels
- Tools provide access to further puzzles in extended systems (Bell spaceships, Born rigidity)
- \Rightarrow window to GRT



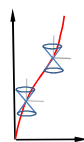
Conceptual refinements of sync-model:

Accelerating clocks



minimalist
particle model:

clock moving along
a parameterized
timelike curve



- Acceleration can be handled in SRT
- Use sequence of instantaneous frames co-moving with accel. particle
- Apply differential Lorentz transformation (Rindler)

$$dx' = \gamma(dx + v dt), \quad dt' = \gamma(dt + v dx/c^2)$$

$$dx = 0, c = 1 \rightarrow dx' = \beta \gamma dt, \quad dt' = \gamma dt$$



Interpretation of resulting relations in terms of one map – two clocks:

- dt – proper time, dt' – coordinate time
- Map distance covered in proper time $x(t) \propto \int \beta \gamma(t) dt$
- Elapsed coordinate time $t_{co} \propto \int \gamma(t) dt$
- Proper velocity (momentum/mass) $v_{prop} = \beta \gamma(t)$
- Unusual notion \Rightarrow common in rocket engineering and space flight
- v_{prop} not limited, α_{prop} links relativistic acceleration with N2
- Force felt by accelerated observer: $F = m \alpha_{prop}$

Summary of math & phys. meaning ($t_{pr}=\tau$, $t_{co}=t$, include c)

$$\begin{aligned} \vec{v}_{co} &= \frac{d\vec{x}}{dt} & \gamma &= \frac{dt}{d\tau} & \vec{a} &= \frac{d\vec{v}_{co}}{dt} & \text{coordinate acceleration} \\ \vec{v}_{pr} &= \frac{d\vec{x}}{d\tau} = \gamma \frac{d\vec{x}}{dt} = \frac{\vec{p}}{m} & \vec{a} &= \frac{d\vec{v}_{pr}}{d\tau} = \frac{d}{d\tau} \left(\frac{d\vec{x}}{d\tau} \right) & \text{proper acceleration } \alpha & \text{- mixed derivatives} \\ d\gamma &= \gamma^3 \beta d\beta & d(\beta\gamma) &= \gamma^3 d\beta & \vec{a} &= \gamma^3 \vec{a} & \text{prop. acc. } \alpha \text{ is absolute!} \end{aligned}$$

Meaning of proper time derivatives (1+1-dim) – 2-acceleration components:

$$\begin{aligned} \text{Temporal part} \quad \frac{d\gamma}{d\tau} &= \gamma^3 \beta \frac{d\beta}{d\tau} \propto A^0 & \text{Spatial part} \quad \frac{d(\beta\gamma)}{d\tau} &= \gamma^3 \frac{d\beta}{d\tau} \propto A^1 \\ |\alpha| &= \sqrt{-(A^0)^2 + (A^1)^2} \propto \gamma^2 \frac{d\beta}{d\tau} & \text{Components in} & \text{comoving frame} & (0, |\alpha|) \end{aligned}$$

REM:
Motion changes
measure of time
Acceler. changes
rate of time

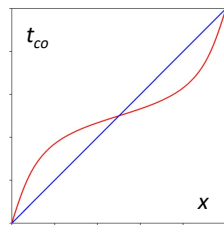
Sync: a conceptual playground to study accelerated systems in SR

- A model, how to divide up energy between spatial & temporal periodicity
- Provides us with tangible tools to explore and to acquaint with abstract concepts
- Represents an intuitive access towards more challenging notions - e.g. accelerations
- Presents a prelude to some phenomena of GRT

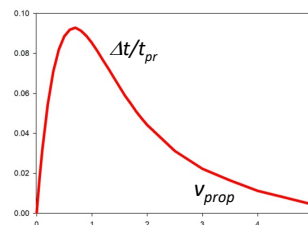


Sync-Model as mechanical “Spacetime Engine”

- Proper velocity – normalized phase rate ($c=1$) $\frac{\dot{\phi}}{2\pi} = \frac{(\beta\gamma)^2}{\gamma + \cos(\omega_B t)} = \beta\gamma(t)$ this is t_{pr} !!!
- Covered distance $x(t) = \int \beta\gamma(t) dt$
- Coordinate time: no direct correspondence in model, must be calculated $t_{co} = \int \gamma(t) dt$



- No uniform motion, $v \neq \text{const.}$
- Jerks on the microscale
- Explore conceptual challenge - different concepts of acceleration : coordinate-proper-temporal-spatial



- Different proper times
- Different ‘rates of aging’
- Micro-version of twin paradox



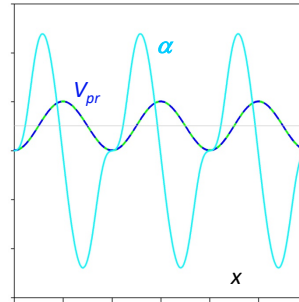
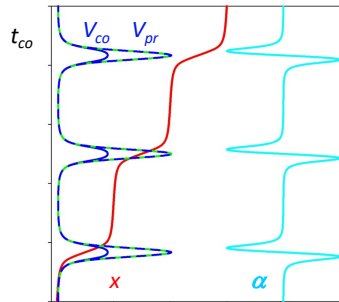
Particle in QFT: never at rest - temporal & spatial modulation

Worldline $x(t_{co})$, $\omega_0 = 2\pi$, $c=1$

- Proper velocity $v_{pr} = \beta\gamma$
- Coordinate velocity $v_{co} = \beta$
- Proper acceleration $\alpha = \frac{dv_{pr}}{dt_{co}}$

Compare with spatial dependence of

- Proper velocity $v_{pr}(x) = \gamma - \cos(2\pi x)$
- harmonic washboard
- Proper acceleration $\alpha = \frac{d\gamma}{dx}$



Spatial modulation of momentum: lattice model as an intuitive picture

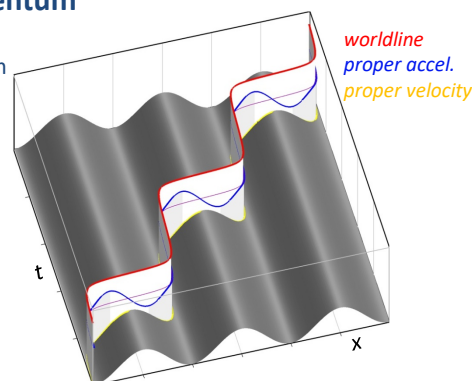
Sync model and quasi-relativity in systems with spatial periodicity of momentum

Lattice models – periodical quasi-momentum

- Historical note:
Heisenberg's lattice world (1930) - discarded because fixed lattice ('vacuum') is not Lorentz invariant
- Sync: emergent modulation of p & α
- Spatial & temporal modulation maintain Lorentz-invariance

Modern versions:

- Quasi-relativity in narrow band semiconductors
- 2 different concepts of effective mass
- Dirac cones in graphene
- Photonic lattices, metamaterials with engineered periodicity



momentum related ($v_{eff} = \text{const.}$) $\vec{p} = \gamma m \vec{v}$
acceleration related $\vec{F} = m \vec{a} = \gamma^3 m \vec{a}$

Why does model work? Does it represent (which?) elements of reality?

Unable to answer comprehensively – same class of ultimate questions as
“What is a particle?”
 Sync resonates with approaches from different domains & theoretical backgrounds

How massless entities become massive (Confinement-models)

- Transversal waves in elastic string \Rightarrow become massive by adding transversal springs
- Waveguide models
- Wave propagation in cochlea
- Photons confined in a resonator

Accelerated charged particles should radiate (the perennial problem of radiation reaction)

- Nonradiating states
- Near field effects (emission & absorption in near field, nothing emitted into far field)
- Virtual particle creation and annihilation

Acceleration and the vacuum

- Unruh-effect
- Particle creation in vacuum (particle number is not conserved)
- Radiation from accelerating mirrors – parametric oscillator models

Just for fun: Mass without mass a (classical) taste of Higgs

- Synchronization creates spectrum with a gap
- Emergence of rest energy $E_0 = \hbar \omega_c$
& invariant rest mass analog: $m = E_0/c^2$
- Background field gives mass to massless particles: $\Rightarrow v < c$
- Classical model provides an intuition of the Higgs mechanism
- Upgrades the popular "crowd" or the "molasses" metaphor

Matter waves are fully relativistic:
 Implications for teaching in view of prevailing nonrelativistic introductions?



Celebrity propagates in a clustering crowd



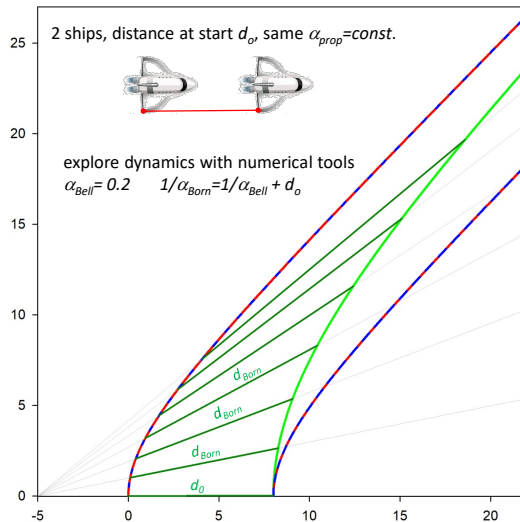
Beats from excited states, resonating with vacuum fields

Sync model opens up a new perspective on "the cornerstone" of quantum physics: matter waves as an emergent **relativistic** phenomenon

How to teach special relativity (1976):

Bell's spaceships as a plea for a dynamical approach

A quiz at CERN cafeteria: Does a thin thread between the spaceships break? Explain!



Both ships in hyperbolic motion.

- Majority vote: no break
- Bell's conclusion: It breaks!
- Correct. ☺
- His explanation- because of "real" Lorentz contraction?

— Simultaneity lines

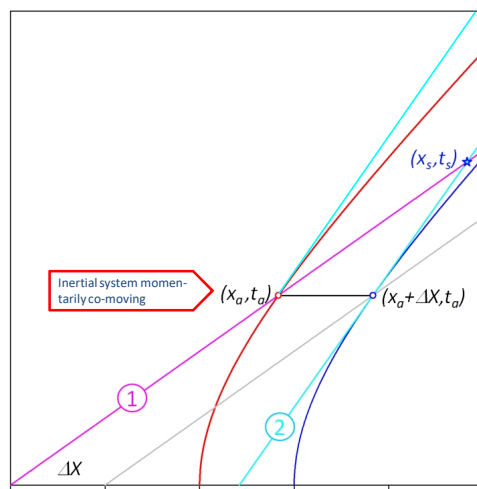
- Reference to Born-rigid motion
- Born-rigidity condition $\alpha = 1/d_{\text{vertex}}$
- Invariant proper distance d_{Born}
- A less philosophically loaded explanation ☺:

It breaks, because $d_{Bell} > d_{Born}$

- the proper length increase is 'real'
 (requires further theor. assumptions)

Algorithm for calculating 'proper length' in accel. systems :

Clarifies the instantaneous & local character of d_{prop}



$$\textcircled{1} \quad \frac{c(t - t_a)}{x - x_a} = \beta_a \quad \text{iterate}$$

$$\textcircled{2} \quad \frac{c(t - t_a)}{x - (x_a + \Delta x)} = \frac{1}{\beta_a}$$

intersect at (x_s, t_s)

$$x_s = x_a + \frac{1}{1 - \beta_a^2} \Delta x$$

$$t_s = t_a + \frac{\beta_a}{1 - \beta_a^2} \Delta x / c$$

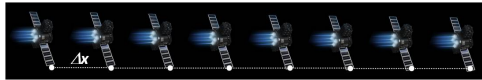
proper distance increment

$$\Delta d = \sqrt{(x_s - x_a)^2 - c^2(t_s - t_a)^2}$$

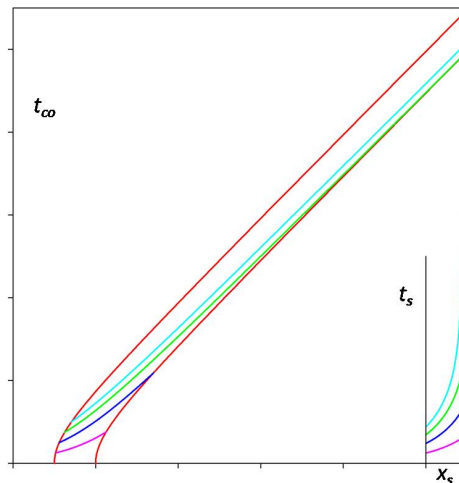
$$\Delta d = \frac{\sqrt{1 - \beta_a^2}}{1 - \beta_a^2} \Delta s = \gamma_a \Delta s$$

sum of increments Δd is an appropriate measure of L_{prop}

Extending the system and refining proper distance:



The accelerating space fleet



Proper distance :

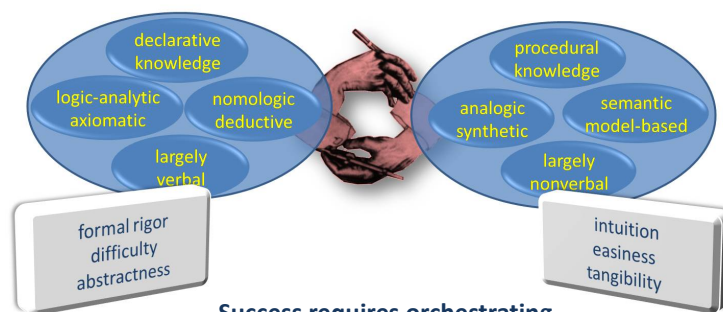
- Local concept in accelerated systems
- Clock rates differ between bow and stern
- Implications for communication?
- Depending on acceleration & time the bow becomes invisible from the stern
- A horizon emerges
- In SR, the existence of horizons in accelerated systems presents a glimpse of GRT



Outlook:

a highly personal account of guiding ideas from philosophy, cognitive science & pedagogy

The challenge of **teaching, learning & doing** physics



Success requires orchestrating
an intelligent interplay of complementary forms of
knowing

