KPK

The Karlsruhe Physics Course

for the secondary school A-level

The Teacher’s Manual

Electromagnetism

The Karlsruhe Physics Cours – *The Teacher’s Manual*

A textbook for the secondary school A-level

- 🟩 **Electromagnetism**
- 🟥 Thermodynamics
- 🟪 Oscillations, Waves, Data
- 🟦 Mechanics
- 🟧 Atomic Physics, Nuclear Physics, Particle Physics

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The Karlsruhe Physics Course
Issue 2015
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I

Physical Foundations

1. The field as an object

The field as a concept of modern physics can be imagined in two ways: on the one hand as a particular state of the so-called vacuum. This was also the idea of Faraday and Maxwell, whom we can consider as the inventors of the field concept. What today carries the inapt name „vacuum“ was at Faraday’s and Maxwell’s time called ether. The other possibility to imagine a field is to consider it as an entity, a kind of object, that resides in space, where we do not further ask for the nature of space, but only for that of the field. We can compare these two conceptions with two ideas we can have about a wave pulse running over the surface of the sea: Either we say that the wave is no more than a particular state of the water, or one insists in the fact that apart from the water, something new has emerged, something that can be considered independently from the water.

In our treatment of the electromagnetic waves we decided to adopt the second model or interpretation. The reason is simple: We describe the rest of the world, i.e. that part which we call matter, in the same way. Indeed, the material part of the world could also be described as a particular state of the vacuum, but obviously one would not do so in a physics course for beginners.

Actually, in the textbooks yet another idea about the field is frequently adopted: A field is no more than the distribution of a physical quantity in space: the field strength. That means that a field is not a physical system. It is rather a mathematical tool that allows to calculate forces which act on a body, and the bodies appear to be the only really existing entities in the world. This idea of a field may be useful for certain purposes. However, it obscures the view on other aspects of the field. It becomes difficult to understand that a field has yet other properties, just as material objects.

Here a physical quantity is confounded with a physical system. It is like saying: „A mass hangs on a spring“, an habit that we, the teachers, are used to tackle. However, a sentence like „between the plates of the capacitor there is a field E“ can be found in many textbooks.

It is hardly better to introduce the electric field as the space or region of space, where forces act on charged bodies. One might try to explain to somebody what we understand by „air“: Also air could be defined as a region of space where forces are acting. And as a measure of the air one might then introduce the pressure p. One would then formulate: „The air p between the walls of the container…“.

Here again our recommendation to the teacher: Do not confound the introduction of the concept field with the definition of the field strength.

2. The stuff the field is made of

In the Junior Highschool version of the KPK we were not consequent in one point: If the word „field“ is used as a name for a physical system or in other words, for an entity that is extended in space, then we need another name for the „stuff“ the entity is made of. At the time the Junior High version was written, we didn’t have an idea about how to call it and we had tried to circumvent the problem. However, this is somewhat like trying to explain to somebody (a Martian for example) what is a „sea“ without using the word „water“. That is why we finally decided to employ the word „field stuff“. When writing the text, and also when teaching it to the real pupils this turned out to be a real deliverance. Indeed, sometimes a distorted formulation, one that is difficult to understand, is caused by the fact that we cannot pronounce an idea that we have in mind because the appropriate word is missing.

3. Mechanical stress in fields

We have to give substance to the field concept. We have to discuss properties that do not only manifest at its boundaries, i.e. the places where it is connected to material bodies. Since the field „transmits“ forces, it must be under mechanical stress, just a material object that transmits a force. These stresses are well-known since the introduction of the field concept by Faraday and the functional dependence on the field strength is known since Maxwell. Nevertheless, in the text books they are treated rather stepmotherly; sometimes they are called „fictitious“ stresses.

4. Forces and force-laws

Forces that are transmitted by fields, i.e. momentum currents that flow through fields, manifest mathematically in laws of three very distinct types.

In Coulomb's law

$$\vec{F} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q_1 \cdot Q_2}{r^2} \cdot \frac{\vec{r}}{|\vec{r}|} \qquad (1)$$

only two charges appear; there is no electric field strength.

In the relation

$$\vec{F} = Q_1 \cdot \vec{E}_2 \qquad (2)$$

there is only Q_1. The other charge is represented by the field strength $\vec{E}_2$. In

$$|\vec{F}| = A \cdot \frac{\varepsilon_0}{2} |\vec{E}| \qquad (3)$$

finally there is only one field strength.

Each of the three equations can be used to describe any single given force. However, the practicality of them is very different, according to the type of problem to solve, and according to the role they play in the conceptual structure of physics.

Regarding the first criterion, i.e. the type of problem: According to the symmetry of the problem one or the other of the three equations is the most appropriate for calculating $\vec{F}$. Coulomb's law is suitable if we have to do with two point charges. Also equation (2) would be appropriate. However, to calculate the force in this case by means of equation (3) would require a cumbersome integration. Equation (2) is convenient if a point charge is in a field that would have a simple field strength distribution as long as the point charge is not there, for instance if this field would be homogeneous. Both equations (1) and (3) would require a complicated integration. Equation (3) finally is appropriate in the case that the *resulting field* has a simple structure. An example is the homogeneous field of a plate capacitor.

Thus, from a practical point of view, each of the three laws has its reason to exist. The assessment is different when asking for the best way to introduce forces (or momentum currents) within the field.

It is not advisable to use the Coulomb law for this purpose, since it suggests an action-at-a-distance interpretation of forces. The medium or whatever we will call the „something" between the charged bodies does not appear in the law. The law of equation (2), that often is used to introduce the field strength, is in our opinion the cause of some confusion concerning the concept of a field. When saying that a body with electric charge Q_1 is placed within a field of the field strength $\vec{E}_2$, one is actually not saying the truth, because as soon as the charged body is in its position the field strength is no longer $\vec{E}_2$; the presence of the body may change it substantially.

In our course we prefer to emphasize the tensional and compressional stress within the field and these are best represented by equation (3). In this way the local causes interpretation of electric (and also magnetic) interactions is obtained. The idea of the field being a really existing physical system is promoted. It requires a verbal description that corresponds to a very concrete idea of the field. So one would say that a field pulls on a charged body, instead of saying that a force is acting on it.

5. Introduction of the field strength

The intention to take the field seriously as a physical system precludes the customary introduction of the field strength: the electric field strength via $F = Q \cdot E$, and the magnetic field strength over $F = Q \cdot v \cdot B$.

Both equations prevent us from considering the field that is actually present, since the field strength that has to be substituted in these equations is that of the field without the „test charge". Normally, when measuring something, one is allowed to believe that one measures a value that is actually realized by the considered system. In our case that is not true. Notice that we are not in the well-known situation that the measuring procedure perturbs the measured value and that the error can be made arbitrarily small by choosing an appropriate measuring instrument. A voltmeter must have in internal resistance that is sufficiently great, a thermometer must have a heat capacity that is… etc. In our case the measurement is correct even if the charge value of the test charge is very large. We therefore emphasize in our course that this way of measuring the field strength is only one method among others.

6. The „real“ field

Often a physical quantity is confused with the physical system to which it belongs. This can be experienced particularly clearly in discussions about which of the two quantities *H* and *B* is the more fundamental one. Today most textbook authors are in favor of *B*. We will later see that this is awkward for a treatment of magnetostatics. At the moment we are interested in the justification for favoring *B*. It is argued, *B* (and by some authors *H*) is the *real* field or the *fundamental* field, whereas the other one is a kind of „auxiliary quantity“. In our opinion such arguments should have no place in physics. Neither *B* nor *H* *is* the magnetic field. Both are quantities with which we can describe the field. And there are a lot more of such quantities: the magnetic vector potential, the scalar magnetic potential, the energy density, the mechanical stress; and finally we can construct or define as many other quantities as we want. Non of these quantities *is* the field. Of course the question is reasonable to ask which of these quantities is appropriate for the description of a certain phenomenon. It turns out that depending on the phenomenon to be described or the question to be answered a different choice will be made. For this reason a compromise has to be made. If we use few different quantities, the description of a phenomenon may become cumbersome. If we introduce too many, the whole teaching will become too complex.

7. Analogies in electromagnetism

In electromagnetism there are several internal symmetries or analogies. Analogy means that, starting from a valid relation between physical quantities one obtains another valid relation by a purely formal replacement of the physical quantities according to certain given rules. In the table the most important physical quantities that correspond to one another are listed for three of these analogies.

1.

electric field strength $\vec{E}$	magnetic field strength $\vec{H}$
electric potential ϕ	magnetic scalar potential ϕ_m
electric charge Q	magnetic charge Q_m
electric flux density $\vec{D}$	magnetic flux density $\vec{B}$
electric field constant ε_0	magnetic field constant μ_0

2.

electric field strength $\vec{E}$	magnetic flux density $\vec{B}$
electric potential ϕ	magnetic vector potential $\vec{A}$
electric charge Q	electric current I
electric field constant ε_0	reciprocal of magnetic field constant $1/\mu_0$

3.

voltage U	electric current I
electric charge Q	magnetic flux Φ
capacitance C	inductance L
electric field strength $\vec{E}$	magnetic field strength $\vec{H}$

Taking profit of an analogy in general contributes to the economy of learning and is thus a good idea. However, there is also a danger, that can be seen particularly well in electromagnetism.

One problem is, that analogy 1 and analogy 2 (in our table) are in competition with each other, and one should avoid to use both of them when teaching to beginners. One has to decide for one of them. Today, most authors and teachers choose the second one. We will explain later on, why we prefer the first one.

A second problem in the context of analogies and symmetries: Treating one of them rigorously with all its consequences, our presentation may become very esthetic. However, it can happen that the resulting presentation becomes somewhat unworldly. This can be seen with our analogy 1: If one decides to introduce the electric potential right at the beginning together with the electric field strength, as is usually done, one might tend to also introduce the magnetic scalar potential. However, an electric potential difference can easily be measured, whereas measuring a magnetic potential difference is intricate.

8. What belongs to electromagnetism in school

What is treated in a common textbook for the upper secondary school is sufficient to pass successfully the bachelor exam in physics at the university. In our opinion it goes beyond what should be learnt at a mainstream school and beyond what can be expected from a high school student.

This observation is not in contradiction with the eternal complaints of the universities about the insufficient preparation of the school leavers.

The fact that electromagnetism has spread so much may have several causes. Simplifying seems to be possible only if the goal is a lower secondary course that is restricted to qualitative statements. If quantitive results are to be obtained, so it seems, the whole of Maxwell's theory must be treated. Each stone that is taken out leads to a collapse of the whole building.

However, if we insist that the students are able to solve those problems that are given in our final exams, the potential for a simplification is small.

We would like to recommend to distance from these high teaching objectives. Calculating the trajectory of a charged particle in a mass spectrometer cannot be considered a goal of a general education. The only justification to treat such questions would be that by doing so the students acquire other more fundamental insights. We doubt that this is the case. Finally, isn't it true for every physics course that we solve those problems that we are able to solve, and not necessarily those that we want to solve? Since we have to take account of the actual practice of the final exams, our electromagnetism has become larger than we would like it to be.

So, the omissions that we have allowed ourselves were no more than a poor compromise.

If we had been completely free in our choice we would have concentrated to present the electromagnetic field as one of the two main constituents of the world, namely field and matter. We would not have reduced our treatment of the field to its mechanical properties, but just as we do with matter, we would have put emphasis also on the thermal and chemical properties of the fields.

Remarks

1. The electric field

1.2 The electric potential

We recommend to introduce the electric potential together with the voltage. This simplifies the understanding of the voltage and has advantages for the treatment of the electric field: the field will be introduced by means of a quantity that the students know already.

1.9 Charge and charge carriers

We emphasize that one has to distinguish between the *physical quantity* „electric charge“ and the *physical system* „charge carrier“. Otherwise it may happen that the students identify the electric charge and the electrons. Electrons, ions and other particles carry not only charge, but also the (substance-like) quantities: mass, amount of substance, angular momentum, momentum, entropy etc. The electron for instance „carries“:

electric charge	$Q = e = -1.602 \cdot 10^{-19}\,C^{-19}$,
mass	$m = m_e = 9.11 \cdot 10^{-31}\,kg$,
angular momentum	$L = h/4\pi = 0.527 \cdot 10^{-34}\,Js$,
amount of substance	$n = 1/N_A = 1.66 \cdot 10^{-24}\,mol$.

The values of the momentum and the entropy depend on the state of the electron.

1.13 The electric field strength

The equation

$$\vec{E} = \frac{\vec{F}}{Q}$$

is the first relation the students get to know, in which the electric field strength appears. It thus plays the role of a defining equation for the field strength. However, it should not be the vehicle that serves to create a pictorial idea of the field. The statement that precedes the equation does not interpret the absolute value of the field strength as a measure of the force that is exerted on al test body, but as a measure for something that is there as long as the test body is not there. And we do not interpret the direction of the field strength vector as the direction of the force on the test body, but as the direction in which we have a tensional stress within the field without the test body.

1.14 Graphical representation of electric fields

1. Our mental representations are considerably shaped by pictures. If we visualize fields mainly by field line pictures, as it is usually done, an idea of a field is created that is characterized by the properties of lines. We have made the experience that physics students at the university, in an oral examination for instance, employ the word „field line“ when they are supposed to speak about the physical system „field“. They identify the graphical representation with the object that is represented. We do not want to criticize such a behavior in general, since it is our intention to create an pictorial idea of the object „field“. In this particular case, however, an idea of the field is created that often causes incorrect conclusions. The field line picture supports the idea that the field is pulling on a body on which the field lines end. However, the transverse direction comes off badly. The pressure perpendicular to the field lines appears to be less real.

There are other reasons why field line picture can mislead. Suppose we want to get an idea about „how much field“ there is in a given space element. A reasonable measure for it is the energy density, i.e. essentially the square of the field strength (just as we can consider a measure for the density of matter the square of the wave function, and not the wave function itself).

We consider the electric field of a charged conducting sphere. The energy density decreases with the 4th power of the distance from the center. A three-dimensional field line model suggests a second power decrease, because the field line density decreases this way. A two-dimensional projection of the sphere with its field lines even suggests a linear decrease. The impression results, that the field reaches far away into the surrounding space. That is why we prefer to represent the field by a shading that is limited so a rather small proximity of the body.

2. We want to warn against some wrong conclusions that might be drawn by someone who is not familiar with field lines and field surfaces.

Field surfaces tell us about the *direction* of the compressional stress; however, they are not isobaric lines, i.e. lines of constant pressure. Consider an electrically charged conducting sphere. The field pulls at the surface outwards. But on what does the field itself hang? Since the field lines go straight away „to infinity“ one might suspect that the tensional stress is „conducted away“ to infinity. (We know that „infinity“ has often to help us to solve a problem.) This would require that the tensional stress decreases with the square of the distance. However, it does not so. Like the energy density it decreases with the forth power of the distance. Actually and fortunately the field does not reach so far. As the spring-and-ring model in the students' text shows, the field hangs on itself: the tension in the radial direction causes a compressional stress in the direction of a circumference.

3. In class we use the designations „homogeneous“ and „isotropic“, and we discuss examples of homogenous and isotropic media. In this regard, a temperature distribution can be homogenous and isotropic. A piece of veined wood can be homogeneous without being isotropic. The electric field turns out to be a „material“ that is not isotropic.

4. On the blackboard and in the students' exercise books many pictures of fields with their field lines and field surfaces, that also appear as lines on the two-dimensional paper, are drawn. We agree upon two colors for the two types of lines.

1.21 Surfaces of constant potential

1. A central experiment in our approach to the field concept is the measuring of equipotential surfaces in water, i.e. a liquid that is a poor conductor, by moving around a probe that is connected to a voltmeter. The experiment is important, because we to not want to define the potential by moving „a point charge in a field“ and measuring the energy that is required to displace the charged body and dividing it by the charge. In this case one would again have the situation that the potential is measured in a field that would be there if the point charge were not there.

2. We briefly want to explain, why and under which conditions this experiment gives the correct result. In the stationary state we have everywhere within the water

$$\mathrm{div}\,\vec{j} = 0$$

where $\vec{j}$ is the electric current density.

The relation between the current density and the electric field strength $\vec{E}$ is:

$$\vec{j} = \sigma \cdot \vec{E}$$

Here, σ is the electric conductivity. Inserting in the previous equation we obtain

$$\mathrm{div}\,(\sigma \cdot \vec{E}) = 0$$

If σ is independent of the position, it can be written outside the div operator

$$\sigma\,\mathrm{div}\,\vec{E} = 0$$

and we obtain

$$\mathrm{div}\,\vec{E} = 0$$

This is the equation for the electric field without the water. Thus, the condition that the potential distribution in the water is the same as in the purely electrostatic case is that σ has to be independent of the position within the water. It is easy to find examples where the original potential distribution is distorted by a conductivity that depends on the position.

1.23 The energy of the electric field

If the integral has already been introduced in the math class, the formula for the energy can be obtained in a more elegant way.

1.27 How to load electrically charged particles with energy – electron beams

1. The potential difference $\phi_2 - \phi_1$ in the equation $dE = (\phi_2 - \phi_1) \cdot dQ$ is that of the field without the test body. The traditional wording is still based on the old idea of an action at a distance, in wich the potential energy and also the potential is attributed to the test body. In Maxwell's electromagnetism however the potential is a quantity of the field, in our case of the field without the test charge.

2. We would prefer a better name for the so-called particle accelerators or colliders. Unfortunately no other names are in use, and we did not decide to introduce a new one. The machines are used to charge the particles with momentum and energy. The name accelerator is not convenient since a long time, because for a modern machine the particle have the terminal velocity of c already at the entrance. They cannot be accelerated further. „Colliding“ is an important process, but it points to the experiment that is carried out after the particles are charged with momentum.

2. The magnetic field

2.1 Magnetic charge and magnetic field

Sometimes, the path of the historical development was weird. Often, concepts that are not needed survive for centuries. Sometimes good concepts disappear due to a misunderstanding. An example for the latter phenomenon is the magnetic charge. The argument against it is: It does not exist, because there are no magnetic monopoles. However, already this formulation is mistakable. One should better say: there are not particles that carry a net magnetic charge. Then it would become clear that one cannot conclude that magnetic charge does not exist.

That there are no magnetically charged particles is an observational fact. Apparently the world is made this way. But the possibility cannot be ruled out that one day such particles will be discovered or produced.

If a physical quantity „magnetic charge“ exists or does not exist depends only on whether we define it and are able to measure its values. As many elder textbooks of electromagnetism show, the definition is very simple. The magnetic charge at the surface of a magnet can be introduced in perfect analogy to the bound electric charge on the surface of a dielectric.

So, the question is not, whether magnetic charge exists or not, but rather whether its introduction is advantageous or not. The answer to this question is simple: It is advantageous. The gap that arose when magnetic charge disappeared from most of the textbooks, is best seen in the cumbersome and nebulous way in which magnetic poles are described. Positive and negative values of the magnetic charge are described by the specifications north and south. The simple fact, that the total charge of any body is zero, can only be expressed indirectly by describing an experiment in which the property manifests itself, for instance by showing that two new poles are created when a bar magnet is broken through.

2.2 Magnetization

Usually magnetization is introduced in such a way that the impression results that the quantity is a subject for an advanced level, i.e. for university students only. We believe, that on the contrary magnetization is the quantity that is most appropriate for a first approach to magnetism. In electrostatics one usually considers a charge distribution (a distribution of charged bodies) as given, and one asks for the electric field. Correspondingly, in magnetostatics one would consider as given a distribution of magnetic poles. It might be felt as a disadvantage that there are no bodies that carry a net magnetic charge. But this is also an advantage: There is a very simple relationship between the magnetization and the distribution of the magnetic charge. Magnetic charge is located where magnetization lines begin or end. It turns out that this fact is very helpful for the understanding of magnetism at the secondary school. Among the three quantities $\vec{H}$, $\vec{B}$ and $\vec{M}$ it is that one, whose distribution can most easily be found out.

2.6 Soft magnetic materials

The statements about soft magnetic materials are correct only as long as the material does not get into the state of saturation. This fact must not necessarily be mentioned in class – just as we hardly mention that Hooke’s law is no longer correct when a spring is overstretched. One could say that as soon as saturation begins, the material is no longer soft magnetic.

2.7 Electric current and magnetic field

We have formulated that the field surfaces of the magnetic field end an electric currents. This does not mean that the field pushes only on the surface of a conducting wire. As is well-known, the magnetic field reaches inside the conductor. Its field strength decreases from the surface of a wire until the axis linearly. Thus, the field affects the whole volume of the conductor, similar to the electric field of a homogeneously charged sphere or the gravitational field of a massive body.

2.12 Magnetic field strength, magnetization and magnetic flux density

In class it is not worthwhile to treat the magnetization algebraically. Therefore, our definition of this quantity is rather brief. It is needed only for the transition form the field strength to the flux density.

2.13 The coil – the inductance

Usually, the inductance is introduced in the context of the phenomenon of induction, namely via the equation

$$U = L\frac{dI}{dt}$$

Its name suggests that the quantity has something to do with induction. When proceeding in this way, however, *L* appears rather unintuitive. It is like introducing the electric capacitance by means of the equation

$$I = C\frac{dU}{dt}$$

Although this equation is correct, it hardly is appropriate for a first understanding of the capacitance.

Just as it is simpler to get an idea of the capacitance by means of

$$Q = C \cdot U$$

one gets a more direct access to the inductance by defining it by means of the equation

$$\Phi = \frac{1}{n}L \cdot I$$

Just as the capacitance tells us whether for a given voltage there is much or few charge on the plates of a capacitor, the inductance tells us, whether a coil attains a great or small magnetic flux for a given electric current.

2.15 „Discharge“ of the coil

In this section we discuss the phenomenon that usually is called self-induction. The concept of self-induction is in our opinion a somewhat unfortunate idea. It suggests (and sometimes it is said explicitly) that within a coil a voltage or an electric field is created – what is incorrect. To the integral

$$\oint \vec{E}\, d\vec{r} = -N\dot{\Phi}$$

contribute only sections of the closed path that are outside of the coil. That the name „self-induction“ is not a good choice can also be seen if one tries to construct the analog concept for the capacitor.

2.16 How the magnetic field presses on an electric current

In this section we discuss what normally is treated under the heading „Lorentz force“. Of course, we have to justify why we refuse an important equation its traditional name. A determinative in front of the name of a physical quantity force suggests, that we deal with a force of a particular kind, a force that appears only if an electrically charged body moves through a magnetic field. However, that is not what we want to express. In the framework of our presentation of electromagnetism this force is nothing more than an application of a more general statement, that the students have learned before: It is a manifestation of the compressional stress that exists in every magnetic field. When giving a proper name to this force, only because a field is caused by a moving body, one should consequently give names to many other up to now nameless forces. Instead of facilitating the insight that all forces are the same quantity the impression of a great complexity would be the result.

3. The interplay between electric and magnetic fields

3.2 Electromagnetic induction

1. The discussion of reference frame changes is instructive, but usually it is complicated. As an example consider rotational movements in mechanics. The description in a rotating reference frame is interesting but intricate. Forces appear that do not exist in the non-rotating frame. The situation is similar in electromagnetism. When the reference frame is changed new fields appear: a purely electric field becomes electric and magnetic and vice versa. What in one reference frame is described by the first of Maxwell's equations finds its explanation in another frame by the second equation, and vice versa. When discussing one phenomenon not only in one but in any arbitrarily chosen reference frame, the teaching time increases considerably. In view of the permanent lack of time we have decided to exclude questions that have to do with a change of the reference frame. They will be treated bundled and treated in a special section of the mechanics volume. We believe that thereby no deficit will show up with respect to the treatment of the phenomenon of induction, in particular since the integral formulation of the law of induction (or Maxwell's second equation) obscures the reference frame problem in a tricky way.

2. In the upper secondary physics class often Lenz's law is formulated. Then with Lenz's law one tries to justify the minus sign in Faraday's law. The argument is as follows: An iron core is introduced into a coil that is connected to a battery. One observes, that the electric current decreases for a short moment. One now argues: $d\Phi/dt$ is positive, the induced emf is negative, thus there must be a minus sign in Faraday's law. However, this argument contains a fallacy: How do we obtain the sign of the rate of change of the magnetic flux?

We have prescinded from introducing Lenz's law as well as from writing a minus sign in the law of electromagnetic induction.

If one makes a statement about the sign in an equation, it should be possible to check the statement by applying the equation. The equation tells us that the induced emf has the opposite sign of the rate of change of the magnetic flux. To check the sign one should be able to measure both these quantities including the algebraic sign.

Imagine the following change of a flux density is realized: The $\vec{B}$ vector points in the positive *x*-direction and its magnitude, and thus, its *x* component increases. Which is now the sign of $d\Phi/dt$? Will the sign remain the same when the coordinate system is inverted, so that the $\vec{B}$ vector now points in the negative *x*-direction? Our flux is surrounded by an electric conductor with a voltmeter, placed in the *y*-*z* plane. How do we read on the voltmeter whether the emf is positive or negative?

Since our students don't learn how to answer these questions, the minus sign that we had so tediously worked out is without any value.

Instead of Lenz's law we introduce the „left-hand rule", formulated in analogy to the already known right-hand rule.

3.5 The transformer

The two conditions

$$\frac{U_1}{U_2} = \frac{n_1}{n_2}$$

and

$$n_1 \cdot I_1 = n_2 \cdot I_2$$

are valid simultaneously only for a certain interval of the load of the transformer. This can be seen with particular clarity, when considering the two extreme cases:

open circuit: $R_{\text{load}} = \infty$

short circuit: $R_{\text{load}} = 0$.

In the open circuit case the second equation cannot apply, since I_2 = 0 A, but $I_1 \neq 0$ A. In the short circuit case we have $U_1 \neq 0$ V, but U_2 = 0 V, thus the first equation cannot be valid.

Also the equation for the energy currents P_1 and P_2 is correct only for this interval of R_{load} . The derivation of the boundaries of this interval is too intricate for the school. Since normally a transformer is used for a load that lies within this interval, we limit the treatment to the simple description in which both above equations are valid.

3.7 Superconductors

1. The properties of superconductors which are discussed in the students' text are those of a type-I superconductor. Type-II superconductors are more complicated. Their relation to type-I superconductors is similar to that of a body that under pressure deforms inelastically to one that deforms elastically. Notice that high-temperature superconductors are typical type-II superconductors. For them it is not true that the magnetic field cannot penetrate the superconducting material.

3.9 Electromagnetic waves

1. It is a tradition to introduce electromagnetic waves via the oscillating circuit. We did not follow this tradition for several reasons.

The explanation aims from the beginning at the complicated field of the dipole antenna. In this case we have to do not only with in intricate field distribution but also with the difficult distinction between the near and the far field. Both is not necessary for the understanding of an electromagnetic wave. There are wave types that are simpler. In order to explain the basics of wave propagation we prefer to limit to these simple cases: the square wave and the sine wave.

2. The traditional introduction begins by explaining how to generate a wave. However, it is much more difficult to understand the process of creation than the wave itself. It is like introducing sound waves by explaining the working principle of a clarinet. The clarinet is a resonator in which an energy current flows back and forth and a small fraction of this current is leaking out. Also the dipole antenna is a resonator, and also here energy is flowing back and forth in the vicinity of the antenna, and only a small fraction of it is being emitted.

3. We insist in giving a representation of the wave in three-dimensional space, see figures 3.29 and 3.35 in the student's manual. It is a popular method of representing a wave by drawing a space coordinate *z* to the right, in the direction of propagation of the wave, the *x* component of the electric field strength upwards and the *y* component of the magnetic field strength forwards. This representation is familiar to every physicist. However, in our opinion it is somewhat treacherous, and for our students too difficult. So, there is the question: Does the vertical axis have another dimension (that of an electric field strength) than the forward axis (magnetic field strength)? Or are the electric and the magnetic field strengths both represented as vector quantities in a plane perpendicular to the *z* direction? The worst drawback however is that the students misunderstand the whole image as a representation in the position space. Students who have only seen this representation, are completely perplexed when being asked for the field distribution in three dimensions.

III

Solutions to problems

1. The electric field

1. 3 The zero-point of the electric potential

1. Point 1: 4.5 V, Point 2: 0 V, Point 3: –4.5 V

2. Point 1: 0 V, Point 2: –12 V, Point 3: 0 V

3. Voltmeter 1: 18 V, Voltmeter 2: 9 V, Voltmeter 3: 9 V

5. Circuit in bicycle, car, airplane, rocket, satellite

1.4 Electrotechnical problems

1. Clockwise, beginning with the straight section of the conductor: 0 V; 4.5 V; 9 V; 5 V

2. Lamp at the left: 1.6 A, lamp at the right: 0 A

3. Sections A: –20 V; B: 0 V; D: 40 V.
The battery generates 60 V. When the switch is open all the potentials of the points A to D are 0 V.

4. (a) $\phi_P = 12$ V, $U_{L1} = 12$ V, $U_{L2} = 0$ V
(b) $\phi_P = 6$ V, $U_{L1} = 6$ V, $U_{L2} = 6$ V

The strength of the current in lamp L1 is greater when the switch is closed, because in this case the voltage over L1 is greater.

The strength of the current in lamp L2 is greater when the switch is open. When the switch is closed the current is 0 A.

5. (a) $\phi_1 = 0$ V, $\phi_2 = 150$ V, $\phi_3 = \phi_4 = 75$ V
(b) $\phi_1 = \phi_4 = 0$ V, $\phi_2 = \phi_3 = 150$ V

When the switch is open only the right lamp lights up.

1.5 Characteristic curves – the electric resistance

1. $U = 20$ V
$I = 4$ mA

$$R = \frac{U}{I} = \frac{20\text{ V}}{0{,}004\text{ A}} = 5\text{ k}\Omega$$

2. $R = 2$ kΩ
$U = 120$ V

$$I = \frac{U}{R} = \frac{120\text{ V}}{2000\ \Omega} = 60\text{ mA}$$

3. $R = 1$ MΩ
$I = 0.1$ mA

$$U = R \cdot I = 10^6\,\Omega \cdot 10^{-4}\text{A} = 100\text{ V}$$

4. $U = 35$ V
$I = 5$ A
$U_1 = 10$ V

$$R_1 = \frac{U_1}{I} = \frac{10\text{ V}}{5\text{ A}} = 2\ \Omega$$

$$U_2 = U - U_1 = 35\text{ V} - 10\text{ V} = 25\text{ V}$$

$$R_2 = \frac{U_2}{I} = \frac{25\text{ V}}{5\text{ A}} = 5\ \Omega$$

5. $U = 12$ V
$R_i = 100$ Ω

$\phi_{\text{above}} = 12$ V, $\phi_{\text{middle}} = 6$ V, $\phi_{\text{below}} = 0$ V

$U_1 = U_2 = 6$ V, $U_3 = 12$ V

$$I_{1,2} = \frac{U}{R_1 + R_2} = \frac{12\text{ V}}{200\ \Omega} = 60\text{ mA}$$

$$I_3 = \frac{U}{R_3} = \frac{12\text{ V}}{100\ \Omega} = 120\text{ mA}$$

$I_{\text{batt}} = I_{1,2} + I_3 = 180$ mA

6.

(a) $R_{\text{total}} = \frac{U}{I} = \frac{U}{I_1 + I_2} = \frac{U}{\frac{U}{R} + \frac{U}{R}} = \frac{R}{2}$

Total resistance = half of the resistance of the single resistors

(b) $R_{\text{total}} = \frac{U}{I} = \frac{U_1 + U_2}{I} = \frac{RI + RI}{I} = 2R$

Total resistance = twice the resistance of the single resistors

Similar rules are valid for the spring constant when connecting springs in parallel and in series, however the other way round (or in the same way for the reciprocal values of the spring constants).

1.6 The resistance of voltmeter and ammeter

Voltmeter and ammeter are connected incorrectly. The ammeter causes a short circuit of the battery. Thereby it may break. The voltmeter interrupts the circuit.

1.7 The electric conductivity

1. Cross sectional area: $A = 2$ mm²
Length of forward and return line: $l = 100$ m
Copper wire: $\sigma = 5.59 \cdot 10^7\ (\Omega \cdot \text{m})^{-1}$

$$R = \frac{100\text{ m}}{5.59 \cdot 10^7\ (\Omega\text{m})^{-1} \cdot 2 \cdot 10^{-6}\text{ m}^2} = 0.9\ \Omega$$

2.

$$\frac{l_1}{\sigma_1} = \frac{l_2}{\sigma_2}$$

$$l_1 = l_2 \frac{\sigma_1}{\sigma_2} = 1\text{ m} \cdot \frac{5.59 \cdot 10^7}{10^{-13}} \approx 6 \cdot 10^{20}\text{m}$$

The conductor could cross the whole Milky Way.

3. By increasing the concentration of the salt

1.10 Charge current and charge carrier current

1. The electric current flows to the right. The current strength is

0.5 A + 0.3 A = 0.8 A.

2. 2 C/(1,6 · 10^{-19} C) = 1.25 · 10^{19}

1.12 The electric field

1. When the two spheres are brought in contact negative charge carriers flow form B to A, so that B will end up with a positive net charge. Now both spheres are positively charged and the field pushes them away from each other.

2. An attraction or repulsion is also observed if the charged bodies are made of a material that cannot be magnetized, aluminum for instance.

3. When in contact with B, A will charge with electricity that it receives from B. Now sphere A is pushed away from B, so that it will touch C. Here A first discharges and then charges again with electricity from C. Therefore, the field pushes A back to B, etc. Thus A swings back and forth between B and C.

1.13 The electric field strength

1. $F = Q \cdot |\vec{E}| = 1.6 \cdot 10^{-19}\text{C} \cdot 80\,000\text{ V/m} = 1.28 \cdot 10^{-19}\text{N}$

2. $|\vec{E}| = \frac{F}{Q} = \frac{0.02\text{ N}}{10 \cdot 10^{-9}\text{C}} = 2 \cdot 10^6\text{ V/m}$

1.15 Rules for drawing electric fields

1. (orthogonal to the field lines)

2. (orthogonal to the field surfaces)

3. (Charges are inside of the three closed field surfaces as well as inside of the bulged field surface at the left side below; field lines orthogonal to the field surfaces)

1.16 Four important electric fields

1. The fields are not different at all. In both cases the field lines must begin somewhere inside of the sphere. Since both the field and the sphere are spherically symmetric, the field lines can only run radially away. The field surfaces are concentric spherical surfaces.

2. Apart from the sign of the charge and the orientation of the field lines, the field as well as the charge has two mirror axes. If the sign of the charge and the orientation of the field lines are taken into account, the mirror image at the vertical axis transforms into a „reflection + change of sign + change of direction of arrows".

3. The charge and the field have the same symmetry: a vertical and a horizontal mirror axis through the center of the figure.

1.17 Calculation of electric field strengths

1. For $r > R$ the field strength is zero everywhere.

2. In the equation
$F = Q \cdot |\vec{E}|$

Q is the charge of a body A, and $|\vec{E}|$ is the strength of the field at the position of A as long as A is not yet there and that is caused by another body B. Therefore we write:
$F = Q_A \cdot |\vec{E}_B|$

In the equation
$$|\vec{E}(r)| = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q}{r^2}$$

Q is the charge of body B and $|\vec{E}|$ is the strength of the field, that is belongs to this charge. We therefore write:
$$|\vec{E}_B(r)| = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q_B}{r^2}$$

and obtain:
$$F = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q_A \cdot Q_B}{r^2}$$

Here r is the distance between the charged bodies A and B. The analogue mechanical law is Newton's law of gravitation.

1.18 Several charged bodies – the addition of vectors

1. See Fig. 4

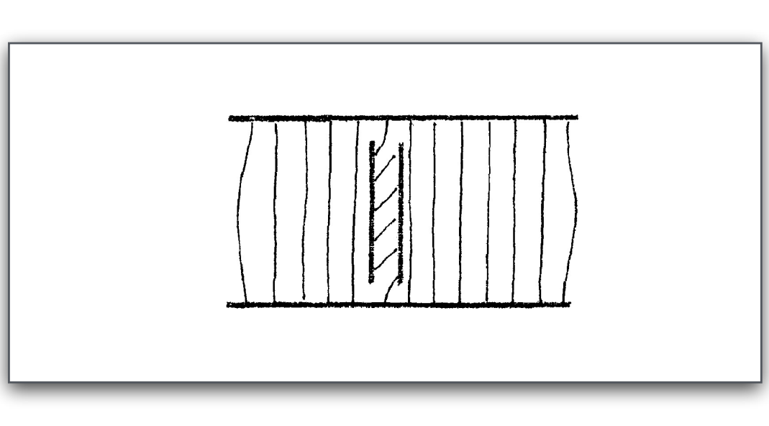

Fig. 4
For 1.18, exercise **1**

2. Upper plate: Field lines start vertically upwards and downwards. Lower plate: Field lines coming from above and from below end on the plate.

Above the upper plate and below the lower plate the field strengths add up to zero; there is no field stuff.

3.

A: Field strength is zero
B: Field strength vector points down right (45°)
C: Field strength vector points vertically down
D: Field strength vector points top right (45°)

1.19 Pressure and tension within the electric field

1. The easiest way is to consider the vertical midplane: There is pure tensional stress in the horizontal direction. This is as if the left part of the figure (left sphere + left half of the field) were connected to the right one (right sphere + right half of the field) by stretched springs.

One can also consider one of the two spheres, the left one for example. Here, the field pulls outwards at all sides of the sphere. Since the field strength is greater at the right side of the sphere than at the left side, a net pull to the right results.

2. Analogous to exercise **1**. In the midplane there is pressure in the horizontal direction. The field pulls more strongly on the left side of the left sphere, than on its right side.

3.

$$|\vec{E}| = \sqrt{\frac{2F}{{}_0A}}$$

$$= \sqrt{\frac{2 \cdot 0.0025\text{ N}}{8.85 \cdot 10^{-12}\text{ C/(V·m)} \cdot 0.24\text{ m}^2}} = 4.85 \cdot 10^4\text{ V/m}$$

4. Field lines are radial, field surfaces show up in the two-dimensional figure as concentric circles.

1.20 Capacitor and capacitance

1. $Q = I \cdot t = 20\ \mu\text{A} \cdot 10\text{ ms} = 200\text{ nC}$
$C = Q/U = 200\text{ nC}/60\text{ V} = 3.3\text{ nF}$

2. $Q = C \cdot U = 16\ \mu\text{F} \cdot 10\text{ V} = 160\ \mu\text{C}$

3. $d = \varepsilon_0 A/C$
$= 8.854 \cdot 10^{-12}\text{ C/(V·m)} \cdot 0{,}5\text{ m}^2/(10^{-6}\text{ F})$
$= 4.4 \cdot 10^{-6}\text{ m} = 4.4\ \mu\text{m}$

4. Two parallel capacitors with the same plate distance and the same plate surface can be considered as one single capacitor, with twice the plate surface. Therefore the total capacitance is twice the capacitance of a single capacitor.

The total capacitance of n capacitors of equal capacitances connected in parallel is n times the capacitance of one single capacitor.

5. Two capacitors with the same plate distance and the same plate surface connected in series can be considered one single capacitor with twice the distance between the plates of a single one. Therefore, its capacitance is half that of a single capacitor.

The total capacitance of n capacitors of equal capacitances connected in series is $1/n$ times the capacitance of a single capacitor.

1.21 Surfaces of constant potential

1.

$$|\vec{E}| = \frac{U}{d} = \frac{2000\text{ V}}{0.5\text{ cm}} = 400\,000\text{ V/m}$$

2.

Point P: $|\vec{E}| = \frac{5\text{ V}}{0.6\text{ mm}} = 8300\text{ V/m}$

Point Q: $|\vec{E}| = \frac{5\text{ V}}{0.2\text{ mm}} = 25\,000\text{ V/m}$

1.22 More about the capacitor

$C' = 3C$
(1) $Q = \text{const}$
$Q' = Q, U' = U/3, |\vec{E}'| = |\vec{E}|/3$
(2) $U = \text{const}$
$Q' = 3Q, U' = U, |\vec{E}'| = |\vec{E}|$

1.23 The energy of the electric field

1.
(a) $Q = I \cdot t = 0.01\text{ A} \cdot 8\text{ s} = 0.08\text{ C}$

(b) $E = \frac{Q^2}{2C} = \frac{0.08^2}{2 \cdot 16 \cdot 10^{-6}}\text{ J} = 200\text{ J}$

2.

$$E = \frac{CU^2}{2} = \frac{80 \cdot 10^{-6} \cdot 300^2}{2}\text{ J} = 3.6\text{ J}$$

$$|\vec{E}| = \frac{U}{d} = \frac{300\text{ V}}{8 \cdot 10^{-6}\text{m}} = 3.75 \cdot 10^7\text{ V/m}$$

$$\rho_E = \frac{\varepsilon_0}{2} \cdot |\vec{E}|^2$$

$$= \frac{8.854 \cdot 10^{-12}\text{ C/(V·m)}}{2} \cdot (3.75 \cdot 10^7\text{ V/m})^2$$

$$= 6.23 \cdot 10^3\text{ J/m}^3$$

3.

$$E = \frac{CU^2}{2} \Rightarrow C = \frac{2E}{U^2} = \frac{2 \cdot 1.6\text{ J}}{10^8\text{ V}^2} = 32\text{ nF}$$

$$Q = C \cdot U = 32\text{ nF} \cdot 10^4\text{ V} = 320\ \mu\text{C}$$

4.

(a) $C = \varepsilon_0 \frac{A}{d} = 8.854 \cdot 10^{-12}\text{ C/(Vm)} \frac{2\pi \cdot 0.0245\text{ m} \cdot 0.12\text{ m}}{0.001\text{ m}} = 0.163\text{ nF}$

(b) $Q = C \cdot U = 0.163\text{ nF} \cdot 2000\text{ V} = 0.326\ \mu\text{C}$

(c) $E = \frac{CU^2}{2} = \frac{0.163\text{ nF} \cdot (2000\text{ V})^2}{2} = 0.326\text{ mJ}$

(d) $\rho_E = \frac{E}{A \cdot d} = \frac{0.326\text{ mJ}}{2\pi \cdot 0.0245\text{ m} \cdot 0.12\text{ m} \cdot 0.001\text{ m}} = 17.6\text{ J/m}^3$

5. Increasing plate distance:

(a) $Q' = Q$, $C' = C/3$
With $E = Q^2/2C$ we obtain $E' = 3E$.

(b) $U' = U$, $C' = C/3$
With $E = U^2 \cdot C/2$ we obtain $E' = E/3$.

Increasing plate surface:

(a) $Q' = Q$, $C' = 3C$
With $E = Q^2/2C$ we obtain $E' = E/3$.

(b) $U' = U$, $C' = 3C$
With $E = U^2 \cdot C/2$ we obtain $E' = 3E$.

6.

before		after	
A	B	A	B
Q_0	0	$Q_0/2$	$Q_0/2$
U_0	0	$U_0/2$	$U_0/2$
$\lvert\vec{E}_0\rvert$	0	$\lvert\vec{E}_0\rvert/2$	$\lvert\vec{E}_0\rvert/2$
$\frac{C}{2}Q_0^2$	0	$\frac{C}{2}\frac{Q_0^2}{4}$	$\frac{C}{2}\frac{Q_0^2}{4}$

$$\underbrace{\frac{C}{2}\frac{Q_0^2}{4} \quad \frac{C}{2}\frac{Q_0^2}{4}}_{\frac{C}{2}\frac{Q_0^2}{2}}$$

The energy has decreased to half its initial value. The missing energy was needed to create entropy immediately after connecting the two capacitors.

Analogous mechanical processes: inelastic collision; connecting two flywheels with friction clutch.

1.24 Discharge curve of the capacitor

1.

$$U + RC\frac{dU}{dt} = 0$$

Tentative solution:

$U(t) = f \cdot t^2$

$\frac{dU}{dt} = 2ft$

Introducing into the differential equation:

$ft^2 + 2RCft = 0 \quad \Rightarrow \quad t + 2RC = 0$

The result is a contradiction.

2.

Dictance sphere - table: $d \approx 0.1$ m

Surface of the sphere: $A \approx 5 \cdot 10^{-2}\text{ m}^2$

$\varepsilon_0 \approx 10^{-11}$ C/(Vm)

$$C = \varepsilon_0 \frac{A}{d} \approx 10^{-11} \frac{5 \cdot 10^{-2}}{10^{-1}} = 5 \cdot 10^{-12}\text{ F}$$

Length of the shaft: $l \approx 0.1$ m

Cross section of the shaft: $A \approx 1\text{ cm}^2 = 10^{-4}\text{ m}^2$

Conductivity of Perspex: $\sigma \approx 10^{-13}\ (\Omega\text{m})^{-1}$

$$R = \frac{1}{\sigma}\frac{l}{A} \approx \frac{1}{10^{-13}}\frac{10^{-1}}{10^{-4}} = 10^{16}\ \Omega$$

$\tau = R \cdot C = 10^{16}\ \Omega \cdot 5 \cdot 10^{-12}\text{ F} = 5 \cdot 10^4\text{s}$

The time constant is of the order of hours.

1.25 Fields and electric conductors

1. The field surfaces are cross-sectional areas of the conductor. Their distances are greater where the cross-sectional area of the conductor is greater.

2. The interior of the metal plate is field-free. There is tensional stress in the plate orthogonal to the plate surface.

3. See figure 5

4. See figure 6

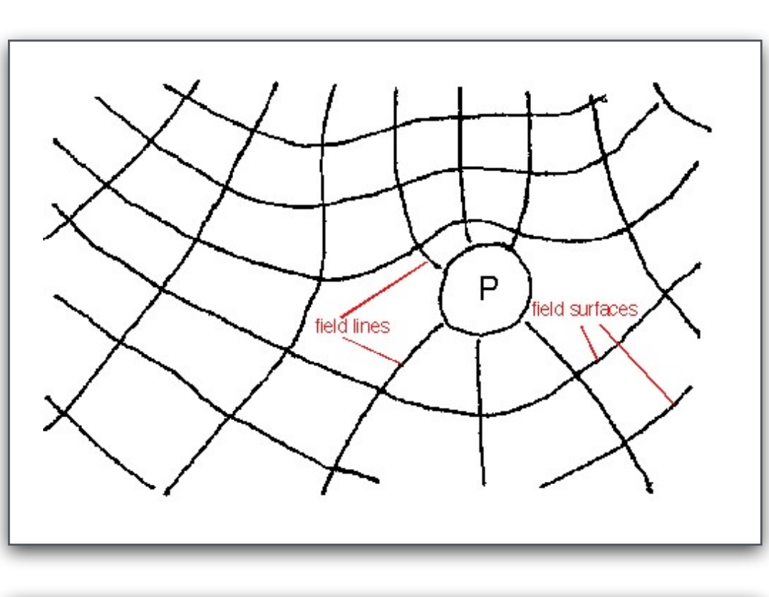

Fig. 5
For 1.25, exercise **3**

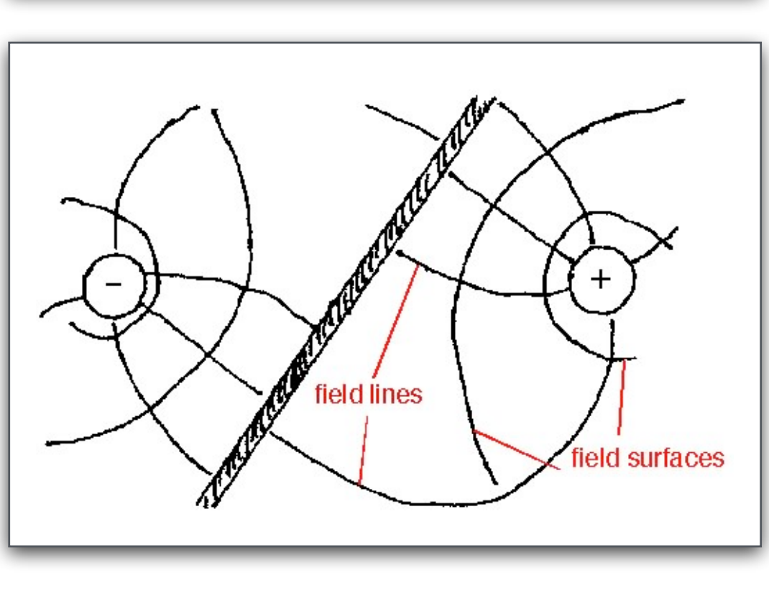

Fig. 6
For 1.25, exercise **4**

1.26 The electric current density – Ohm's law locally

1.

$$|\vec{E}| = \frac{U}{d} = \frac{1.5\text{ V}}{100\text{ m}} = 0.015\text{ V/m}$$

$$j = \sigma \cdot |\vec{E}| = 5.59 \cdot 10^7 \Omega^{-1}\text{m}^{-1} \cdot 0{,}015\text{ V/m}$$
$$= 8.38 \cdot 10^5\text{ A/m}^2$$
$$I = j \cdot A = 8.38 \cdot 10^5\text{ A/m}^2 \cdot 10^{-6}\text{ m}^2 = 0.838\text{ A}$$

2.

$$j = \frac{I}{A} = \frac{5\text{A}}{3 \cdot 10^{-6}\text{ m}^2} = 1.67 \cdot 10^6\text{ A/m}^2$$

$$|\vec{E}| = \frac{j}{\sigma} = \frac{1.67 \cdot 10^6\text{ A m}^{-2}}{5.59 \cdot 10^7\ \Omega^{-1}\text{m}^{-1}} = 0.03\text{ V/m}$$

capacitor: $|\vec{E}| = \frac{U}{d} = \frac{1000\text{ V}}{0.005\text{ m}} = 200\,000\text{ V/m}$

3.

$$j = \frac{I}{A} = \frac{0.5\text{ A}}{10^{-6}\text{ m}^2} = 5 \cdot 10^5\text{ A/m}^2$$

$$|\vec{E}_{Cu}| = \frac{j}{\sigma_{Cu}} = \frac{5 \cdot 10^5\text{ A m}^{-2}}{5.59 \cdot 10^7\ \Omega^{-1}\text{m}^{-1}} = 0{,}0089\text{ V/m}$$

$$|\vec{E}_{Fe}| = \frac{j}{\sigma_{Fe}} = \frac{5 \cdot 10^5\text{ A m}^{-2}}{1.02 \cdot 10^7\ \Omega^{-1}\text{m}^{-1}} = 0.049\text{ V/m}$$

$$U_{Cu} = |\vec{E}_{Cu}| \cdot d = 0.0089\text{ V m}^{-1} \cdot 2\text{ m} = 0.018\text{ V}$$

$$U_{Fe} = |\vec{E}_{Fe}| \cdot d = 0.049\text{ V m}^{-1} \cdot 1\text{ m} = 0.049\text{ V}$$

4. At place A the current density is smaller than at B. The filament will bum through at B.

1.27 How to load electrically charged particles with energy – electron beams

1. $Q = 50 \cdot 1.6 \cdot 10^{-19}$ C
$m = 10^{-5}$ kg
$h = 1000$ m
$U = \Delta\phi = 2 \cdot 10^7$ V

$E_{\text{grav}} = m \cdot g \cdot h = 10^{-5}\text{ kg} \cdot 9.8\text{ N/kg} \cdot 1000\text{ m} = 0.098\text{ J}$

$E_{\text{electr}} = \Delta\phi \cdot Q = 2 \cdot 10^7\text{ V} \cdot 50 \cdot 1.6 \cdot 10^{-19}\text{ C} = 1.6 \cdot 10^{-10}\text{ J}$

The water drop transmits only a small fraction of the energy that it receives from the gravitational field to the electric field. The greater part is used for the creation of entropy.

2. $P = U \cdot I = 50\,000\text{ V} \cdot 50\ \mu\text{A} = 2.5\text{ W}$

3.

(a) $E = Q \cdot (\phi_2 - \phi_1) = 1.6 \cdot 10^{-19}\text{ C} \cdot 1.2 \cdot 10^6\text{ V} = 1.92 \cdot 10^{-13}\text{ J}$

(b) $E = \frac{mv^2}{2} \Rightarrow v = \sqrt{\frac{2E}{m}}$

With $m = 9.1 \cdot 10^{-31}$ kg we get

$$v = \sqrt{\frac{2 \cdot 1.92 \cdot 10^{-13}\text{J}}{9.1 \cdot 10^{-31}\text{kg}}} = 0.65 \cdot 10^9\text{ m/s}$$

This is much more than the terminal velocity c. The equation for the kinetic energy is no longer valid.

1. The magnetic field

2.2 Magnetization

1. See Fig. 7a

2. See Fig. 7b

3. See Fig. 7c

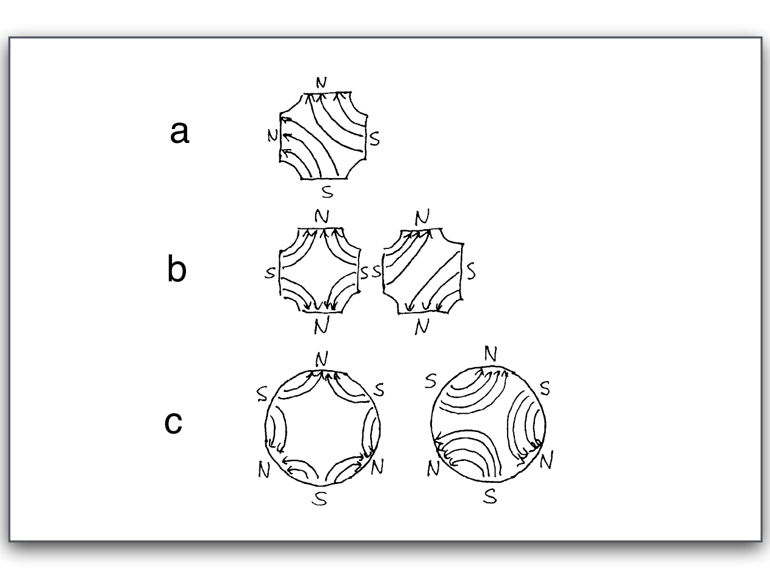

Fig. 7
For 2.2, exercises**1**, **2** and **3**

4. Break the ring into two pieces. If it is magnetized poles will show up at the fracture areas.

2.3 The magnetic field strength

1.
(a) $F = Q_m \cdot H = 10^{-5}\ \mathrm{Wb} \cdot 6.4\ \mathrm{A/m} = 6.4 \cdot 10^{-5}\ \mathrm{N}$
The vector of the momentum current, that flows to the positive pole, points in the direction of the field lines, that which flows to the negative pole points in the opposite direction.

(b) The field pulls the positive pole in one direction, the negative pole in the other. Result: the compass needle will be rotated. It will oscillate and finally settle down in the direction of the field.

2. See figure 8

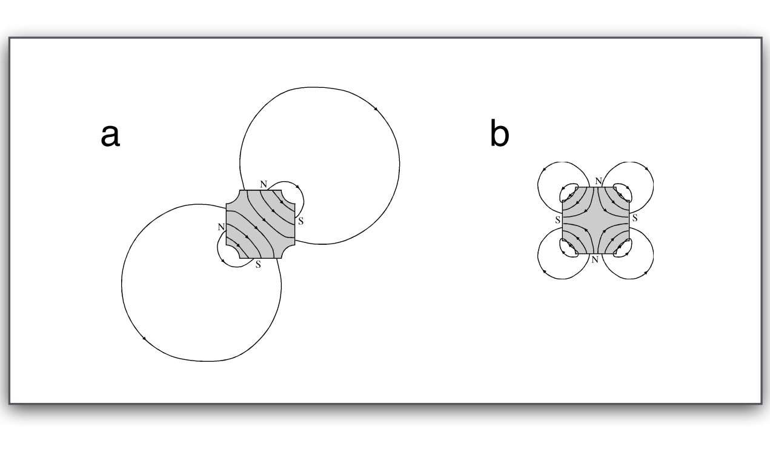

Fig. 8
For 2.3, exercise **2**

2.5 Four important magnetic fields

1. (a) Inside the magnet the magnetization is homogeneous. The lines run towards the north pole at the left, and away from the south pole at the right. Field lines and surfaces see Fig. 9a.

(b) The field lines enter the pole surfaces perpendicularly. Therefore, the field pulls at the poles. It pulls them toward each other.

2. (a) Inside the magnet the magnetization is homogeneous. The lines run from the outside towards the two north poles. Field lines and surfaces see Fig. 9b.

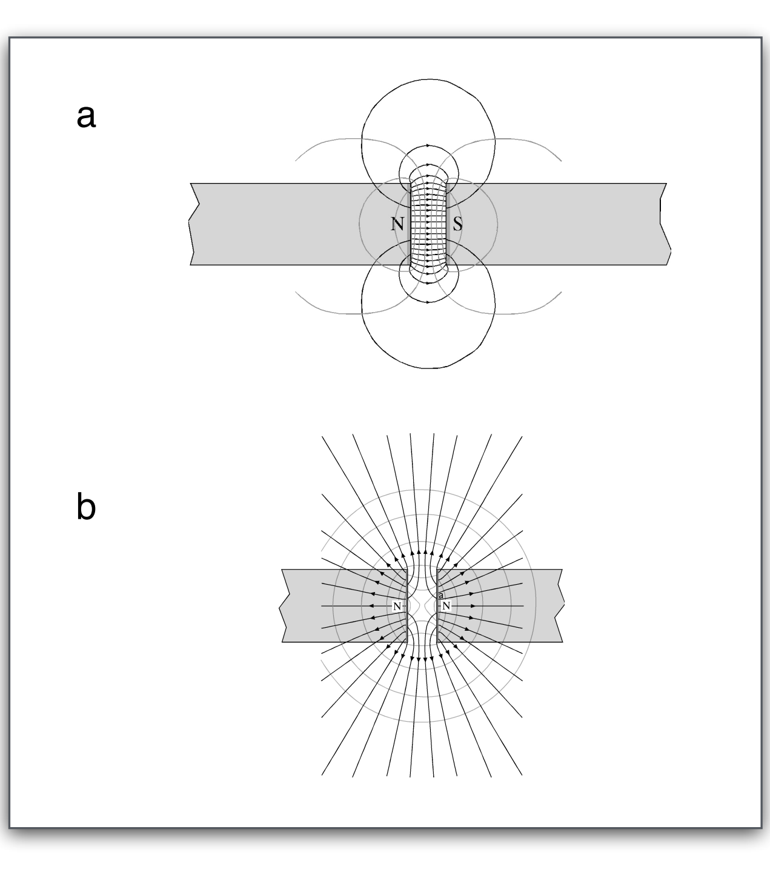

Fig. 9
For 2.5, exercises **1** and **2**

(b) Between the poles the field surfaces end almost perpendicularly on the pole surfaces. Thus, the field presses on the poles. The field lines depart from the left side of the left pole and from the right side of the right pole almost perpendicularly. Thus, the field pulls the poles away from each other.

3. Magnetization lines run from inside to outside. Field lines from outside to inside, but only within the material. The regions inside of the inner surface and outside of the outer surface are field-free.

4. The magnet has no poles. The magnetic field strength is zero everywhere. There are neither field lines nor field surfaces.

2.6 Soft magnetic materials

1. See Fig. 10. The field lines enter the soft magnetic piece form above and from below perpendicularly. Therefore, the field pulls from above and from below. The metal plate is under tensional stress in the vertical direction.

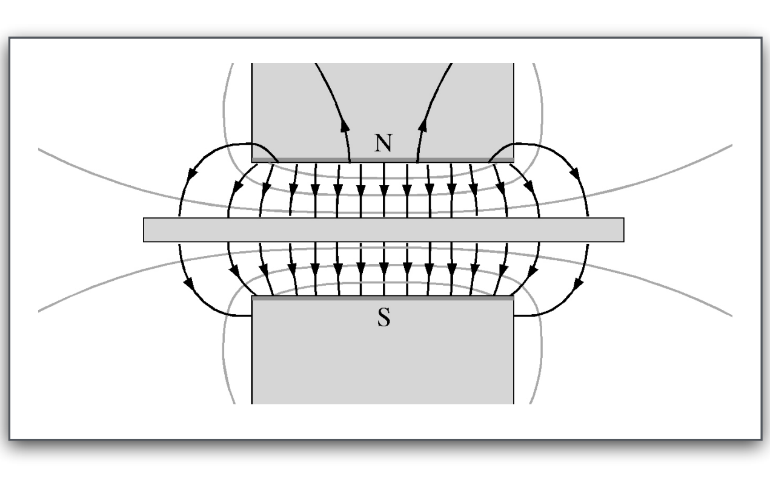

Fig. 10
For 2.6, exercise **1**

2. In the immediate neighborhood of the poles of the permanent magnet poles of the opposite sign appear within the soft magnetic material. The corresponding antipoles are located at the inner side of the soft magnetic parts. The upper soft iron bar has negative magnetic charge (south pole) on its lower surface, the lower bar has positive charge (north pole) an its upper surface. In its two-dimensional cross section the field looks the same as the electric field of a plate capacitor.

Advantage of such a magnet: The field is nearly homogeneous. Disadvantage: When another magnet or a piece of a soft magnetic material is brought in its field, the magnetic charges are easily displaced.

2.7 Electric current and magnetic field

1. See Fig. 11. Two conditions have to be met:

– the field surfaces must end on the current;

– the field image must have a rotational symmetry.

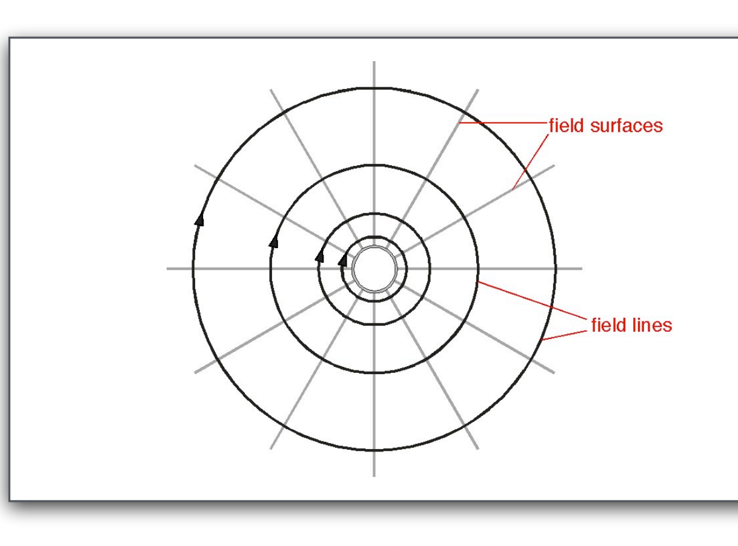

Fig. 11
For 2.7, exercise **1**

From this the shape of the field lines and field surfaces outside of the pipe can be deduced.

If field surfaces would run from the conductor towards the inside the should meet in the center of the pipe. That would mean that in the middle there is pressure perpendicularly to each field surface, what is not correct. Thus, the inside of the pipe must be field-free.

Since the field surfaces of the exterior field end perpendicularly on the surface of the pipe the field presses on the pipe from the outside.

2. See Fig. 12a

3. See Fig. 12b

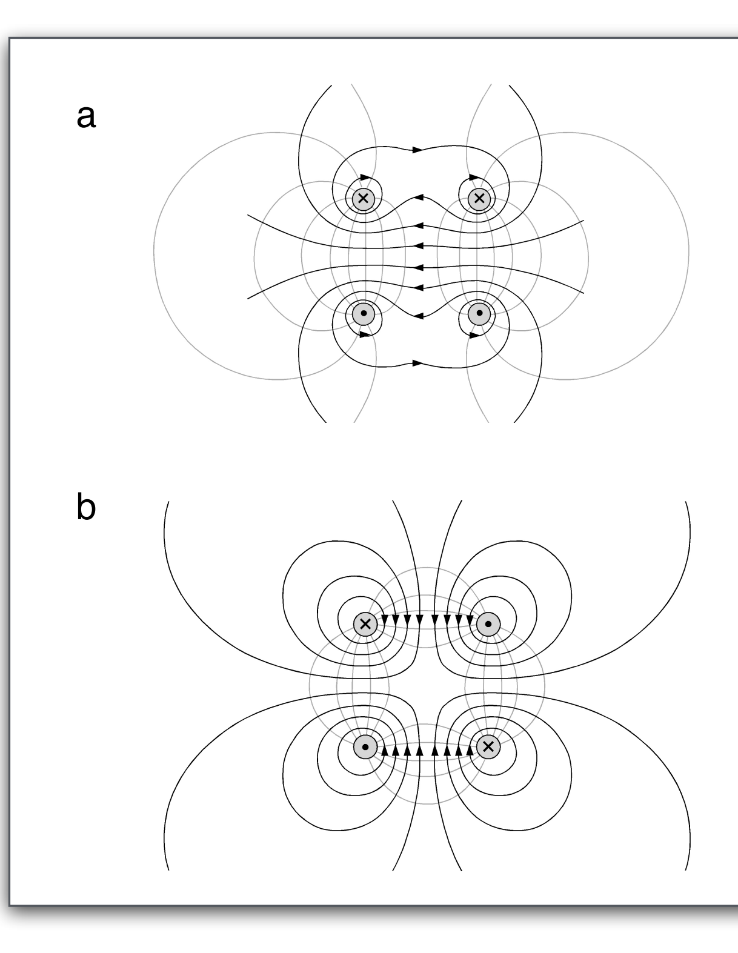

Fig. 12
For 2.7, exercises **2** and **3**

2.8 Calculation of magnetic field strengths

1. The equations have a similar structure. In one of them there is the voltage in the numerator, in the other the current. In the denominator there is a length in both cases.

Regarding equation (1.3): Consider two capacitors connected in parallel. For each of them equation (1.3) is valid. Both together can be considered a capacitor with twice the plate surface. For this new capacitor *U* and *d* have the same value as for the single capacitors. With equation (1.3) we obtain the same field strength. If the surface area appeared in the equation, the large capacitor would have a field strength different from that of a single capacitor – what obviously would not be correct.

Regarding equation (2.3). Consider two coils connected in series. They should have a quadratic cross section. For each of them equation (2.3) is valid. Now the coils can be placed one at the side of the other, so that they are equivalent to one coil with twice the cross-sectional area. (On the sides where the coils touch each other there are two currents flowing in opposite directions. Their contribution to the field therefore cancels out.) For this large coil the current and the length have the same value as for the initial separate coils. According to equation (2.3) we obtain the same field strength. If the surface area appeared in the equation, the large coil would have a field strength different from that of a single coil – what obviously would not be correct.

2. $\left|\vec{H}\right| = \dfrac{n \cdot I}{l} = \dfrac{3000 \cdot 0.8\ \mathrm{A}}{0.6\ \mathrm{m}} = 4000\ \mathrm{A/m}$

3. $n = \dfrac{l \cdot \left|\vec{H}\right|}{I} = \dfrac{0.15\ \mathrm{m} \cdot 3000\ \mathrm{A/m}}{0.5\ \mathrm{A}} = 900$

4.
(a) See Fig. 13

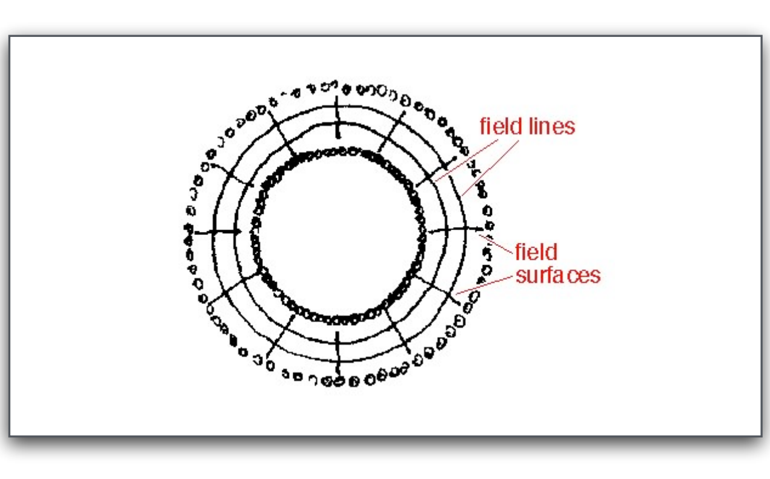

Fig. 13
For 2.8, exercise **4**

(b) $\left|\vec{H}\right| = \dfrac{n \cdot I}{\pi \cdot d} = \dfrac{1000 \cdot 2.5\ \mathrm{A}}{\pi \cdot 0.5\ \mathrm{m}} = 1590\ \mathrm{A/m}$

5.
(a) $\left|\vec{H}\right| = \dfrac{I}{l} = \dfrac{16\ \mathrm{A}}{\pi \cdot 0.002\ \mathrm{m}} = 2546\ \mathrm{A/m}$

(b) $\left|\vec{H}\right| = \dfrac{16\ \mathrm{A}}{\pi \cdot 0.02\ \mathrm{m}} = 254.6\ \mathrm{A/m}$

6. (a) We first consider only the cylinder. Outside of it the field is the same as that of a wire with the same current. Inside there is no field.

Next we ask for the field of the complete cable, that consists of a wire and a cylinder. Outside of the cable the contributions of the wire and the cylinder to the field are equal an opposite. Therefore they cancel each other; the field strength is zero A/m. Inside remains only the contribution of the wire.

(b) $\left|\vec{H}\right| = \dfrac{I}{l} = \dfrac{0.5\ \mathrm{A}}{\pi \cdot 0.001\ \mathrm{m}} = 159\ \mathrm{A/m}$

2.10 Pressure and tension within the magnetic field

1.

$A = 1.5\ \mathrm{mm}^2 = \pi r^2$

$r = \sqrt{\dfrac{A}{\pi}} = \sqrt{\dfrac{1.5\ \mathrm{mm}^2}{\pi}} = 0.69\ \mathrm{mm}$

Magnetc field strength at the surface:

$\left|\vec{H}\right| = \dfrac{I}{l} = \dfrac{16\ \mathrm{A}}{2\pi \cdot 0.00069\ \mathrm{m}} = 3691\ \mathrm{A/m}$

Pressure at the surface:

$$\sigma_\perp = \frac{\mu_0}{2}\left|\vec{H}\right|^2$$
$$= -\frac{1.257 \cdot 10^{-6}\ \mathrm{Wb/(A{\cdot}m)}}{2} \cdot (3691\ \mathrm{A/m})^2 = 8.56\ \mathrm{Pa}$$

2.
(a)

$d = 0.01\ \mathrm{m}, \quad I = 10\,000\ \mathrm{A}$

$\left|\vec{H}\right| = \dfrac{I}{\pi \cdot d} = \dfrac{10\,000\ \mathrm{A}}{\pi \cdot 0.01\ \mathrm{m}} = 3.18 \cdot 10^5\ \mathrm{A/m}$

(b)

$$\sigma_\perp = \frac{\mu_0}{2}\left|\vec{H}\right|^2$$
$$= -\frac{1.257 \cdot 10^{-6}\ \mathrm{Wb/(A{\cdot}m)}}{2} \cdot (3.18 \cdot 10^5\ \mathrm{A/m})^2$$
$$\approx 64\,000\ \mathrm{Pa} = 0.64\ \mathrm{bar}$$

The gas pressure is much greater than the pressure of the magnetic field. The field cannot keep the gas together.

3. Imagine a vertical plane in the middle of the field. In this plane all the field surfaces are perpendicular to the plane. It follows that the two halves of the field press against each other.

4. Imagine a vertical plane in the middle of the field. In this plane all the field lines are perpendicular to the plane. It follows that the two halves of the field pull at each other.

2.11 Electromagnets

3. $d = 0.01\ \mathrm{m}$, $A = 0.01\ \mathrm{m}^2$, $n = 1000$, $I = 2\ \mathrm{A}$

(a) $\left|\vec{H}\right| = \dfrac{n \cdot I}{d} = \dfrac{1000 \cdot 2\ \mathrm{A}}{0.01\ \mathrm{m}} = 2 \cdot 10^5\ \mathrm{A/m}$

(b) $E = \rho_E \cdot V = \dfrac{\mu_0}{2}\left|\vec{H}\right|^2 \cdot A \cdot d = \ldots = 2.51\ \mathrm{J}$

(c) The field strength increases by a factor of ten, the volume decreases to one tenth. Thus, the energy increases by a factor of ten.

4. Both can occur. If the electromagnet is sufficiently strong (compared with the permanent magnet) there will be repulsion. If it is weak (if the electric current is small), a south pole will be created at that end of the electromagnet, that had initially a north pole– due to electrostatic induction –, with the result that there is attraction.

2.13 The coil – the inductance

1.

$n = 500, \quad A = 0.001\ \mathrm{m}^2, \ l = 0.08\ \mathrm{m}$

$L = \mu_0 \cdot n^2 \cdot \dfrac{A}{l} = \ldots = 3.93 \cdot 10^{-3}\ \mathrm{H}$

2. The inductance decrease to half of its initial value.

2.14 The energy of the magnetic field

1.

$\left|\vec{H}\right| = 40\ \mathrm{A/m}, \quad V = 1\ \mathrm{m}^3$

$E = \dfrac{\mu_0}{2} \cdot \left|\vec{H}\right|^2 \cdot V = \ldots = 1.01 \cdot 10^{-3}\ \mathrm{J}$

2.

$A = 0.0004\ \mathrm{m}^2, \quad d = 0.005\ \mathrm{m}, \quad \left|\vec{H}\right| = 120\,000\ \mathrm{A/m}$

$E = \dfrac{\mu_0}{2} \cdot \left|\vec{H}\right|^2 \cdot V = \ldots = 0.0181\ \mathrm{J}$

3.

$L = 0.01\ \mathrm{mH}, \quad I = 2.5\ \mathrm{A}, \quad l = 0.1\ \mathrm{m}, \quad A = 0.0004\ \mathrm{m}^2$

(a) $E = \dfrac{L}{2} I^2 = \ldots = 0.000\,031\ \mathrm{J}$

(b) $\rho_E = \dfrac{E}{V} = \ldots = 0.78\ \mathrm{J/m}^3$

4.

$L = 0.2\ \mathrm{mH}, \quad R = 500\ \Omega, \quad U = 200\ \mathrm{V}$

$E = \dfrac{L}{2} I^2 = \dfrac{L}{2}\left(\dfrac{U}{R}\right)^2 = \ldots = 0.000\,016\ \mathrm{J}$

5. (a) First the connections of the coil are bridged. Since thereby the power supply is also bridged, the power supply has to withstand a short. Next the coil with its two terminals still connected is separated from the power supply.

(b) The capacitor, that is separated from the power supply, loses its energy, because it will discharge. It discharges because the insulation between its plates is not perfect. The coil, that is separated from the power supply loses its energy because the electric current will cease to flow. The current ceases to flow because the wire of the coil is not a perfect conductor. In a superconducting coil the current will not decay.

2.15 „Discharge“ of the coil

1.

$R = 500\ \Omega, \quad t_{1/10} = 4\ \mathrm{ms}$

$$I(t) = I_0 \cdot e^{-t/\tau} \quad \Rightarrow \quad -\frac{t}{\tau} = \ln\frac{I(t)}{I_0} \quad \Rightarrow \quad \frac{t}{\tau} = \ln\frac{I_0}{I(t)}$$
$$\Rightarrow \quad \frac{0.004\ \mathrm{s}}{\tau} = \ln 10$$
$$\Rightarrow \quad \tau = \frac{0.004\ \mathrm{s}}{\ln 10} = 0.00174\ \mathrm{s}$$

With $\tau = \dfrac{L}{R}$ we obtain

$L = \tau \cdot R = 0.00174\ \mathrm{s} \cdot 500\ \Omega = 0.87\ \mathrm{H}$

2.

$$E(t) = \frac{L}{2} I(t)^2$$
$$I(t) = I_0 \cdot e^{-t/\tau}$$
$$E(t) = \frac{L}{2} \cdot I_0^2 \cdot e^{-2t/\tau}$$

The energy in the coil decreases twice as fast as the electric current.

2.16 How the magnetic field presses on an electric current

1.

$I = 200\ \mathrm{A}, \quad H = 40\ \mathrm{A/m}, \quad \Delta s = 1\mathrm{m}$

$F = I \cdot \Delta s \cdot B = I \cdot \Delta s \cdot \mu_0 \cdot H$

$= 200\ \mathrm{A} \cdot 1\ \mathrm{m} \cdot 1.257 \cdot 10^{-6}\ \mathrm{Vs/Am} \cdot 40\ \mathrm{A/m}$

$= 0.01001\ \mathrm{N}$

2.

$H = 2400\ \mathrm{A/m}, \quad r = 0.1\mathrm{m}, \quad m = 0.911 \cdot 10^{-30}\ \mathrm{kg}, \quad e = 1.60 \cdot 10^{-19}\ \mathrm{C}$

$$r = \frac{m \cdot v}{e \cdot B}$$
$$v = \frac{r \cdot e \cdot B}{m} = \frac{r \cdot e \cdot \mu_0 \cdot H}{m}$$
$$E = \frac{m}{2} v^2 = \frac{(r \cdot e \cdot \mu_0 \cdot H)^2}{2m}$$
$$= \ldots = 1.279 \cdot 10^{-15}\mathrm{J} = \frac{1.279 \cdot 10^{-15}}{1.60 \cdot 10^{-19}}\ \mathrm{eV} = 8\ \mathrm{keV}$$

3.

$d = 0.005\ \mathrm{m}, \quad B = 1.2\ \mathrm{T}, \quad I = 200\ \mathrm{mA}$

$U_{H1} = 0.12 \cdot 10^{-3}\ \mathrm{V}, \ U_{H2} = 0.36 \cdot 10^{-6}\ \mathrm{V}$

$U_H = v \cdot B \cdot d \quad \Rightarrow \quad v = \dfrac{U_H}{B \cdot d}$

$v_1 = \ldots = 0.02\ \mathrm{m/s}$

$v_2 = \ldots = 0.000\,06\ \mathrm{m/s}$

The first sample contains less mobile electrons. In order to achieve the same current strength as in sample 2, the electrons have to move faster.

4.

$v = 1.1\ \mathrm{m/s}, \quad d = 0.002\ \mathrm{m}, \quad B = 3\ \mathrm{T}$

$U_H = v \cdot B \cdot d = \ldots = 0.066\ \mathrm{V}$

The fact that the water contains positive and negative ions has no consequence for the Hall voltage.

3. The interplay between electric and magnetic fields

3.2 Electromagnetic induction

1.

$$U = n \cdot \frac{d\Phi}{dt} = n \cdot A \cdot \frac{dB}{dt}$$

$$= 200 \cdot 0.0008\ \text{m}^2 \cdot \frac{0.3\ \text{T}}{2\ \text{s}} = 0.024\ \text{V}$$

2. See Fig. 14

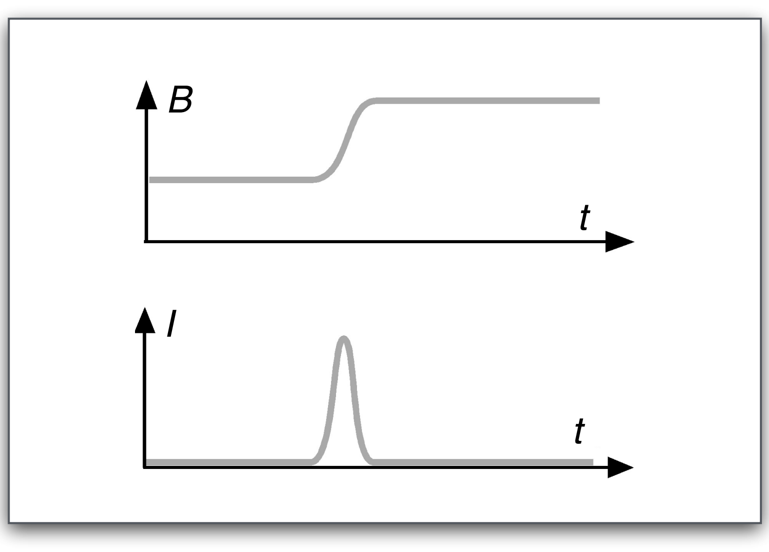

Fig. 14
For 3.2, exercise **2**

3.

(a) $\left|\vec{H}\right| = \frac{n \cdot I}{l} = \frac{2000 \cdot 10\ \text{A}}{0.5\ \text{m}} = 40\,000\ \text{A/m}$

(b) $\left|\vec{B}\right| = \mu_0 \left|\vec{H}\right| = 1.257 \cdot 10^{-6} (\text{Wb/Am}) \cdot 40\,000\ \text{A/m} = 5.03 \cdot 10^{-2}\ \text{T}$

(c)

$$U = n' \cdot A' \cdot \frac{dB}{dt} = n' \cdot A' \cdot \frac{B}{t} \Rightarrow$$

$$t = \frac{n' \cdot A' \cdot B}{U}$$

$$= \frac{500 \cdot 1.5 \cdot 10^{-3}\text{m}^2 \cdot 5.03 \cdot 10^{-2}\text{T}}{100\ \text{V}} = 0.38\ \text{ms}$$

4.

$$U = n \cdot A \cdot \frac{dB}{dt} = 1 \cdot 2 \cdot 10^{-4}\text{m}^2 \cdot 0.2\ \text{T/s} = 4 \cdot 10^{-5}\ \text{V}$$

$$I = \frac{U}{R} = \frac{4 \cdot 10^{-5}\ \text{V}}{200\ \Omega} = 2 \cdot 10^{-7}\text{A}$$

3.3 The generator

2. Yes. The coil can be imagined to be composed of many small quadratic coils. Each of these coils generates a sine tension of the same frequency.

3. No. In the relation $\Phi = B \cdot A$ A changes as the sine of the time. Φ is a sine function of t only if B is constant, i.e. if the field is homogeneous.

3.4 Alternating voltage and alternating current

1. $U_0 = \sqrt{2} \cdot U_{\text{eff}} = \sqrt{2} \cdot 230\ \text{V} = 325\ \text{V}$

2. In the P-t diagram draw a horizontal straight line for the P value $0.5 \cdot U_0 \cdot I_0$. For each time interval for which P is by a certain amount above this line there is another one for which it is by the same amount below. The average value of P for two such corresponding time intervals is $0.5 \cdot U_0 \cdot I_0$. As a consequence, also the total time average has this value.

3. No. The average value of the voltage is 0 Volt, that of the current strength is 0 Ampere. The resulting average of the energy current would be 0 Watt. This cannot be correct, since the energy current has positive values.

3.5 The transformer

1. $5 \cdot 230\ \text{V} = 1150\ \text{V}$ and $(1/5) \cdot 230\ \text{V} = 46\ \text{V}$

2. The number of turns of the primary coil is 20 times that of the secondary coil. In the primary coil there is a current of 0.1 A.

3. $I_2 = (1/10) \cdot I_1 = 10\ \text{mA}_1$

$U_2 = 10 \cdot U_1 = 10 \cdot 230\ \text{V} = 2300\ \text{V}$

4. The input conductors must have a great diameter, so that the resistance is not too great. The output conductors must be well insulated so that no spark will jump over.

5. (a)

$$I = \frac{P}{U}$$

$$I_1 = \frac{60\,000\,000\ \text{W}}{3000\ \text{V}} = 20\,000\ \text{A}$$

$$I_2 = \frac{60\,000\,000\ \text{W}}{300\,000\ \text{V}} = 200\ \text{A}$$

(b)

$$U = R \cdot I$$

$$U_1 = 0.05\ \Omega \cdot 20\,000\ \text{A} = 1000\ \text{V}$$

$$U_2 = 0.05\ \Omega \cdot 200\ \text{A} = 10\ \text{V}$$

(c)

$$U_{01} = 3000\ \text{V} - 2000\ \text{V} = 1000\ \text{V}$$

$$U_{02} = 300\,000\ \text{V} - 20\ \text{V} = 299\,980\ \text{V}$$

(d)

$$P_1 = 2 \cdot U_1 \cdot I_1 = 2 \cdot 1000\ \text{V} \cdot 20\,000\ \text{A} = 40\ \text{MW}$$

$$P_2 = 2 \cdot U_2 \cdot I_2 = 2 \cdot 10\ \text{V} \cdot 200\ \text{A} = 0{,}004\ \text{MW}$$

3.7 Superconductors

1. See Fig. 15. In the pipe circular currents are flowing (center of the circle on the axis of the pipe). The direction of the current in the region of the magnet is opposite to that outside (to the left and the right) of the magnet.

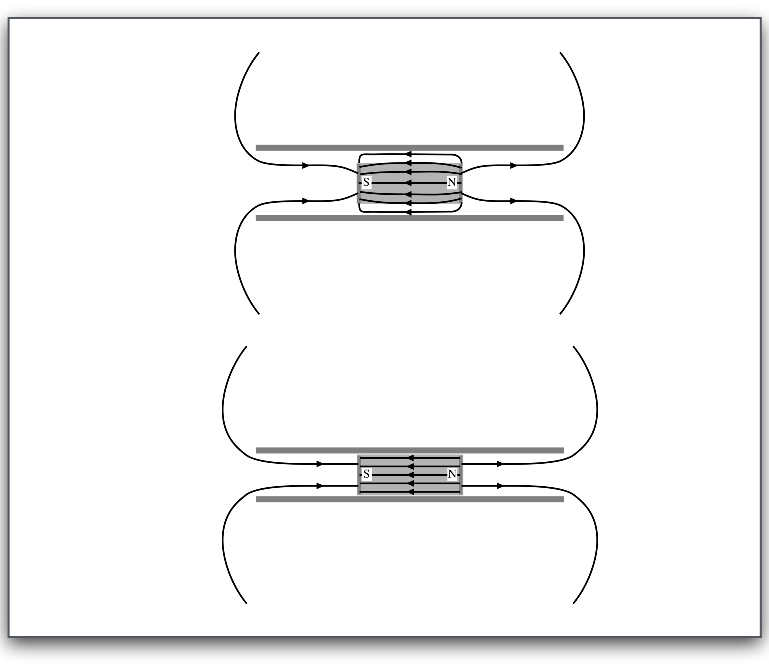

Fig. 15
For 3.7, exercise **1**

2. The magnet will not come out at all. It is suspended by the supercurrent.

3.11 Energy transmission with electromagnetic waves

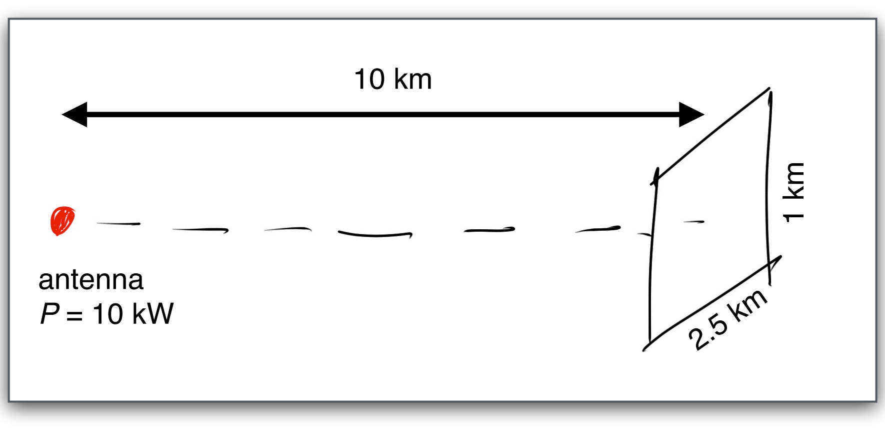

Fig. 16
For 3.11, exercise **1**

1.

$$j_E = \frac{P}{A} = \frac{10^4\ \text{W}}{2.5 \cdot 10^6\ \text{m}^2} = 4 \cdot 10^{-3}\ \text{W/m}^2\ \text{F}$$

For an electromagnetic wave, equation (3.13) holds:

$$\sqrt{\varepsilon_0}\left|\vec{E}\right| = \sqrt{\mu_0}\left|\vec{H}\right|$$

With equation (3.15) we get:

$$j_E = \left|\vec{E}\right| \cdot \left|\vec{H}\right| = \sqrt{\frac{\varepsilon_0}{\mu_0}}\left|\vec{E}\right|^2$$

and

$$\left|\vec{E}\right| = \sqrt[4]{\frac{\mu_0}{\varepsilon_0}}\sqrt{j_E}\ .$$

We calculate:

$$\frac{\mu_0}{\varepsilon_0} = \frac{1.257 \cdot 10^{-6}\ (\text{Vs/Am})}{8.854 \cdot 10^{-12}\ (\text{As/Vm})} = 1.94\sqrt{\frac{\text{V}}{\text{A}}}$$

We thus obtain:

$$\left|\vec{E}\right| = \sqrt[4]{\frac{\mu_0}{\varepsilon_0}}\sqrt{j_E} = 12{,}27\ \text{V/m}$$

and correspondingly:

$\left|\vec{H}\right| = \sqrt[4]{\frac{\mu_0}{\varepsilon_0}}\sqrt{j_E} = 32{,}56\ \text{A/m}$. The energy current density of the light from the Sun is about 500 W/m², i.e. greater by a factor of 100 000.

2. The current is an alternating current. Fig. 17 shows a snapshot. The energy flows perpendicularly to the drawing plane, away from the observer.

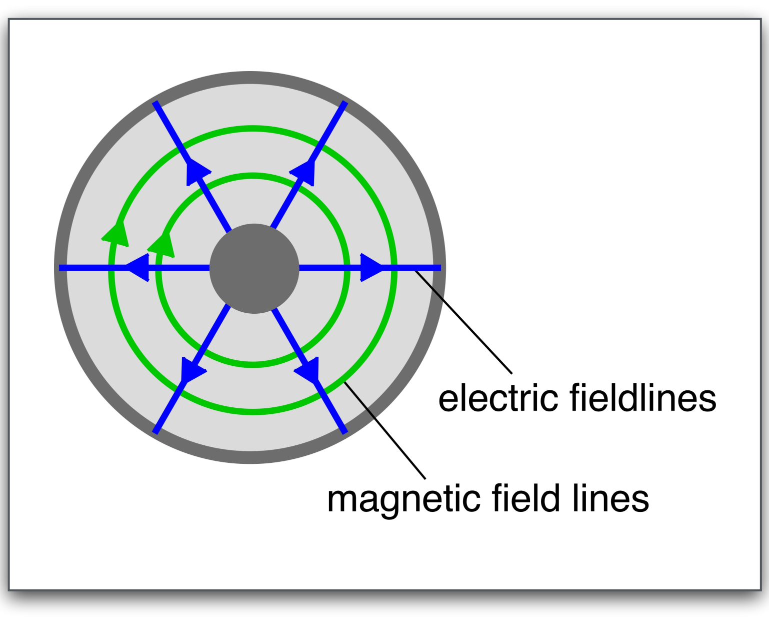

Fig. 17
For 3.11, exercise **2**