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Is an energy current energy in motion?

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Abstract It is an old question whether an energy current can be imagined as energy moving with a well defined velocity. It is shown that in two important systems, namely in the electromagnetic field and in moving matter under stress, the energy current can be decomposed into two parts of opposite directions. Each part can be imagined as energy moving with the velocity of light or with the velocity of sound, respectively.

Zusammenfassung Es wird die Frage behandelt, ob man sich einen Energiestrom vorstellen darf als mit einer wohldefinierten Geschwindigkeit bewegte Energie. Es wird gezeigt, daß der Energiestrom in zwei wichtigen Systemen, nämlich im elektromagnetischen Feld und in bewegter, unter mechanischer Spannung stehender Materie, in zwei Teilströme entgegengesetzter Richtung zerlegt werden kann. Jeden Teilstrom darf man sich vorstellen als Energie, die sich mit Licht- bzw. mit Schallgeschwindigkeit bewegt.

1. Introduction

Physicists call some physical quantities currents or flows, and other quantities, which are related to the former, current densities. However, there is no unambiguous rule which allows us to say if a given quantity is a current or not. In some cases the designation 'current' seems to be near at hand, e.g. for a mass current or an electric current. In other cases the word is used with some reservation, e.g. for energy and entropy currents. Finally, Maxwell's true current (Maxwell 1954) or Hertz's magnetic current (Hertz 1884, Herrmann 1986) which are always divergence-free never established themselves successfully.

One might think that a quantity would be easily accepted as a current whenever it appears in a relation which can be interpreted as a continuity equation. However, Newton's second law

$$\boldsymbol{F} = \mathrm{d}\boldsymbol{p}/\mathrm{d}t$$

shows that this hypothesis is false. If the force $\boldsymbol{F}$ is identified with a momentum current this equation appears to be a continuity equation for the momentum (Weyl 1977, Herrmann and Schmid 1984, 1985). However, this interpretation of Newton's law is not common.

We conclude that there must be another reason for our readiness to interpret a quantity as a current. We suggest this reason to be the following. It must be possible to describe the process under consideration as a 'kinematic displacement of a physical quantity'. By that, we mean that a relation

$$\boldsymbol{j}_X = \rho_X \boldsymbol{v} \tag{1}$$

must exist, where ρ_X is the density of the extensive quantity X, $\boldsymbol{j}_X$ the current density of the current of X and $\boldsymbol{v}$ a velocity. This is the case, for instance, for the flow of a fluid where X is the mass. ρ_X and $\boldsymbol{v}$ are quantities which are not *defined* by (1). Both of them can be measured independently. In particular, $\boldsymbol{v}$ is the velocity of a reference frame in which $\boldsymbol{j}_X$ is locally zero.

However, things are rarely as simple as in the above example. Although the electric current in a wire is often described by a relation of type (1), by the charge density ρ_Q one does not mean the total density of the charge, but only that part of it which belongs to the 'free charge carriers'. Thus, the whole charge density is decomposed in a free part ρ_Q and a rest part ρ'_Q, which is not free. However, in any reference frame in which the wire is not at rest, the charge carriers which are not free are moving, and (1) has to be replaced by a relation with two terms:

$$\boldsymbol{j}_Q = \rho_Q \boldsymbol{v} + \rho'_Q \boldsymbol{v}'.$$

Consequently, in general one must admit that the total

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current density consists of several terms

$$\boldsymbol{j}_X = \sum_i{}' \rho_{Xi} \boldsymbol{v}_i, \quad \text{where} \quad \sum_i{}' \rho_{Xi} = \rho_X. \tag{2}$$

Notice that such a decomposition is always possible, and even in infinitely many different ways. However, it is reasonable only if there is a procedure which allows the velocities $\boldsymbol{v}_i$ to be determined independently one by one. In the example of an electric current, by the way, one can not only distinguish between free and bound charge carriers. One can even distinguish different kinds of free charges: those carried by electrons and those carried by defect electrons.

To summarise, it can be said that a description of type (2) may not exist—but if it exists the idea of a current seems to be particularly suggestive.

Hertz (1894) raised the question of whether the energy current in a drive belt and in an electromagnetic field could be conceived as a movement of energy. This question was also discussed by Mie (1898). Both authors left the question unanswered.

In the present paper we will show that in the two systems referred to by Hertz, i.e. the electromagnetic field and the sound field, an energy current can be represented in the form of equation (2)—and this with only two terms. It will be seen that every electric energy transport can be represented by two electromagnetic waves travelling in opposite directions and every mechanical energy transport, e.g. in a moving bar under stress, can be represented by two sound waves travelling in opposite directions. Thus, in both cases it is possible to imagine the energy current as 'moving energy'. The energy is moving with the velocity of light or with the velocity of sound, respectively.

It will become apparent, that there is an analogy between these cases. The electric field $\boldsymbol{E}$ and the magnetic field $\boldsymbol{H}$ correspond to the mechanical strain ε and the velocity $\beta = v/c_S$ (in units of the velocity of sound c_S).

In § 2 we treat the electromagnetic field. Section 3, which deals with energy transport in stressed bars, is organised in analogy with § 2. In § 4 special cases of mechanical energy transmission are discussed.

2. The electromagnetic field

We restrict our discussion to the case in which the electric field vector is perpendicular to the magnetic field vector. Thereby the mathematics will be easier and the analogy to the mechanical case in § 3 becomes more apparent. The extension to fields for which $\boldsymbol{E}$ is not perpendicular to $\boldsymbol{H}$ is simple but not of interest for our purpose.

We choose the x axis to be in the direction of the $\boldsymbol{E}$ field and the y axis in the direction of the $\boldsymbol{H}$ field. Thus, we have $\boldsymbol{E} = (E, 0, 0)$ and $\boldsymbol{H} = (0, H, 0)$. E and H can independently admit any value. If $E = 0$ and $H \neq 0$ we have a pure magnetic field and if $H = 0$ and $E \neq 0$ the field is purely electric.

2.1. Energy density and energy current density for arbitrary values of the field strengths

The energy density ρ_W in the point under consideration is

$$\rho_W = \tfrac{1}{2}\varepsilon_0 E^2 + \tfrac{1}{2}\mu_0 H^2. \tag{3}$$

The energy current density vector $\boldsymbol{j}_W = (0, 0, j_W)$ points in the z direction. Its z component j_W is:

$$j_W = E \cdot H. \tag{4}$$

We can write down a relation between j_W and ρ_W which is of the form of equation (1):

$$j_W = \rho_W v.$$

However, the factor v has no other meaning than to relate j_W and ρ_W. It does not represent a velocity which can be measured independently of the field strengths.

2.2. Energy density and energy current density for electromagnetic waves

In the following we have to consider particular solutions of the wave equation. As was mentioned above we consider fields for which only the x component of the electric field and the y component of the magnetic are different from zero. Moreover, we restrict ourselves to the case in which the values of both E and H are independent of x and y, i.e. to those solutions of the wave equation which correspond to perturbations which propagate in the positive or negative z direction.

In this case, the wave equations of the electric and magnetic field strengths are:

$$\frac{\partial^2 E(z,t)}{\partial z^2} - \frac{1}{c_L^2}\frac{\partial^2 E(z,t)}{\partial t^2} = 0$$

$$\frac{\partial^2 H(z,t)}{\partial z^2} - \frac{1}{c_L^2}\frac{\partial^2 H(z,t)}{\partial t^2} = 0.$$

Here,

$$c_L = (\varepsilon_0 \mu_0)^{-1/2} \tag{5}$$

is the velocity of light.

The general solutions of these wave equations are

$$E(z,t) = E_1(z - c_L t) + E_2(z + c_L t)$$

$$H(z,t) = H_1(z - c_L t) + H_2(z + c_L t).$$

The magnetic fields H_1 and H_2 are related to the electric fields E_1 and E_2 according to

$$\varepsilon_0^{1/2} E_1 = \mu_0^{1/2} H_1 \tag{6a}$$

and

$$\varepsilon_0^{1/2} E_2 = -\mu_0^{1/2} H_2. \tag{6b}$$

The field strengths E_1 and H_1 belong to a perturbation propagating in the positive z direction, E_2 and H_2 correspond to a perturbation which propagates in the negative z direction.

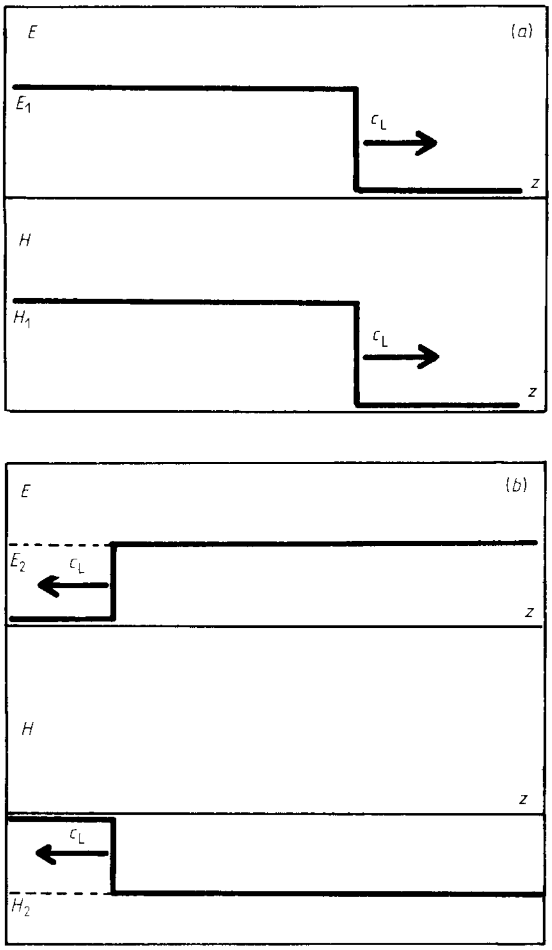


Figure 1 A step-like perturbation is propagating with the velocity of light c_L in: (*a*) the positive, and (*b*) the negative z direction.

As the 'pure' perturbations, marked by indices 1 and 2, play a particular role in what follows, we shall attribute to them a name of their own: we call each of them an electromagnetic wave. Thus, the word 'electromagnetic wave' is not used for *any* solution of the wave equation, but only for those for which only one of the equations (6*a*) and (6*b*) is valid.

Although our discussion is concerned with the field at a single location and at a single instant of time, it may be helpful to imagine an extended field which is homogeneous in space and constant in time. A field for which one of the relations (6*a*) or (6*b*) is valid and which is homogeneous and constant in time can be thought to come about when a step-like perturbation travels in the positive or negative z direction respectively, see figure 1. Notice that the fields are called waves, even though the field strengths are constant in time after the perturbation has passed.

Let us now determine the energy density and the energy current density for each of the two waves 1 and 2. With (6*a*) and (6*b*) one obtains from (3)

$$\rho_{W1} = \varepsilon_0 E_1^2 \qquad (7a)$$

$$\rho_{W2} = \varepsilon_0 E_2^2 \qquad (7b)$$

and from (4)

$$j_{W1} = (\varepsilon_0/\mu_0)^{1/2} E_1^2 \qquad (8a)$$

$$j_{W2} = -(\varepsilon_0/\mu_0)^{1/2} E_2^2. \qquad (8b)$$

With (5) one gets finally

$$j_{W1} = \rho_{W1} c_L$$

$$j_{W2} = -\rho_{W2} c_L.$$

Thus, the energy current densities belonging to the partial solutions 1 and 2 can be represented in the form of equation (1). Therefore, in the particular case that only one of the two solutions is non-zero, the energy flow can be imagined as energy travelling with the velocity of light in the positive or negative z direction, respectively.

2.3. Decomposition of an arbitrary electromagnetic field into two electromagnetic waves

Let us come back to a field with arbitrary values of E and H, i.e. values which are not restricted to agree with (6*a*) or (6*b*). Now, the field strengths E and H can be represented as the sum of the field strengths of two electromagnetic waves, i.e. of one field for which (6*a*) holds and one for which (6*b*) holds:

$$E = E_1 + E_2 \qquad (9a)$$

$$H = H_1 + H_2 = (\varepsilon_0/\mu_0)^{1/2}(E_1 - E_2). \qquad (9b)$$

The field strengths E_1 and E_2 are obtained by solving (9*a*) and (9*b*):

$$E_1 = \tfrac{1}{2}[E + (\mu_0/\varepsilon_0)^{1/2} H]$$

$$E_2 = \tfrac{1}{2}[E - (\mu_0/\varepsilon_0)^{1/2} H].$$

We will show now that, besides the electric and magnetic field strengths, the energy density and the energy current density superpose linearly.

The energy density of the resulting field follows from (3) by means of (9*a*) and (9*b*):

$$\rho_W = \tfrac{1}{2}[\varepsilon_0(E_1 + E_2)^2 + \varepsilon_0(E_1 - E_2)^2] = \varepsilon_0 E_1^2 + \varepsilon_0 E_2^2.$$

It is seen that ρ_W equals the sum of the energy densities of the partial fields (see (7*a*) and (7*b*)).

The energy current density is obtained from (4) by means of (9*a*) and (9*b*):

$$\begin{aligned} j_W &= (E_1 + E_2)(H_1 + H_2) \\ &= (E_1 + E_2)(E_1 - E_2)(\varepsilon_0/\mu_0)^{1/2} \\ &= (E_1^2 - E_2^2)(\varepsilon_0/\mu_0)^{1/2}. \end{aligned}$$

According to (8*a*) and (8*b*) this is equal to the sum of the energy current densities of the partial fields.†

† Besides ρ_W and j_W the components σ_{zz} of the stress tensor of the fields also add linearly, but not the components σ_{xx} and σ_{yy}. However, these facts are not relevant for our considerations.

Finally, with these results we can write

$$j_W = \rho_{W1} c_L + \rho_{W2}(-c_L). \qquad (10)$$

This is an expression for the energy current density in an arbitrary field of the type of equation (2). The velocity c_L is not *defined* by equation (10), but it can be measured independently. Consequently, the energy flow in any arbitrary field can be conceived as a superposition of two partial energy currents, and each of these partial currents can be imagined to be energy moving at the speed of light in opposite directions.

According to (9*a*) and (9*b*) $H=0$ results when $E_1 = E_2$, i.e. when the electric field strength of both waves is the same. Thus, even a purely electric field can be described as a sum of two electromagnetic waves. The net energy current is of course zero in this case. The same is true for a field which is purely magnetic.

The previous discussion referred to one space–time point. However, it can easily be generalised to extended space and time intervals by choosing partial waves with the appropriate distribution in space and time.

3. States of deformation and movement of matter

We will show now that the state of deformation and movement of matter can be decomposed into two sound waves travelling in opposite directions. This is analogous to the decomposition of the electromagnetic field in the previous section.

The system under consideration can be a moving bar which is under stress (e.g. the piston rod of a steam engine), a moving string which is under tension or a fluid under pressure moving in a pipe. To be specific, let us imagine the system to be a bar. Although our discussion is of local nature, i.e. refers to one point of the bar, it may be helpful to imagine the whole bar as a homogeneous field. At any moment in time all points of the bar are in the same state of motion and stress.

Let the bar be oriented in the z direction. We further assume that the deformation and the velocity of the matter of the bar do not have x or y components. Let $u(z, t)$ be the displacement of the matter in the z direction. Then

$$v = \frac{\partial u(z, t)}{\partial t} \qquad (11)$$

is the velocity of the matter (or more accurately: the z component of the velocity vector, the x and y components of which vanish). In the following we often use the dimensionless quantity

$$\beta = v/c_S \qquad (12)$$

as a measure of this velocity, where c_S is the absolute value of the velocity of sound. c_S is related to the modulus of elasticity E and to the mass density ρ_m according to (Landau and Lifshitz 1963)

$$c_S = (E/\rho_m)^{1/2}. \qquad (13)$$

The modulus of elasticity relates the linear stress in the z direction σ to the linear strain ε:

$$\sigma = E\varepsilon. \qquad (14)$$

The linear strain is defined by

$$\varepsilon = \frac{\partial u(z, t)}{\partial z}. \qquad (15)$$

If the matter under consideration is a fluid in a pipe $-\sigma$ has to be replaced by the pressure p, the modulus of elasticity by the compressibility and ε by the relative change of volume $\mathrm{d}V/V$.

β and ε can assume any values, independent from one other, as long as $\beta \ll 1$ and $\varepsilon \ll 1$. If $\beta = 0$ and $\varepsilon \neq 0$ the bar is deformed but at rest. If $\beta \neq 0$ and $\varepsilon = 0$ the bar is moving but relaxed.

3.1. Energy density and energy current density for arbitrary values of strain and velocity

The density of the energy of deformation and of the kinetic energy of the bar is

$$\rho_W = \tfrac{1}{2}E\varepsilon^2 + \tfrac{1}{2}\rho_m v^2 \qquad (16)$$

We will call ρ_W the energy density for short, although the total energy density is greater than ρ_W by the density of the rest energy of the relaxed bar $\rho_m c_L^2$ (c_L is the speed of light). However, the rest energy does not play any role in our discussion.

According to Landau and Lifshitz (1959) the energy current density (again without the rest energy) is

$$j_W = (\rho_W - \sigma)v. \qquad (17)$$

j_W consists of two terms: the term $\rho_W v$ represents energy which is 'transmitted convectively', the term $-\sigma v$ (or pv in a fluid) is what could be described in traditional wording as 'the work, per cross sectional area, which is done by means of the bar'.

In hydrodynamics the sum $\rho_W + p$ is called the enthalpy density. It is apparent that (17) is not an expression of the form of equation (2): the factor $(\rho_W - \sigma)$ (or $(\rho_W + p)$ respectively) in front of v is not the energy density but the enthalpy density.

3.2. Energy density and energy current density for acoustic waves

As for the electromagnetic field, here too we ask for solutions of the wave equation. The wave equation for $u(z, t)$ reads

$$\frac{\partial^2 u(z, t)}{\partial z^2} - \frac{1}{c_S^2}\frac{\partial u(z, t)}{\partial t^2} = 0.$$

The general solution of this equation is

$$u(z, t) = u_1(z - c_S t) + u_2(z + c_S t).$$

With (11), (12) and (15) one gets (Landau and Lifshitz 1959)

$$\varepsilon_1 = -\beta_1 \qquad (18a)$$

and

$$\varepsilon_2 = \beta_2 \qquad (18b)$$

where the index 1 refers to a perturbation travelling in the positive z direction and the index 2 designates a perturbation travelling in the negative z direction. We will call each of the two partial solutions 1 and 2 an acoustic wave.

An acoustic wave corresponding to a state of homogeneous strain and movement can be imagined to originate as follows (figure 2(a)). At time $t=0$ the left end of the bar is set into motion with velocity v (where $|v| \ll c_S$). At the same instant $t=0$ a wavefront begins to travel to the right. The matter of the bar left of the wavefront is moving with uniform velocity v, the matter to the right is at rest. If $v > 0$, then the material at left is compressed, if $v < 0$ it is rarefied.

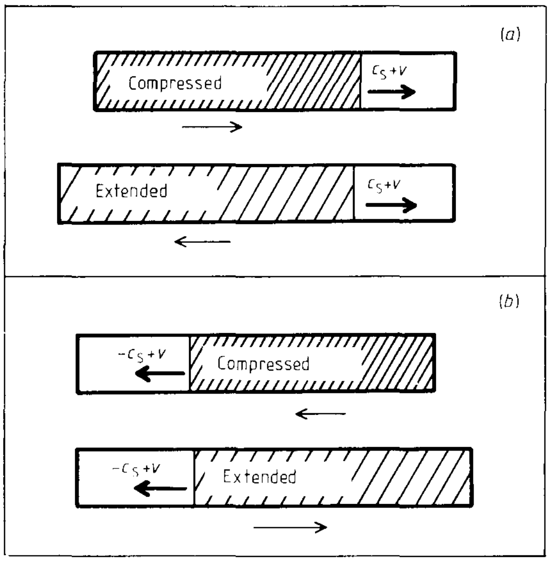
Let us calculate the energy density and the energy current density of each of the two acoustic waves. With (12), (13), (18a) and (18b) equation (16) becomes

$$\rho_{W1} = E\varepsilon_1^2 \qquad (19a)$$

and

$$\rho_{W2} = E\varepsilon_2^2. \qquad (19b)$$

Figure 2 At the time $t=0$ the left (a) or the right (b) end of a bar is set in motion with velocity v. At the same instant a wavefront begins to propagate from the end of the bar with the velocity of sound. The wavefront divides the bar into two regions: on one side matter is at rest, on the other it is moving with the velocity v. The matter of the bar is compressed or extended according to the direction of v. The arrows under each bar indicate the direction of movement of the matter, the arrows within the bar indicate the movement of the wavefront.



For the energy current density (17) we get with (19a), (19b), (14), (12), (18a) and (18b):

$$j_{W1} = (E\varepsilon_1^2 - E\varepsilon_1)c_S\beta_1 = E\varepsilon_1^2(1-\varepsilon_1)c_S$$

and

$$j_{W2} = (E\varepsilon_2^2 - E\varepsilon_2)c_S\beta_2 = -E\varepsilon_2^2(1-\varepsilon_2)c_S.$$

With (18a), (18b) and (12) one gets

$$j_{W1} = E\varepsilon_1^2(c_S + v_1)$$

and

$$j_{W2} = E\varepsilon_2^2(-c_S + v_2)$$

and finally with (19a) and (19b)

$$j_{W1} = \rho_{W1}(c_S + v_1) \qquad (20a)$$

and

$$j_{W2} = \rho_{W2}(-c_S + v_2). \qquad (20b)$$

These equations have the form of equation (1). That means that the energy current in an acoustic wave can be imagined to be energy displaced with the velocity $(c_S + v_1)$ or $(-c_S + v_2)$ respectively. It is easy to understand that the velocities in (20a) and (20b) are not c_S or $-c_S$ alone: the energy does not propagate in a medium at rest but in a medium which is itself moving with the velocity v_1 or v_2 respectively. That is why the wavefront in figure 2(a) propagates with the velocity $c_S + v$.

Notice that c_S is a positive quantity. The direction of propagation of the acoustic wave follows from the sign in front of c_S. On the contrary, the velocities v_i (v_1 or v_2) can take positive and negative values. If the values of v_i and the velocity of sound have the same sign, then the matter is compressed by the wave, figure 2(a), otherwise it is rarefied, figure 2(b).

3.3. Decomposition of an arbitrary state of deformation and movement in two acoustic waves

Let us come back now to states with arbitrary values of ε and β, i.e. values which are not restricted to obey one of the equations (18a) or (18b). For every such state ε and β can be written as a sum of the deformations ε_i or the velocities β_i, respectively, of two acoustic waves travelling in opposite directions:

$$\varepsilon = \varepsilon_1 + \varepsilon_2 \qquad (21a)$$

and

$$\beta = \beta_1 + \beta_2 = -\varepsilon_1 + \varepsilon_2. \qquad (21b)$$

The deformations ε_1 and ε_2 as well as the velocities β_1 and β_2 can be obtained by solving (21a) and (21b):

$$\varepsilon_1 = -\beta_1 = \tfrac{1}{2}(\varepsilon - \beta) \qquad (22a)$$

and

$$\varepsilon_2 = \beta_2 = \tfrac{1}{2}(\varepsilon + \beta). \qquad (22b)$$

We will show now that besides the deformations ε_i and the velocities β_i the energy density and the energy current density superpose linearly.

The energy density of the resulting state follows from (16) by using (12), (13), (21*a*), and (21*b*):

$$\rho_W = \tfrac{1}{2}(E\varepsilon^2 + \rho_m v^2) = \tfrac{1}{2}E(\varepsilon^2 + \beta^2)$$
$$= \tfrac{1}{2}E[(\varepsilon_1 + \varepsilon_2)^2 + (-\varepsilon_1 + \varepsilon_2)^2] = E(\varepsilon_1^2 + \varepsilon_2^2).$$

It is seen that ρ_W is equal to the sum of the energy densities of the acoustic waves composing the state (see (19*a*) and (19*b*)).

With (12), (13), (14) and (16) the energy current density (17) becomes:

$$j_W = \tfrac{1}{2}Ec_S(\varepsilon^2 + \beta^2)\beta - Ec_S\beta.$$

With (21*a*) and (21*b*) it follows

$$j_W = E\varepsilon_1^2(\beta + 1)c_S + E\varepsilon_2^2(\beta - 1)c_S$$

and finally with (12), (19*a*) and (19*b*)

$$j_W = \rho_{W1}(c_S + v) + \rho_{W2}(-c_S + v). \quad (23)$$

According to (20*a*) and (20*b*) this is equal to the sum of the energy current densities of the acoustic waves 1 and 2. These waves propagate in a medium which itself moves with the velocity $v = v_1 + v_2$. Thus, we have found for j_W an expression of the form of equation (2). The energy flow in matter which is moving and deformed can be conceived as a linear superposition of the energy flow of two acoustic waves. The energy flow in each wave can be imagined to be energy moving at the speed of sound (in a moving medium).

Again our discussion, which has been of local nature, can be generalised to extended space–time intervals.

4. Special cases of the energy transmission in material media

In the various technical applications the deformations ε_1 and ε_2 (as well as the velocities β_1 and β_2) are related to one another in a characteristic way. This will be discussed now.

First let us introduce an approximation. We insert (14) and (16) into (17) and rearrange:

$$j_W = \varepsilon E(-1 + \tfrac{1}{2}\varepsilon)v + \tfrac{1}{2}\rho_m v^2 v. \quad (24)$$

When energy is transmitted with bars, strings or hydraulic liquids we always have $|\varepsilon| \ll 1$, so that (24) simplifies:

$$j_W = (-\varepsilon E + \tfrac{1}{2}\rho_m v^2)v = (-\sigma + \tfrac{1}{2}\rho_m v^2)v \quad (25a)$$

or

$$j_W = (p + \tfrac{1}{2}\rho_m v^2)v \quad (25b)$$

respectively.

The approximation introduced here is an expression of the fact that the energy of deformation which is 'transmitted convectively' $\tfrac{1}{2}E\varepsilon^2 v$ is very small compared with the work done $-\sigma v$ as long as $|\varepsilon| \ll 1$.

Moreover, for most energy transmissions of technical importance one of the two terms in the parenthesis of (25*a*) or (25*b*) is negligible with respect to the other. To convince ourselves let us consider some examples.

In the jet of a Pelton turbine the first term in (25*b*), i.e. the pressure of the water, is negligible with respect to the second, the density of the kinetic energy. However, in the pipes conducting the water to the turbine, the velocity is low (in order to minimise energy losses by friction). Here, the second term in the parenthesis of (25*b*) is negligible with respect to the first. For hydraulic circuits in agricultural and in building machines we have approximately: $p = 100$ bar $= 10^7$ Pa, $\rho_m = 1000$ kg m^{-3} and $v = 1$ m s^{-1}. It follows that the term $\tfrac{1}{2}\rho_m v^2 \simeq 10^3$ Pa is four orders of magnitude smaller than p.

Let us discuss the two cases $|\varepsilon E| \ll \tfrac{1}{2}\rho_m v^2$ and $|\varepsilon E| \gg \tfrac{1}{2}\rho_m v^2$ in more detail. These two conditions can be translated into simple statements about the partial waves 1 and 2. For this reason we write (25) with the help of (12), (13) and (21) in the form:

$$j_W = E[-(\varepsilon_1 + \varepsilon_2) + \tfrac{1}{2}(-\varepsilon_1 + \varepsilon_2)^2]v.$$

Now, the two special cases are characterised by $|\varepsilon_1 + \varepsilon_2| \ll \tfrac{1}{2}(\varepsilon_2 - \varepsilon_1)^2$ and $|\varepsilon_1 + \varepsilon_2| \gg \tfrac{1}{2}(\varepsilon_2 - \varepsilon_1)^2$. Since only the order of magnitude of each side of the inequalities matters, the factor $\tfrac{1}{2}$ can be omitted and $(\varepsilon_2 - \varepsilon_1)^2$ can be replaced by $|\varepsilon_2 - \varepsilon_1|$. Thus, we obtain

$$|\varepsilon_1 + \varepsilon_2| \ll |\varepsilon_1 - \varepsilon_2| \quad \text{and} \quad |\varepsilon_1 + \varepsilon_2| \gg |\varepsilon_1 - \varepsilon_2|$$

and this is equivalent to

$$\varepsilon_1 \simeq -\varepsilon_2 \quad \text{and} \quad \varepsilon_1 \simeq \varepsilon_2.$$

In each of these special cases we can make further distinctions: both ε_1 and ε_2 can be positive or negative, the absolute value of ε_1 can be greater or smaller than or equal to the absolute value of ε_2.

The possibilities which result from all these distinctions are summarised in table 1. Some of them are important for particular technical applications: i.e., (i) a drive belt under tension which is moving in the positive or negative z direction or which is at rest corresponds to columns 9, 8 and 7, respectively; (ii) a hydraulic liquid which is flowing in the positive or negative z direction or which is at rest corresponds to columns 10, 11 and 12 respectively; (iii) the jet of a Pelton turbine which is flowing in the positive or negative z direction corresponds to columns 1 and 4, respectively.

5. Conclusion

It has been shown that the energy current in the electromagnetic field can be decomposed into two energy currents travelling with the velocity of light in opposite directions and that the energy current in moving matter under stress can be decomposed in two energy currents travelling with the velocity of sound in opposite directions.

It has often been discussed whether it is meaningful to say that in crossed static $\boldsymbol{E}$ and $\boldsymbol{H}$ fields an energy current is flowing (Feynman *et al* 1964, Lai 1981,

Table 1

$\varepsilon_1 \simeq -\varepsilon_2$					
$\varepsilon_1 < 0, \varepsilon_2 > 0$ movement to the right			$\varepsilon_1 > 0, \varepsilon_2 < 0$ movement to the left		
$\lvert\varepsilon_1\rvert = \lvert\varepsilon_2\rvert$ relaxed	$\lvert\varepsilon_1\rvert > \lvert\varepsilon_2\rvert$ compressed	$\lvert\varepsilon_1\rvert < \lvert\varepsilon_2\rvert$ extended	$\lvert\varepsilon_1\rvert = \lvert\varepsilon_2\rvert$ relaxed	$\lvert\varepsilon_1\rvert > \lvert\varepsilon_2\rvert$ extended	$\lvert\varepsilon_1\rvert < \lvert\varepsilon_2\rvert$ compressed
1	2	3	4	5	6

$\varepsilon_1 \simeq \varepsilon_2$					
$\varepsilon_1 > 0, \varepsilon_2 > 0$ extended			$\varepsilon_1 < 0, \varepsilon_2 < 0$ compressed		
$\lvert\varepsilon_1\rvert = \lvert\varepsilon_2\rvert$ at rest	$\lvert\varepsilon_1\rvert > \lvert\varepsilon_2\rvert$ movement to the left	$\lvert\varepsilon_1\rvert < \lvert\varepsilon_2\rvert$ movement to the right	$\lvert\varepsilon_1\rvert = \lvert\varepsilon_2\rvert$ at rest	$\lvert\varepsilon_1\rvert > \lvert\varepsilon_2\rvert$ movement to the right	$\lvert\varepsilon_1\rvert < \lvert\varepsilon_2\rvert$ movement to the left
7	8	9	10	11	12

Gough 1982). We have claimed in this paper that it is possible to speak about energy currents even in a static electric field alone or in a static magnetic field alone. We have also claimed that the state of a stressed bar at rest or the state of the relaxed water jet of a turbine can be characterised by two energy currents travelling in opposite directions. Is it not the fact that these statements are completely unrealistic? Indeed, the examples just cited represent cases in which our description appears to be unnatural and unnecessary. Actually, we do not mean that our statement represents a deeper insight about the physical reality. All we are saying is that it is possible to imagine these states as two waves.

This decomposition is of similar nature to, say, the Fourier decomposition of a periodic non-sinusoidal process: the Fourier components do not represent the deeper truth about the process, but sometimes it is helpful to discuss the problem in terms of these components.

References

Feynman R P, Leighton R B and Sands M 1964 *The Feynman Lectures on Physics* vol 2 (New York: Addison-Wesley)

Gough W 1982 *Eur. J. Phys.* **3** 83–7

Herrmann F 1986 *Am. J. Phys.* to be published

Herrmann F and Schmid G B 1984 *Am. J. Phys.* **52** 146

—— 1985 *Eur. J. Phys.* **6** 16

Hertz H 1884 *W. Ann. Phys. Chem.* **23** 84

—— 1894 *Untersuchungen über die Ausbreitung der elektrischen Kraft – Gesammelte Werke* vol 2, 2nd edn (Leipzig: Barth) p 293. 'If a steam engine drives a dynamo with the help of a drive belt and this dynamo in turn feeds an arc lamp via a wire loop, it is of course a conventional and not objectionable manner of speaking to say that energy is transferred from the steam engine through the drive belt into the dynamo and then from the dynamo with the help of a wire to the lamp. But does it make clear sense from a physical point of view to maintain that the energy is moving from point to point along the stretched drive belt against the drive belt's direction of motion? And if not, can it then make more sense to say that energy is moving in the wires or—according to Poynting—from point to point within the interstitial space between both wires? The conceptual cloudiness which appears here still requires a lot of clearing up.'

Lai C S 1981 *Am. J. Phys.* **49** 841–3

Landau L D and Lifshitz E M 1959 *Theory of Elasticity* (London: Pergamon) pp 12, 247

—— 1963 *Fluid Mechanics* (London: Pergamon) p 110

Maxwell J C 1954 *A Treatise on Electricity and Magnetism* vol 2 (New York: Dover) Article 610, p 253

Mie G 1898 *Sber. k. Akad. Wiss.* **107** 1128. 'If we use for the energy the image of a fluid, let the vector f be the intensity of the real current. Here, it has to be noticed, that, whereas the current of a material fluid has necessarily a well defined velocity, it is not the same in our theory on energy transmission. I let the question open if it is possible to give any meaning to the word velocity of the energy'.

Weyl H 1977 *Die Naturwissenschaften* vol 12 (Darmstadt: Wissenschaftliche Buchgesellschaft) p 32