Energy density and stress: A new approach to teaching electromagnetism

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By introducing the electromagnetic field in the customary way, ideas are promoted that do not correspond to those of contemporary physics: on the one hand, ideas that stem from pre-Maxwellian times when interactions were still conceived as actions at a distance and, on the other hand, ideas that can be understood only from the point of view that the electromagnetic field is carried by a medium. A part of a course in electromagnetism is sketched in which, from the beginning, the electromagnetic field is presented as a system in its own right and the local quantities energy density and stress are put into the foreground. In this way, justice is done to the views of modern physics and, moreover, the field becomes conceptually simpler.

I. INTRODUCTION

The way the electromagnetic field is customarily introduced can be criticized in two respects. The first reproach has to do with the idea of what is to be understood by a field. Although modern physics did not change anything in the mathematical formalism developed by Maxwell, our ideas about the nature of the electromagnetic field have changed significantly since the time of Maxwell and Faraday. One can say that our ideas about the field have become simpler. However, in teaching the electromagnetic field one usually does not take advantage of this fact.

The second criticism has to do with the way we deal with one important part of the Faraday–Maxwell theory: with statements about the mechanical stress within the field. Usually, Maxwellian stress is introduced, if it is introduced at all, very late in the curriculum, and it appears to be a kind of curiosity, something that is not really important for the understanding of the electromagnetic forces. Instead, the so-called force laws are put into the foreground. The domain of application of these laws is, however, less general than Maxwell's stress laws since each of the force laws refers to a particular geometry. Moreover, the verbal formulation of the force laws is strongly reminiscent of an action-at-a-distance picture of electromagnetic interactions. On the contrary, Maxwell's expression of the stress tensor represents a local, field theoretical statement in agreement with the modern view of the electromagnetic field.

In Sec. II A we will compare the concept of the field from Faraday and Maxwell's time to that of modern physics. In Sec. II B the relation between Maxwell's stress tensor and the force laws is discussed. In Secs. III and IV part of an electrodynamics course is sketched, which takes into account the criticisms of Sec. II. The course was developed for and tested with physics students in the second semester at the University of Karlsruhe. To prepare the students, the concept of stress had been introduced in their first-semester mechanics course. A simplified version of the course was also taught to junior-high-school pupils of age 15.

II. CRITICISM OF THE CUSTOMARY INTRODUCTION OF THE ELECTROMAGNETIC FIELD

A. The field concept at Maxwell's time and today

Maxwell clearly distinguishes between the field and a medium that carries the field. By a field Maxwell[1] understands "... the portion of space in the neighborhood of electrified bodies, considered with reference to electric phenomena."

It can be seen from many parts of his work that he is convinced that space is filled with a medium that carries the field. We cite the very last sentence of his treatise[2]: "Hence all these theories lead to the conception of a medium in which the propagation takes place, and if we admit this medium as an hypothesis, I think it ought to occupy a prominent place in our investigations, and that we ought to endeavour to construct a mental representation of all the details of its action, and this has been my constant aim in this treatise."

As we know today, the validity of Maxwell's theory does not depend on the existence of such a medium. Rather, it has survived the modifications of our ideas about such a medium caused by the theory of relativity.

With regard to the modern concept of the field it has first to be noticed that the word "field" is currently used in two distinct meanings.[3] On the one hand, we speak about a field when describing a distribution in space of a local physical quantity. In this sense we have, for instance, temperature, pressure, or mass density fields. On the other hand, in modern field theory the word "field" is used as a name for particular physical systems. "Electromagnetic field" is a name of a system just as "rigid body" and "perfect gas" are names of systems.

Unfortunately, one does not always distinguish between these two meanings and it is this mixing of meanings that leads many students to the idea that the electromagnetic field is nothing more than the distribution in space of the quantities "electric field strength" and "magnetic field strength" (or "magnetic induction").

In this article, we will use the word field exclusively in the sense of field theory, i.e., as a name of a physical system. If we want to express the fact that the magnitude **E** is distributed in space in a certain way we do not speak about "the field **E**" but about "the distribution of the field strength **E**(**r**)."

Of course, the system "electromagnetic field" is far more than just a field strength distribution. It is a thing, in which not only the field strengths have definite values but many other variables too, as, for instance, energy, entropy, momentum, and mechanical stress. Some variables have the value zero—the electric charge, for instance—but that does not mean that they have no value at all.

Like every other physical system, the electromagnetic field can exist in different states. In certain states its entropy is zero, in others it is not. There are states in which a temperature and a chemical potential are defined and there are other states for which these quantities are not defined. Among the quantities of the field there are two that have been exclusively invented to describe the electromagnetic field and are not defined for any other system: the electric and the magnetic field strengths.

The reason for the importance of these two quantities is that they are very practical: Various other dynamic quantities (but not all) can easily be calculated if the field strengths are known, e.g., the energy density, the momentum density, and the mechanical stress.

It is often convenient to treat the set of states with $\mathbf{H} = 0$ as a separate system, the "electric field," and the states with $\mathbf{E} = 0$ as another system, the "magnetic field."

Thanks to the modern concept of a field as a physical system we can form a simple intuitive picture of a field: Field is a kind of stuff, like air or water, and a certain quantity of that stuff is a kind of thing like other familiar things. It is no more necessary to distinguish between the medium that carries the field and the field itself. Unfortunately, in teaching electromagnetism one usually does not take advantage of this fact. Of course, one does not speak anymore about a medium that carries the field. But that makes things even worse. The medium that Maxwell imagined made the field understandable at his time. Instead of telling students that things are easier today, that a field is a thing of its own right, one still gives them a description that stems essentially from Maxwell, but without adding that this description was understood to go together with a field-carrying medium. In Maxwell's time it made sense to describe the electric field as "the portion of space in the neighborhood of electrified bodies" because of the medium that Maxwell admitted. Without such a medium, this sentence which every student knows today appears as a kind of sophism.

Here is another example of how we deal today with the field concept[4]: "The electric field attaches to every point in a system a local property ... ." This sentence is not wrong if by "electric field" the magnitude "electric field strength" is meant. If, however, a physical system is meant, then the statement is as curious as the following one: "A gas attaches to every point in a system a local property." If we are convinced of the existence of a system, we do not speak in such terms. We say, rather, the system, a gas, for instance, "has properties." And we should also say that a field "has properties." Notice the word "has" and notice the plural of the word property.

B. Mechanical stress and the force laws

According to Maxwell, forces between bodies that are charged or traversed by an electric current are mediated by a medium that fills the whole space. Thus the medium itself is submitted to mechanical stress.[5] This stress is described by the tensor

$$\sigma_{ik} = \left[E_i D_k + H_i B_k - \tfrac{1}{2}(\mathbf{ED} + \mathbf{HB})\delta_{ik} \right].$$

If the field is purely electric or magnetic the tensor takes upon the simple forms

$$\sigma_{ik} = (E_i D_k - \tfrac{1}{2}\,\mathbf{ED}\,\delta_{ik})$$

and

$$\sigma_{ik} = (H_i B_k - \tfrac{1}{2}\,\mathbf{HB}\,\delta_{ik}),$$

respectively. Here, δ_{ik} is the Kronecker delta. The mechanical stress σ_{ik}, which can also be interpreted as the momentum current density,[6–8] must also exist if the modern field concept is admitted. Then it is not a stress within the field-carrying medium but within the field itself.

However, if one does not introduce the field as a thing in its own right but as a kind of mathematical construction, it is hard to imagine that it could be mechanically stressed and, as a consequence, the idea of pressure or tension appears to be a very abstract idea that may be accepted by an advanced student but not by a beginner. Therefore, traditionally the beginner learns various force laws, such as Coulomb's law for electric charges, Coulomb's law for magnetic poles, and Lorentz's law. Of course, these laws are correct. However, to introduce them as the basis of electromagnetic interactions has disadvantages.

One disadvantage refers to the teaching of Coulomb's law as a basic law. Coulomb's law is valid only for a very particular charge distribution: for two point charges. Thus it describes only a very special case. Moreover, it promotes the old pre-Maxwellian idea of an action at a distance, since in it the force is expressed solely by the charges, i.e., the sources of the field. It suggests statements like the following: "Like charges repel each other" and "Unlike charges attract each other," which can indeed be found in many textbooks and which are strongly reminiscent of the old action-at-a-distance times.

Apart from these laws, which contain only the sources of the fields, there is another type of force law, in which a field strength appears but which, nevertheless, are no real local-causes laws. What we mean are the relations

$$\mathbf{F} = Q \cdot \mathbf{E}$$

and

$$\mathbf{F} = Q(\mathbf{v} \times \mathbf{B}).$$

Neither of these laws promotes the idea that a field is a system in its own right: The field strengths that appear in these equations are not the strengths of the fields that really exist but those of the fields that would exist if the body on which the force $\mathbf{F}$ is exerted would not be there. Using these equations promotes the unfortunately widespread idea that a field is not more than a field strength distribution, and the field strength is not more than a practical mathematical tool for the calculation of forces. Like Coulomb's law, these relations have the disadvantage that they tell us only something about the forces on a body but not about the forces within the field.

An up-to-date approach to electromagnetism would proceed in the following way: Besides Maxwell's field equations, expressions for the mechanical stresses in the electric and magnetic fields are introduced. These expressions are, just like Maxwell's equations, local and very general. From these the forces on charged, magnetized, or current-carrying bodies are deduced: forces on point charges, magnetic poles, electric and magnetic dipoles, on current-carrying wires, etc.

One might object that this introduction requires tensor calculus, unknown to beginners. We will show that this is not the case. The forms of the stress tensors of the electric and the magnetic fields are so simple that it is easy to grasp the whole stress state without knowing tensor calculus. All the student has to understand is that in the direction of the field lines there is tension and perpendicular to them there is pressure of the same amount. By the way, we deal currently with pressure in gases and liquids without pointing out that pressure is equal to the diagonal elements of the stress tensor, which, in this case, however, is even more singular than in the case of the electromagnetic field.

In our course, besides the expressions for mechanical stress, we derive the expression for energy density. Moreover, we discuss the momentum density and the energy current density within the field. However, we will not enter into this particular subject here, since a special discussion about these quantities has been going on for many years.[9–13]

III. SKETCH OF THE COURSE

A. Energy and mechanical stress within the electric field

Bodies A and B are pushed away one from the other by a spring in Fig. 1(a), and by the air in the cylinder in Fig. 1(b). The two vehicles in Fig. 1(c) are pulled together by a person by means of a rope. In all these cases, momentum is being transferred from one body to another and in all these cases there is a connection between the bodies that makes the momentum transfer possible: a spring, air, and a rope, respectively.

In Fig. 1(d), momentum is also being transferred, namely, from the electrically charged body A to the electrically charged body B and, of course, in this case a connection must also exist between A and B. This connection is invisible—like the air in the case of Fig. 1(b). We call this connection the "electric field." An electric field can be "strong" or "weak," i.e., there can be a lot or a little of it in a given domain of space, in the same way as there can be more or less air in a given domain of space. Whereas air can only push, i.e., exert pressure, an electric field can both push and pull.

We study the field between two charged bodies A and B by means of a probe: another electrically charged body. This test body should be of small size and its charge should be small in order not to disturb the charge distribution on A and B.

We observe the following:

(1) In every point of space a direction can be attributed to the field: the direction of the force that acts on the test body.

(2) If a probe of charge Q_{P0} is placed at a particular point and is then replaced by a probe with charge $2Q_{P0}$, $3Q_{P0}$, ..., the force on the probe increases by the same factor, i.e., by 2, 3, etc. Thus the factor relating the charge of the test body with the force on it is characteristic for the *field without the test body*. We call this factor the strength of the field at the location of the probe. Thus the field strength **E** is defined by

$$\mathbf{F}=\mathbf{E}Q_P. \quad (1)$$

(3) If a second charge distribution is added to the first one, the field strengths in every point of space add vectorially.

Next, we investigate the field strength distribution in space. We represent this distribution by field lines and we state that charges are the place where field lines begin or end so that the following relation must hold,

$$Q=\epsilon_0 \oint \mathbf{E}\, d\mathbf{A}.$$

The integral extends over a closed surface and Q is the total charge within this surface. By means of this relation we can calculate a field strength distribution if a charge distribution is given, for instance, for a charge distribution that is spherically symmetric or for that of a parallel-plate capacitor.

We are now ready to explore the mechanical properties of the system "electric field" and ask for their dependence on the field strength. Since we ask for local properties only, the spatial distribution of the field strength does not matter and we can consider the simplest distribution that can be imagined without restricting the validity of our results: the homogeneous field of the parallel-plate capacitor, Fig 2. Let ξ and η be the length and width of the plates, respectively, and ζ their distance. Thus the area of the plates is $\xi\eta$ and the volume of the field $\xi\eta\zeta$.

1. The mechanical stress in the direction of the field strength

One plate (A) pulls, by means of the field, on the other plate (B). Thus there must be a tension within the field in

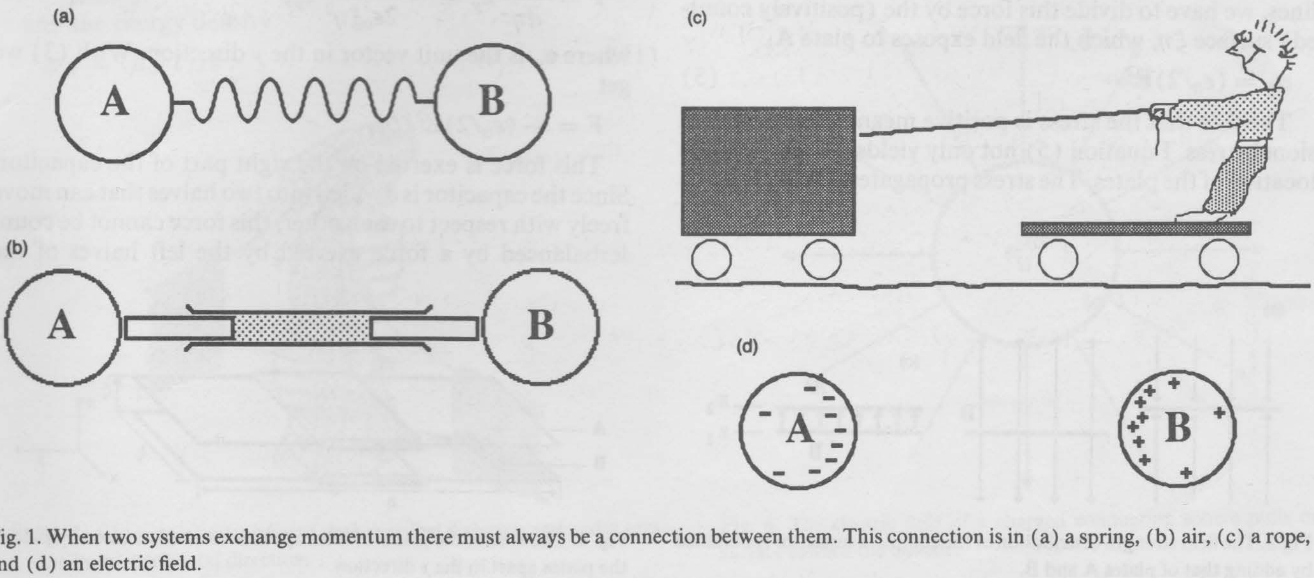

Fig. 1. When two systems exchange momentum there must always be a connection between them. This connection is in (a) a spring, (b) air, (c) a rope, and (d) an electric field.

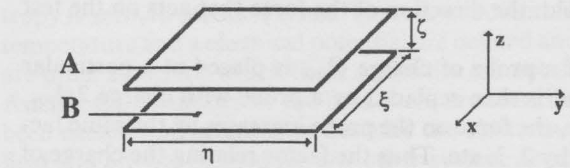

Fig. 2. Parallel-plate capacitor.

the direction of the field lines. We shall first calculate the value of this tension. For that purpose we consider the charge dQ_B of any surface element of B as a test charge in the field of A and obtain the force $d\mathbf{F}_B$ that A exerts on it,

$$d\mathbf{F}_B = \mathbf{E}_A dQ_B.$$

The total force $\mathbf{F}_B$ that A exerts on B is

$$\mathbf{F}_B = \mathbf{E}_A Q_B. \quad (2)$$

Here, $\mathbf{E}_A$ is the strength of the field created by plate A alone, i.e., of the field that would be present if plate B would have been taken away. If only one of the plates A or B would be present, we would have the fields represented in Fig. 3(a) and (b), respectively. The field strengths in the region between z_1 and z_2 would be

$$\mathbf{E}_A = -(Q_A/2\epsilon_0\xi\eta)\mathbf{e}_z$$

and

$$\mathbf{E}_B = (Q_B/2\epsilon_0\xi\eta)\mathbf{e}_z,$$

respectively, where $\mathbf{e}_z$ is the unit vector in the z direction.

Since the absolute value of the charges of the plates is the same, i.e., $Q_B = -Q_A = Q$, we have

$$\mathbf{E}_A = \mathbf{E}_B.$$

The field strength within the capacitor is obtained by adding $\mathbf{E}_A$ and $\mathbf{E}_B$, as in Fig. 3(c),

$$\mathbf{E} = 2\mathbf{E}_A = (Q/\epsilon_0\xi\eta)\mathbf{e}_z. \quad (3)$$

We find the force that plate A exerts on plate B as a function of the strength of the field $\mathbf{E}$ between the plates by substituting $\mathbf{E}_A$ and $Q_B = Q$ in Eq. (2) by means of (3),

$$\mathbf{F}_B = \zeta\eta(\epsilon_0/2)\mathbf{E}^2\mathbf{e}_z. \quad (4)$$

This is at the same time the force plate A exerts on the field. To obtain the stress $\sigma_\parallel$ within the field, in the direction perpendicular to the plates, i.e., parallel to the field lines, we have to divide this force by the (positively counted) surface $\xi\eta$, which the field exposes to plate A,

$$\sigma_\parallel = (\epsilon_0/2)\mathbf{E}^2. \quad (5)$$

The fact that the stress is positive means that it is a tensional stress. Equation (5) not only yields the stress at the location of the plates. The stress propagates throughout the whole field and, because of the homogeneity, has the same value at any location between the plates. Since (5) is a local expression, we can generalize our result: *In every point of an electric field there is a tensional stress in the direction of the field strength. Its amount is* $(\epsilon_0/2)\mathbf{E}^2$.

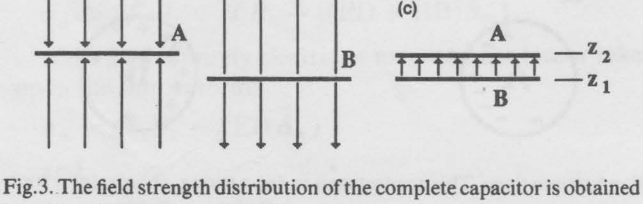

Fig.3. The field strength distribution of the complete capacitor is obtained by adding that of plates A and B.

2. The energy density

Next we shall calculate the energy content of the field. To this purpose we imagine a capacitor of fixed charge whose plates have, at the beginning, a distance of zero. The plates are oriented perpedicularly to the z axis, as in the case of the capacitor of Fig. 2. We now displace plate A in the z direction until the plate distance is ζ. In this process, the field volume increases whereas the field strength remains constant. The energy W necessary to separate the plates can be calculated by means of

$$W = \mathbf{F}_A\zeta\mathbf{e}_z.$$

The force $\mathbf{F}_A$, which has to be exerted on plate A in order to separate it from B, is the same as the force calculated by means of (4), i.e., the force that A exerts on B. We thus obtain

$$W = \xi\eta\zeta(\epsilon_0/2)\mathbf{E}^2 \quad (6)$$

and, with (3),

$$W = Q^2\zeta/2\epsilon_0\xi\eta. \quad (7)$$

The energy density ρ_W is obtained from (6) by dividing by the field volume $\xi\eta\zeta$,

$$\rho_W = (\epsilon_0/2)\mathbf{E}^2.$$

It is seen that the energy density has the same value as the mechanical tension. We resume: *The electric field contains energy. The energy density is* $(\epsilon_0/2)\mathbf{E}^2$.

3. The mechanical stress perpendicular to the direction of the field strength

We consider a capacitor whose plate's area can be increased by pulling two opposite edges of every plate apart, Fig. 4. The force $\mathbf{F}$ that has to be exerted on the right half of the capacitor in order to increase its extension in the y direction, with Q remaining constant, is obtained by deriving the energy (7) with respect to η:

$$\mathbf{F} = \frac{dW}{d\eta}\mathbf{e}_y = -\frac{Q^2\zeta}{2\epsilon_0\xi\eta^2}\mathbf{e}_y,$$

where $\mathbf{e}_y$ is the unit vector in the y direction. With (3) we get

$$\mathbf{F} = -(\epsilon_0/2)\mathbf{E}^2\xi\zeta\mathbf{e}_y.$$

This force is exerted on the right part of the capacitor. Since the capacitor is divided into two halves that can move freely with respect to each other, this force cannot be counterbalanced by a force exerted by the left halves of the

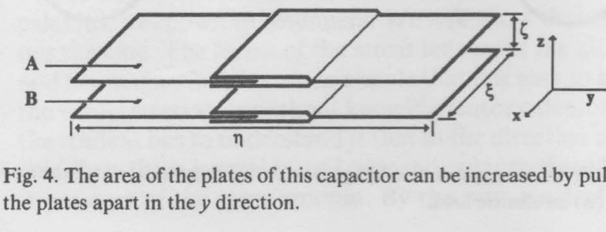

Fig. 4. The area of the plates of this capacitor can be increased by pulling the plates apart in the y direction.

plates directly on the right halves. It has to be mediated by the field, i.e., the outside force acts on the right halves of the plates, these exert a pressure on the field, and the field pushes against the left halves of the plates. Since the field is homogeneous, we can calculate the local stress within the field in the y direction by dividing the force by the (positively counted) cross section $\xi\zeta$ of the capacitor,

$$\sigma_\perp = -\mathbf{F}/\xi\zeta = -(\epsilon_0/2)\mathbf{E}^2. \quad (8)$$

The fact that $\sigma_\perp$ is negative means that we have to do with a compressional stress.

An analogous reasoning would lead us to the result that a pressure of the same amount is exerted in the x direction. We can thus resume: *In every point of an electric field there is a compressional stress in the directions perpendicular to the field strength. Its amount is* $(\epsilon_0/2)\mathbf{E}^2$.

Accordingly, in every point of a field we have pressure and tension at the same time. How can that be imagined? Are there other systems standing simultaneously under tension and pressure? Of course, there are. To see it, take any deformable object, a sponge, for instance, with both hands, Fig. 5, pull your hands apart, and squeeze the object at the same time with your fingers. In this case you can give any value to the pressure and the tension independently. The electric field, on the contrary, has the peculiarity that at a fixed point in space pressure and tension always are of the same amount.

B. Energy and mechanical stress within the magnetic field

The procedure is similar to that of the electric field. Instead of the homogeneous electric field of a parallel-plate capacitor, we consider the magnetic field in the slit of a permanent ring magnet (the magnetization is tangential to the circumference, the slit is radial). The force law from which we start and which corresponds to Eq. (1) is

$$\mathbf{F} = \mathbf{H}Q_m,$$

where $\mathbf{H}$ is the magnetic field strength and Q_m is the magnetic pole strength. The reasoning is exactly the same as in the electric case and one gets the mechanical stresses

$$\sigma_\parallel = (\mu_0/2)\mathbf{H}^2 \quad (9)$$

and

$$\sigma_\perp = -(\mu_0/2)\mathbf{H}^2 \quad (10)$$

and the energy density

$$\rho_W = (\mu_0/2)\mathbf{H}^2. \quad (11)$$

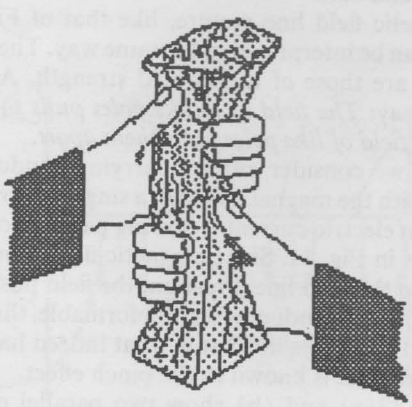

Fig. 5. A sponge is under tension in the vertical direction and under pressure in the horizontal direction.

Thus, also in the magnetic field, there is tension in the direction of the field strength and pressure perpendicular to it and the absolute values of pressure, tension, and energy density are the same

IV. READING FIELD LINE PICTURES

Usually, a field line picture is interpreted in the following way: The picture tells us the force that is exerted by the field on a test charge (or a magnetic test pole), where by "field" one means the field that is present before introducing the probe. This way of interpreting a field line picture is useful, for instance, in the case of an electron-optical arrangement in which one wants to know what will happen to an electron by considering the field distribution without the electron.

There are cases, however, where the problem is different: Given is a field distribution and wanted are the forces on the matter that may be electrically charged, magnetized, or traversed by an electric current and to which the fields are attached. In this case, the above-mentioned interpretation of the field line pictures is not useful since the stress distribution of the really existing field is desired. However, these stresses and forces can easily be deduced from the field line picture by means of the results that have been obtained in Sec. III. Before discussing examples we shall formulate some rules.

Since an electric charge is always the starting or ending point of field lines, it follows from Eq. (5) that the electric field exerts a tensional stress on a charge and on the matter carrying the charge. We thus have **Rule 1**: The electric field pulls on electric charges.

Since on magnetic poles **H**-field lines begin or end, we have, analogously, **Rule 2**: The magnetic field pulls on magnetic poles.

Magnetic fields are not only attached to the poles of magnetized matter but also to electric currents. The magnetic field is connected to a current in such a way that the field lines surround the current. Thus the current and the current-carrying matter are exposed to the pressure of the field and we have **Rule 3**: The magnetic field pushes on electric currents.

Applying these rules we shall now interpret some field line pictures.

Figure 6 shows a charged conducting sphere with its

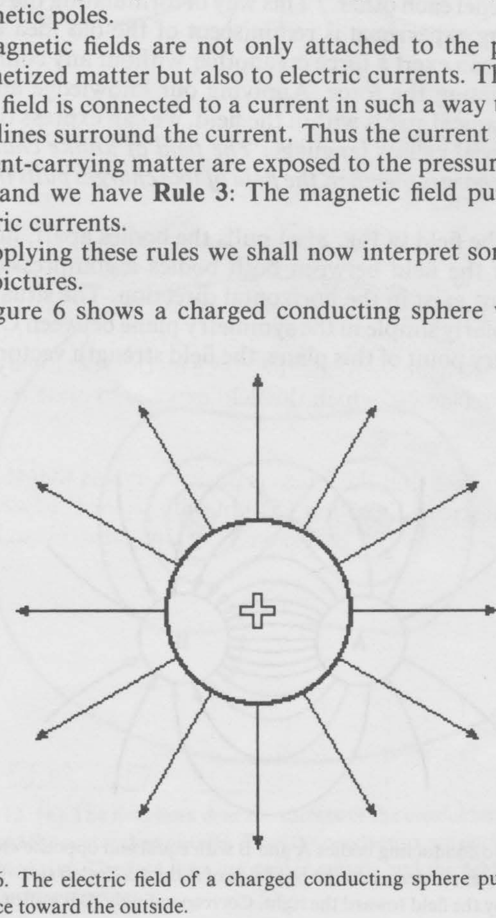

Fig. 6. The electric field of a charged conducting sphere pulls on the surface toward the outside.

electric field. According to Rule 1, the electric field pulls on the charges and, since the charges cannot leave the surface, it pulls on the surface of the sphere. This pull lowers the surface tension of a charged water drop, an effect that can be demonstrated experimentally.[14] The force per area that is exerted by the field on the surface is easily calculated by inserting the field strength at the surface

$$|\mathbf{E}| = Q/4\pi\epsilon_0 r^2$$

into Eq. (5),

$$\sigma_{\parallel} = Q^2/32\pi^2\epsilon_0 r^4.$$

The traditional way of describing the situation is as follows. One picks out a small surface element and considers it in the field of the rest of the surface charges. The surface element is repelled by these remaining charges. This description is mathematically correct. It is, however, not very natural because the space between the surface element under consideration and the remaining charges, i.e., the interior of the sphere, is free of field.

As a second example let us analyze the arrangements of Figs. 7 and 8(a). In order to infer from the field line picture to the force on one of the charged bodies, body A, for instance, we have to consider the field line density at the surface of A. The field pulls on A in all directions. Thus body A is exposed to mechanical tension. However, the field lines are denser on the right side of A than on the left side. Thus the field pulls more toward the right than toward the left and a net force to the right results. By analogous arguments, we get the net forces on body B in Fig. 7 and on bodies C and D in Fig. 8(a).

It is usual to describe the behavior of charged bodies by a sentence like the following: "Like charges attract, unlike charges repel each other." This way of formulating the corresponding experience is reminiscent of the old idea that one body can exert a force on another without any connection mediating the force. Applying our knowledge about the mechanical stress within the field, we can express these facts in local causes language: *The field of unlike charges pulls the charges together, the field of like charges pulls them apart.*

Since the field in Fig. 8(a) pulls the bodies apart, in the region of the field between both bodies a compressional stress must exist in the horizontal direction. The situation is particularly simple in the symmetry plane between C and D. In every point of this plane, the field strength vector lies

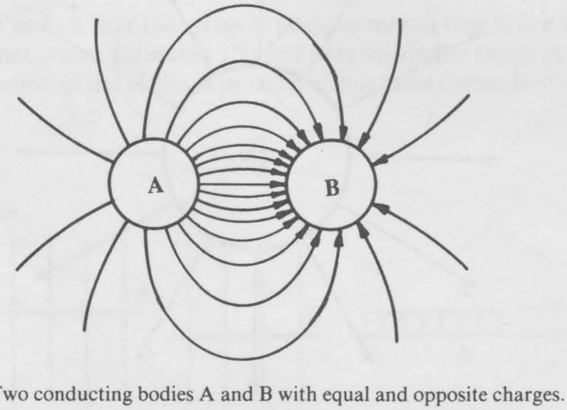

Fig. 7. Two conducting bodies A and B with equal and opposite charges. The field lines are denser on the right side of A than on its left side. Thus A is pulled by the field toward the right. Correspondingly, B is pulled by the field toward the left.

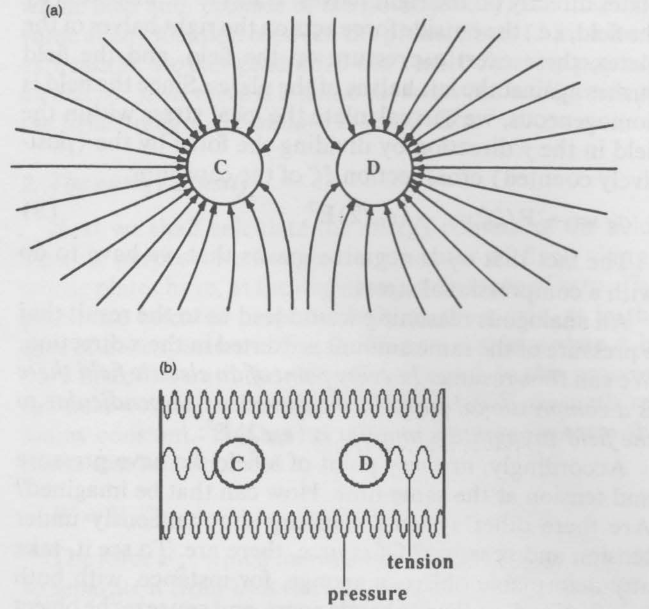

Fig. 8. (a) Two conducting bodies C and D of equal charges. The field lines are denser on the left side of C than on its right side. Thus C is pulled by the field toward the left. (b) A mechanical example of how two bodies C and D are pulled apart via a spring-and-yoke arrangement.

parallel to the plane. It follows that everywhere in this plane there is pure pressure in the direction perpendicular to the plane: The part of the field on the left of the symmetry plane pushes on the part of the field on the right of it. Figure 8(b) shows the stress distribution of the field of Fig. 8(a) schematically: The field is replaced by a system of yokes and springs.

To derive Eq. (8), we considered the homogeneous field within the parallel-plate capacitor, Fig. 4. To understand the situation completely, there remains one question to be answered. The field pushes the plates toward the outside of the capacitor, in a direction parallel to the plates. How does the field stick to the plates? The only way it can do it is through the nonhomogeneous part of the field on both sides of the capacitor, Fig. 9(a). This part of the field, which one might be inclined to consider as due to a mere imperfection of the capacitor, has a component that pulls the plates toward the outside, in the y direction. Schematically, the stress within the capacitor's field is pictured in Fig. 9(b). Like in Fig. 8(b), the field has been substituted by springs and bars.

A magnetic field line picture, like that of Fig. 10, for instance, can be interpreted in the same way. The field lines of Fig. 10 are those of the **H** field strength. Again, it is correct to say: *The field of unlike poles pulls the poles together, the field of like poles pulls them apart.*

Finally, we consider current-carrying conductors and we begin with the magnetic field of a single hollow cylinder in which an electric current is flowing parallel to the cylinder axis, as in Fig. 11. Since magnetic fields push perpendicularly to the field line direction, the field pushes on the conductor. If the conductor were deformable, the magnetic field would compress it. That is what indeed happens in a plasma and what is known as the pinch effect.

Figures 12(a) and (b) show two parallel conductors with currents flowing in opposite directions and in the same direction, respectively. In the case of Fig. 12(a), the

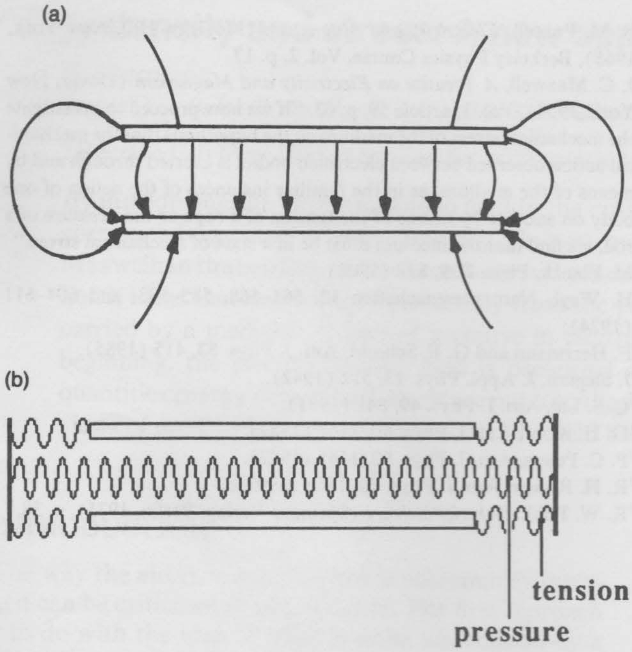

Fig. 9. (a) To pull the plates apart in the direction of the plane of the plates the field grasps the plates on the bords. (b) To demonstrate the field's action the field is substituted by a spring-and-yoke arrangement.

field strength is greater in the region between the conductors than outside. Thus the field pushes more from this central region than from the outside and the resulting forces on the conductors are also oriented toward the outside. Correspondingly, we see that in the case of Fig. 12(b) the field pressure is greater at the outside than in the middle.

Also in this case one often says that the wires repel or attract each other, respectively. Again we can formulate these facts more satisfactorily, thanks to our knowledge about the stress distribution within the fields: *The field of two currents flowing in the same direction pushes the currents together, the field of two currents flowing in opposite directions pushes them apart.*

V. CONCLUSIONS

The main purpose of this article was to show how the electromagnetic field can be introduced in a way that makes it apparent that this field is a system in its own right. The most important means to attain this goal is to speak about the field in the same way as we are used to speaking

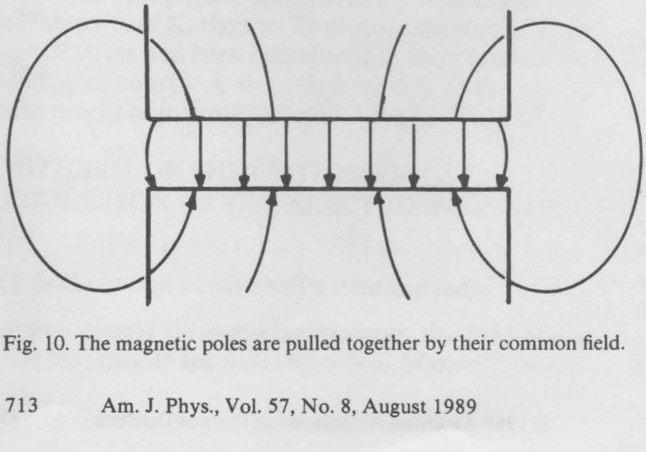

Fig. 10. The magnetic poles are pulled together by their common field.

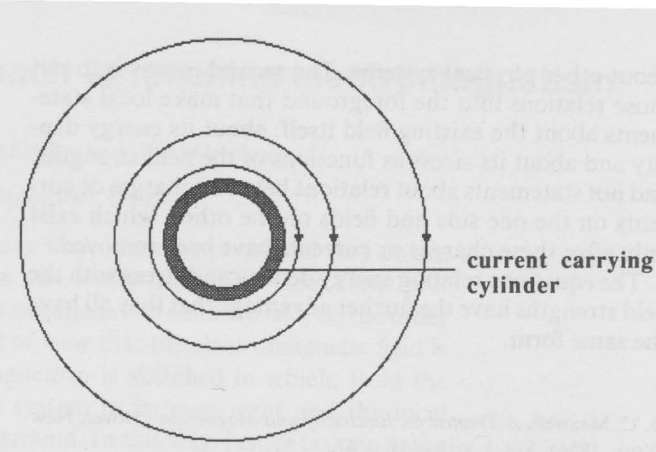

Fig. 11. The magnetic field pushes on the current within the conductor.

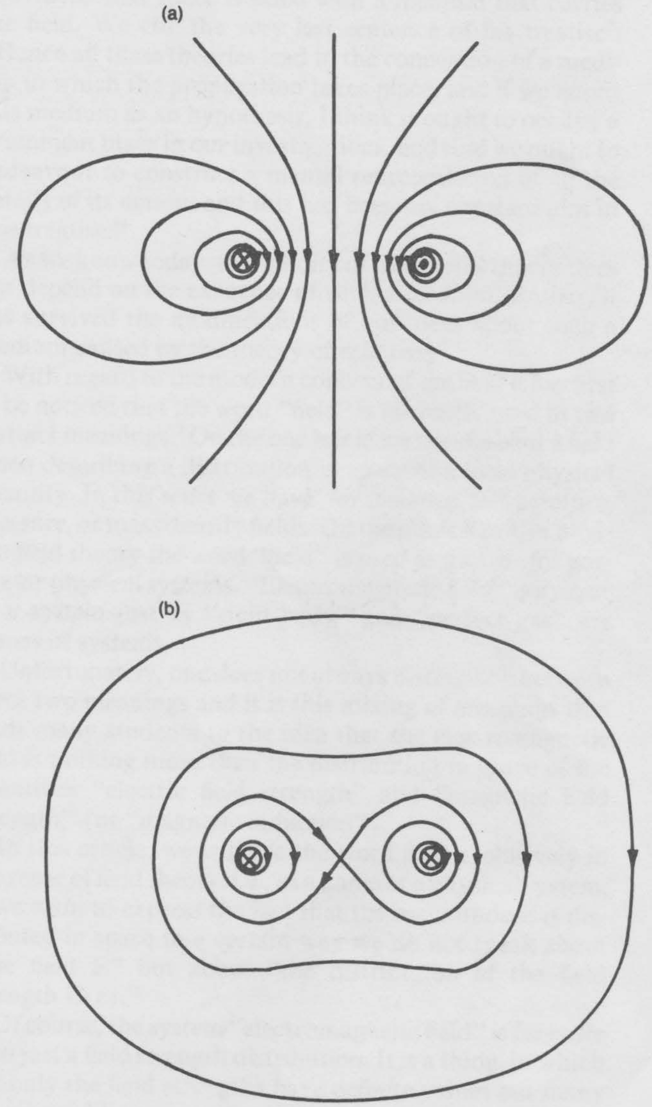

Fig. 12. (a) The field lines near the surface of the conductors are denser toward the center than outside. Thus the conductors are pushed apart by the field. (b) The field lines are denser outside and the field pushes the conductors together.

about other physical systems. The second means is to put those relations into the foreground that make local statements about the existing field itself: about its energy density and about its stress as functions of the field strengths, and not statements about relations between charges or currents on the one side and fields on the other, which exist only after these charges or currents have been removed.

The equations relating energy density and stress with the field strengths have the further advantage that they all have the same form.

[1]J. C. Maxwell, *A Treatise on Electricity and Magnetism* (Dover, New York, 1954), Vol. 1, article 44, p. 47.

[2]J. C. Maxwell, *A Treatise on Electricity and Magnetism* (Dover, New York, 1954), Vol. 2, article 866, p. 493.

[3]G. Falk and W. Ruppel, *Mechanik–Relativität–Gravitation* (Springer-Verlag, Berlin, 1973), pp. 133–135.

[4]E. M. Purcell, *Electricity and Magnetism* (McGraw-Hill, New York, 1965), Berkeley Physics Course, Vol. 2, p. 17.

[5]J. C. Maxwell, *A Treatise on Electricity and Magnetism* (Dover, New York, 1954), Vol. 1, article 59, p. 63. "If we now proceed to investigate the mechanical stress of the medium on the hypothesis that the mechanical action observed between electrified bodies is exerted through and by means of the medium, as in the familiar instances of the action of one body on another by means of the tension of a rope or the pressure of a rod, we find that the medium must be in a state of mechanical stress."

[6]M. Planck, Phys. Z. **9**, 828 (1908).

[7]H. Weyl, Naturwissenschaften **12**, 561–568, 585–593, and 604–611 (1924).

[8]F. Herrmann and G. B. Schmid, Am. J. Phys. **53**, 415 (1985).

[9]J. Slepian, J. Appl. Phys. **13**, 512 (1942).

[10]C. S. Lai, Am. J. Phys. **49**, 841 (1981).

[11]D. H. Kobe, Am. J. Phys. **50**, 1162 (1982).

[12]P. C. Peters, Am. J. Phys. **50**, 1165 (1982).

[13]R. H. Romer, Am. J. Phys. **50**, 1166 (1982).

[14]R. W. Pohl, *Elektrizitätslehre* (Springer-Verlag, Berlin, 1975), p. 38.

