The Physics Teacher, Vol.27, No.4, p272-273

An Electrical Model of a Carnot Cycle

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1989.4

All physical quantities for which a continuity equation can be written are called substance-like quantities.[1,2] Such quantities include, for example, electric charge, momentum(linear or angular), entropy, and the like. The method of presentation remains much the same from one sub-branch of physics to another, for these quantities have something in common: namely, each can be thought to be contained in a physical system and flow from one system to another. Thus knowledge of a single branch of physics already provides an analogy for the ways and means by which processes are described in other branches.

In electrical processes a charge current is caused by an electrical potential difference, in mechanical processes a momentum current is caused by a mechanical potential difference, and in thermal processes an entropy current is caused by a thermal potential difference. Accordingly, entropy, thermal potential (traditionally called temperature), and entropy current are analogous to momentum, mechanical potential (traditionally called velocity and angular velocity), and momentum current (traditionally called force and torque), respectively, and are also analogous to electric charge, electrical potential, and charge current, respectively. Thus knowledge of mechanics already provides an analogy for the methods by which processes are described in thermodynamics and electricity. For example, the Carnot cycle can be described in terms of either mechanical processes or electrical processes. A.J. Nicastro has successfully set up a dynamical model of a Carnot cycle to describe these thermal processes.[3] An electrical model of a Carnot cycle is shown in Fig. 1.

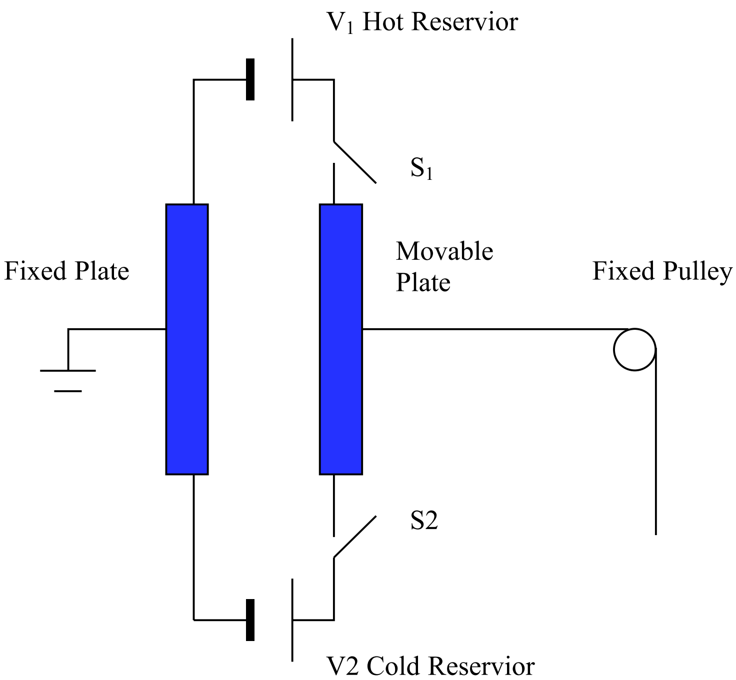

Fig.1 An electrical model of a Carnot cycle

In Fig. 1, the upper source has a higher constant terminal voltage V_1, and the lower one has a constant terminal voltage V_2. The negative terminals of the sources are connected to the fixed plate of a parallel-plate capacitor, and the fixed plate is in turn connected to the Earth. The potential of the fixed plate and the potentials of the negative terminals of the sources are always zero, and the potentials of the positive terminals of the sources are always V_1 and V_2, respectively. The positive terminal of the upper source represents a hot reservoir at a constant temperature T_1, and the positive terminal of the lower source represents a cold reservoir at some lower constant temperature T_2. They are connected to a movable plate of the capacitor by switches S_1 and S_2. A string attached to the movable plate is wrapped around a fixed pulley. The movable plate is the electrical analog of the working substance of a Carnot engine.

When one of the switches is closed, the electrical potential energy representing the heat energy is transferred between the working substance and one of the reservoirs. During heat transfer, there is no finite temperature difference, because each process of the Carnot cycle is completely reversible. When the movable plate takes an amount of charge from the upper source at the higher potential V_1, the movable plate itself must be at V_1. Similarly, when an amount of charge is rejected to the lower source at the lower potential V_2, the movable plate itself must be at V_2. When the movable plate is pulled, increasing the separation of the capacitor plates, work input is supplied. When the movable plate is moved toward the fixed plate, the electrical potential energy taken from the hot reservoir is partly converted into work output.

The following are the four steps of the Carnot cycle described in terms of electrical processes.

(1) The capacitor is char ged to a char ge of magnitude q_1 on each plate. The movable plate is pulled to change the capacitance C (the capacitance of a parallel-plate capacitor is inversely proportional to the separation of the plates) until the potential of the movable plate is equal to V_1 (= q_1/C_1). Then the switch S_1 is closed so that the amount of charge on the movable plate increases to q_2. Meanwhile, the separation of the plates is decreased in such a way that the potential of the movable plate remains constant. The movable plate absorbs electrical potential energy U_1 from the upper source in this step.

(2) The switch S_1 is opened. The separation of the plates decreases still further , and the potential of the movable plate decreases. The switch S_2 is closed when the potential of the movable plate is equal to V_2 (= q_2/C_2).

(3) The movable plate is pulled, increasing the separation of the plates so that the potential of the movable plate remains constant. The switch S_2 is opened when the amount of charge on the movable plate decreases to q_1. So the amount of the charge output is equal to that of the charge input. Electrical potential energy U_2 is rejected to the lower source in this step.

(4) The movable plate is pulled, increasing the separation of the plates until its potential is equal to V_1. Then the cycle repeats.

Now, consider the potential ener gy input U_1 and output U_2. We know that $dU = Vdq$, so

$U_1 = \int v_1 dq = v_1 \int dq = v_1 (q_2 - q_1)$

and

$U_2 = \int v_2 dq + v_2 \int dq = - v_2(q_2 - q_1)$.

From this, we get that

$$U_2/U_1 = - V_2/V_1.$$

So the efficiency to the model

$$\eta = (U_1 + U_2)/U_1 = 1+U_2/U_1 = 1- V_2/V_1.$$

This is analogous to

$$\eta = 1 - T_2/T_1$$

for the efficiency of a Carnot engine.

With the help of this electrical model, some abstract concepts about the Carnot cycle become easier to understand. Pedagogically, this is an analogical method of teaching. To present abstract concepts in a physical process to students, instructors often draw an analogy between the physical process in which the abstract concepts appear and another that has been learned by students. In terms of substance-like quantities, we can draw many effective analogies for our physics teaching.

References

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