Color charge and perceptible color – a suitable analogy?

A Grutsch and F Herrmann

Institut für Theoretische Festkörperphysik, Abteilung für Didaktik der Physik, Universität Karlsruhe, 76128 Karlsruhe, Germany

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Abstract

Elementary texts about the strong interaction between elementary particles suggest that the space of the color charge is three-dimensional. We discuss what should be understood by the term “dimension of a physical quantity” and show that the space of the color charge is two-dimensional.

Zusammenfassung

In Einführungstexten über die starke Wechselwirkung wird oft suggeriert, daß der Raum der Farbladung dreidimensional ist. Wir diskutieren, was man unter der Dimension einer physikalischen Größe verstehen sollte und zeigen, daß die Farbladung eine zweidimensionale Größe ist.

1. Introduction

This article is about a question which may come to the minds of those physics students, physicists or physics teachers who are not specialists in particle physics, but who have a good basic knowledge of physics, including quantum mechanics. Their knowledge about quantum chromodynamics might stem from elementary text books or from popular science magazines such as Scientific American.

The so called color charge of particles which are subject to the strong interaction is a quantum number that is used to characterize the states of quarks, antiquarks and gluons. To avoid confusion with the current meaning of the word “color”, in the following article we shall call this quantity “strong charge”.

In the same way as other quantum numbers strong charge is an extensive quantity. The total value of the strong charge is zero for baryons, antibaryons and mesons. (A baryon consists of three quarks, an antibaryon of three antiquarks, and a meson of a quark and an antiquark.)

It was for these facts that the names of colors were attributed to the various values which the strong charge admits and that the name color charge was coined for the quantity itself. The values which the strong charge admits for the three “color states” of a quark have been called red, green and blue and those for the antiquark are cyan, magenta and yellow or also antired, antigreen and antiblue. Just as for perceptible colors the additive mixing of red, green and blue light of appropriate intensity results in the color impression “white” or as the mixing of light of the colors cyan, magenta and yellow combines to white light, the addition (in the sense of mathematics) of the strong charges red, green and blue or cyan, magenta and yellow yields the value zero. In the same way the value of the strong charge of the mesons seems to have the correct counterpart in this model: The strong charge values of the two quarks of a meson are one of the couples red-cyan, green-magenta or blue-yellow. The mixing of light of these colors yields (provided that the intensities are appropriate), as everybody knows, white light. It is seen that, in this analogy, the perceptible color “white” is put in correspondence to the strong charge value zero and the additive mixing of light is put in correspondence to a mathematical addition.

Thus, in particle physics a model is being used which, at least at a first glance, seems to work well. On closer inspection, however, the model causes some difficulties. And indeed, it may mislead the students instead of helping them. Our analysis showed us that there are several aspects of the model which could be criticized. The first objection to using the color model is that it suggests that the manifold of the values of the strong charge is three-dimensional. The second is of a more general nature: We recommend to avoid giving particular names to particular values of any physical quantity. A third objection refers to a certain use of the color code of the strong charge: One often suggests, that the strong charge of gluons has two values, which is not the case. These three points are discussed in turn in the following three sections.

2. The number of dimensions of a physical quantity

The space of the perceptible colors is three-dimensional. In order to specify a color impression, three numbers are necessary. The color valence of a pixel of the TV screen, for example, is defined by the intensity of three electron beams. Another method to characterize a color valence by means of three numbers is the definition of hue, brightness and saturation. Every person who knows a little about the three-dimensional color space will expect the space of the strong charge also to be three-dimensional. In articles for non-specialists in QCD this is even sometimes said explicitly (Feldman and Steinberger 1991). What is the dimensionality of the space of the strong charge in reality?

Let us put the question in a more general context and first ask what generally is meant when speaking about the dimension of a physical quantity. For pre-quantum-mechanical physics the answer is easy: scalars are one-dimensional, vectors are three-dimensional or, in a relativistic approach, four-dimensional. The question turns out to be more complicated when coming to quantum mechanics. We shall discuss the problem by using the familiar example of angular momentum. Let us consider the spin of a one-electron system. The quantity spin has three components s_x, s_y and s_z. Each of these components is represented by a 2x2 matrix. No two of these three matrices commute with one another, i. e. the system can be in an eigenstate of only one of the three spin components s_x, s_y and s_z. In other words, only one of the matrices can be diagonal in a given representation. One possible representation is given by the Pauli matrices:

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \qquad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

The spin components s_x, s_y and s_z are obtained by multiplying them by $\hbar/2$. This representation corresponds to states of the electron which are eigenstates of s_z.

In order to tell somebody what the angular momentum state of the system is, it is sufficient to communicate one single number, namely the value of that component of the angular momentum to which the system is in an eigenstate. Since the corresponding matrices are 2x2 matrices this component can admit one out of two values: $+\hbar/2$ or $-\hbar/2$. To represent the manifold of possible values graphically a one-dimensional coordinate system is needed, i. e. one single axis, where the eigenvalue can be plotted, Fig. 1.

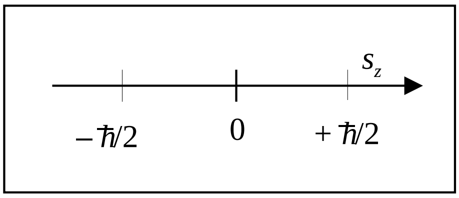

Fig. 1. Values of the spin in a one-dimensional coordinate system

We are in search for the “dimension” of the spin and we are now faced with three candidates:

- The number d of components of the quantity, i. e. the number of matrices that are necessary for representing the quantity. In the case of the spin we have $d = 3$.

- The maximum number n of components, that can admit sharp values, i. e. the number of components which commute. In the case of the spin we have $n = 1$.

- The number of eigenvalues of each component, i. e. the number l of rows and columns of the matrices which represents the quantity. In the case of the spin we have $l = 2$.

Of course, in the sense of classical physics the choice would be in favor of d. In quantum mechanics, however, it seems more reasonable to interpret n as a dimension of the quantity. This is seen clearly in Fig. 1, where the eigenvalues are represented in a one-dimensional coordinate system.

Let us now come back to the quantity we were originally interested in, the strong charge. The strong charge is commonly represented by the eight Gell-Mann matrices (Just as the Pauli matrices are generators of a representation of the rotation group, the Gell-Mann matrices are generators of a representation of the special unitary group SU(3).):

$$\lambda_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \lambda_2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \lambda_4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

$$\lambda_5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix} \quad \lambda_6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \quad \lambda_7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix} \quad \lambda_8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

While the spin has three components, the strong charge has eight: $d = 8$. Two of these eight matrices commute, i. e. two can be simultaneously diagonalized. In the Gell-Mann representation these are the matrices λ_3 and λ_8. A particle described by these matrices finds itself in an eigenstate of the matrices λ_3 and λ_8. Thus, to describe this state of the particle, two numeric values have to be given and we have $n = 2$. The number l of different values that each component λ_3 and λ_8 can admit is equal to three: $l = 3$.

Since $n = 2$, for the graphical representation of a quark's state a two-dimensional coordinate system is needed. In the above proposed meaning of dimensionality the strong charge is two-dimensional. Fig. 2 shows the pairs of eigenvalues $\{f_3; f_8\}$ of the matrices λ_3 and λ_8 for the three quark states (multiplied by a factor of 1/2, as is common practice).

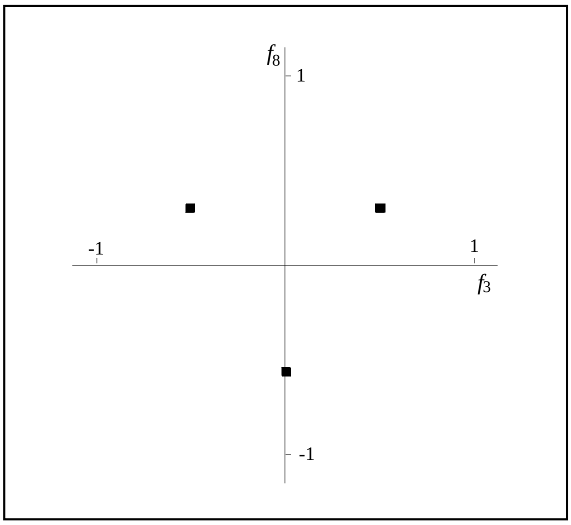

Fig. 2. The values of the strong charge can be represented in a two-dimensional coordinate-system.

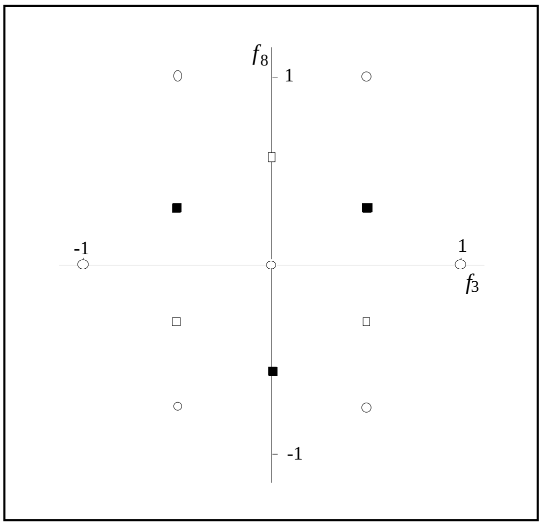

Fig. 3. Quarks ( ), antiquarks ( ) and gluons ( ) in the f_3-f_8-coordinate system of the strong charge. Two of the gluons carry the same charge {0,0}.

The antiquarks are described by eight matrices of another representation of the group SU(3) and the eight gluons by yet another representation of the same group. Since both the antiquark and the gluon matrices are generators of representations of the same group SU(3), in both cases again two matrices commute. As a result, for antiquarks and gluons we equally have $n = 2$. Antiquark and gluon states can be represented in the same two-dimensional coordinate system as quark states. Since the antiquark matrices are 3x3 matrices, there are $l = 3$ antiquark states (an antiquark triplett, or antiquarks "of three different colors"). Furthermore, since the gluon matrices are 8x8 matrices, there are $l = 8$ different gluon states (a gluon octet, or gluons "of eight different colors"). In Fig. 3 the eigenvalue pairs $\{f_3; f_8\}$ of the three quark states, the three antiquark states and the eight gluon states are represented. Since two of the gluons are degenerate they have the same eigenvalue pair {0;0}.

Let us come back to our original question: How many dimensions does the space of the strong charge have? If by dimension the number of values which have to be indicated to characterize a state is meant the answer is "two". Indeed, the operations which can be realized in the coordinate system of Fig. 3 are the very basis of the color model for the strong interaction: the vectorial addition of the three quark states in Fig. 3 gives, just as that of the three antiquark states, zero.

Thus, when applying the color model of the strong charge, one has to pay attention to the fact that only a two-dimensional projection of the space of the perceptible colors has to be taken into consideration, i. e. the plane of the variables hue and saturation. For the third dimension – the brightness – there is no correspondence in particle physics. Whereas the additive mixing of the colors red, green and blue will generally yield a white different from that one gets by mixing of red and cyan, the addition of the corresponding values of the strong charge gives the same result – namely zero.

When the color model of the strong charge is used the student will undoubtedly expect a three-dimensional space for the strong charge. We therefore recommend to refrain from using this analogy. Finally, it is not too difficult to show that the rules that hold for the addition of the values of the strong charge of quarks, antiquarks and gluons are the simple rules of vector addition.

3. Proper names for values of physical quantities

Using the color model can mislead the learner yet in another respect. When the strong charge is introduced but, instead of indicating numerical values of this quantity one operates only with proper names of these values (red, green, blue and so on), one is suggesting that the discussion is not about different values of one single quantity but about different qualities.

It would be the same situation if to the electric charge of the electron one name would be given, say male, and to the charge of the positron another name, female. Then it would be necessary to formulate an additional rule like “male + female = neutral”. Of course, it is possible to proceed in this way and to get consistent results. It is much simpler, however, to take advantage of some modest mathematics and to introduce the charge of the electron and the positron as two different values of one and the same quantity. The values are equal in amount and have opposite sign, and mathematics yields automatically the addition rule. Indeed, in the beginning of the research on electricity the idea that two different qualities of electricity might exist was seriously discussed and even left traces in our present text books. One still can read statements such as: “There are two *different kinds* of electric charge.” This sentence is reminiscent of a statement which is currently found in texts about the strong charge: “There are three kinds of color charge.”

4. The strong charge of the gluons

Finally, we would like to make a critical remark about the way of characterizing gluons. One often reads that a gluon carries two color charges, red and antigreen for instance or green and antiblue ('t Hooft 1980). It is true that a gluon can be characterized unambiguously by indicating two color names. However, this does not mean that the strong charge has two values at the same time. It is like saying that a Li^+ ion has five electric charges: three positive and two negative ones. Nevertheless, this ion can be characterized by giving the number of protons and electrons. Its electric charge, however, is one positive elementary charge, the same as that of a single proton, an Fe^+ ion or many other particles.

5. Conclusion

The color model of particle physics suggests that the three quark states are distinguished by three different qualities and not only by different values of a single physical quantity. Moreover, it suggests that these qualities extend a three-dimensional space. In reality the space of the strong charge is two-dimensional. The impression is further enhanced by characterizing a gluon by two color names instead of one. We propose refraining completely from using the color model in particle physics.

References

Feldman G J and Steinberger J, “The Number of Families of Matter”, *Scientific American*, February 1991, 26-33. “Despite its name, color is an invisible trait. It is to the strong interaction what charge is to the electrical one: a quantity that characterizes the force. But whereas electrodynamic charge has only one state – positive or negative – the color charge has three.”

't Hooft G, “Gauge Theories of the Forces between Elementary Particles,” *Scientific American*, June 1980, 90-116. “Each gluon carries one color and one anticolor.”