Demonstration of angular momentum coupling between rotating systems

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A simple arrangement is discussed which demonstrates "spin–spin" and "spin–orbit" coupling between rotating systems both with and without dissipation.

I. INTRODUCTION

One nice thing about teaching the conservation of angular momentum is that classroom demonstrations[1–5] are relatively simple to carry out and are almost always popular with students. Typical demonstrations include the use of gyroscopes or tops[6,7] and the well-known example[7,8] of a small object rotating in a horizontal plane, say, a table top, at the end of a string which passes through a hollow tube, or hole in the table, and which can be pulled down at the other end, shortening the radius of rotation. In this paper, we present yet another classroom demonstration which has an advantage over the usual demonstrations in so far as it clearly emphasizes, with several variations, the angular momentum coupling both with and without dissipation between rotating systems.

The arrangement described here consists essentially of two flywheels attached to a horizontal bar (see Fig. 1). Each flywheel is free to rotate about its own vertical axis on the bar and the whole arrangement can rotate about a vertical axis through its center of mass.

By means of a coupling device (not shown in Fig. 1), angular momentum can be transmitted from one wheel to the other, or from the wheels to the orbital motion of the entire system. This transmission can be realized both elastically (no energy dissipation) or inelastically (with energy dissipation). Accordingly, this system models a wide range of phenomena. For example, it models the angular momentum transfer (inelastically via the tides) from the rotation of the Earth about its own axis to the orbital motion of the system "Earth + moon." Another example is the spin–spin or spin–orbit (nondissipative, elastic) coupling within an atom.

II. DECOMPOSITION OF THE ANGULAR MOMENTUM

The total angular momentum L of the system shown in Fig. 1 can be decomposed into three parts:

$$\mathbf{L} = \mathbf{L}_A + \mathbf{L}_B + \mathbf{L}_{AB}.$$

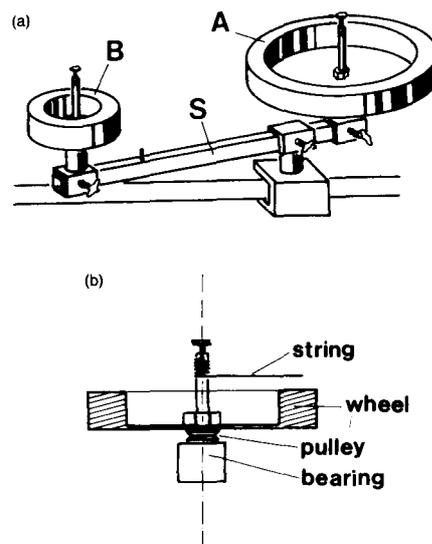Here $\mathbf{L}_A$ and $\mathbf{L}_B$ are the angular momenta of the wheels A

Fig. 1. (a) Two flywheels A and B are attached to a horizontal bar S. Each flywheel is free to rotate about its own vertical axis on the bar and the whole arrangement can rotate about a vertical axis through its center of mass. (b) Detailed illustration of the construction of each flywheel.

and B, respectively, about their own axes and $\mathbf{L}_{AB}$ represents the angular momentum of system AB which is obtained if the masses m_A and m_B of A and B are thought to be concentrated at the centers of mass C_A and C_B of wheels A and B, respectively. Accordingly, system AB consists of two masspoints which are separated by a distance d and which can rotate about their center of mass C_{AB}. $\mathbf{L}_{AB}$ is known as the orbital angular momentum of the entire system. For short, we will call $\mathbf{L}_A$ or $\mathbf{L}_B$ the "spin" of flywheel A or B, respectively.

If there is no coupling between the subsystems A, B, and AB, each of the angular momenta $\mathbf{L}_A$, $\mathbf{L}_B$, and $\mathbf{L}_{AB}$ are constant in time. If angular momentum transfer between A and B is allowed, we will speak of spin–spin coupling. If angular momentum can be exchanged between AB on the one hand and A and/or B on the other, we will speak of spin–orbit coupling.[9]

III. EXPERIMENTAL SET-UP

For the experiments described in the next section, two small flywheels and one larger flywheel as well as two bars of different lengths are required. The wheels A and B and the bar S to which they are attached are interchangeable, i.e., each wheel can be easily detached from its own axis.

The flywheels are made of brass and have the bulk of their mass in the rim. All bearings are roller bearings. The diameters of the wheels are 12 and 24 cm and their moments of inertia are 4.5×10^{-3} kg-m^2 and 2.0×10^{-2} kg-m^2, respectively. A pulley of 10-mm diam is attached to the axis below each wheel (see the inset in Fig. 1). The axis extends somewhat above the upper side of each wheel. This part of the axis can be used during rotation to wind up a string connecting the flywheels across a spring. Of course, the dimensions cited here need not be strictly adhered to. They are presented primarily with the intention of providing a guideline for the reader.

Each bar can be fixed on the central bearing in such a way that it can easily be shifted along its own length. In every experiment, the bar is adjusted in such a way that the center of mass of the entire system, "two wheels plus bar," is situated above the central bearing. Accordingly, the whole system behaves as if it rotated without suspension in free space.

IV. DEMONSTRATIONS

A. No coupling (S is any length; A and B are any size)

Both flywheels A and B and the system AB are set in motion in an arbitrary manner. Each of the three subsystems A, B, and AB then has a certain amount of angular momentum. It is observed that each subsystem keeps its angular momentum. Only a slow decrease of the angular momenta due to friction is observed. The fact that there is no coupling between the subsystems is particularly evident with the initial values $\mathbf{L}_A = \mathbf{L}_B = 0$ and $\mathbf{L}_{AB} \neq 0$, that is, if only the bar S is set into rotation, the center of mass of each wheel rotates around the center of mass of the entire system, but neither wheel rotates about its own axis.

B. Dissipative spin–orbit coupling (S is long, A is large, B is small)

A string is looped around the pulley of A and connected via a stretched spring (spring constant 400 N/m) to the bar S, resulting in a friction brake for the wheel A (see Fig. 2).
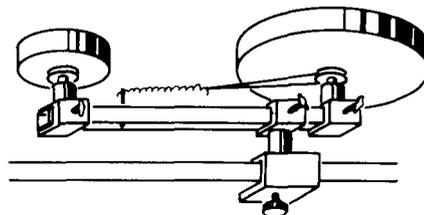

Fig. 2. Demonstration of dissipative spin–orbit coupling.

Then A is set into rapid rotation, i.e., $\mathbf{L}_A$ is given a high initial value. If the whole system is now left to itself, the angular momentum $\mathbf{L}_A$ of A will decrease and the angular momentum $\mathbf{L}_{AB}$ of AB will increase: the system AB begins to rotate and rotates ever more quickly. The angular velocity ω_{AB} increases until $\omega_{AB} = \omega_A$, i.e., rotational equilibrium[10] between A and AB is established. The rate of angular momentum transfer from A to AB can be augmented by increasing the tension of the spring. The experiment is even more surprising if the bar S is not initially at rest but rotates with a low angular velocity in the opposite direction of A. Then, as angular momentum transfers from A to AB, the rotation of the bar decreases, stops, and then begins to rotate in the opposite direction.

This experiment simulates the angular momentum transfer from the rotation of the Earth about its own axis to the orbital rotation of the system "Earth + moon" about the center of mass of this system. Of course, the detailed nature—mechanical, gravitational, electromagnetic, etc.—of the internal forces and torques does not matter to angular momentum conservation. By the way, the angular momentum transfer between the moon and the system "Earth + moon" ended a long time ago: the angular velocities of both systems are the same as is evidenced by the fact that we always see the same face of the moon.

C. Nondissipative spin–spin and spin–orbit coupling (S is short, A is small, B is small)

A string is attached at the upper part of the axis of each wheel A and B and both strings are connected via a spring. The spring is horizontal but relaxed when the strings are not wound up. Next the wheels are rotated by hand in the same sense of direction, e.g., both clockwise and with the same number of revolutions, e.g., 3. As a result, the string winds up on both wheels and the spring is stretched. If we now release the system, it makes a surprising movement: Whereas wheels A and B begin turning about their own axes, e.g., both counter clockwise, the whole system begins to rotate in the opposite direction, e.g., clockwise.

When the spring is relaxed, the strings begin to wind up in the opposite direction as before, all rotations become slower, eventually stop, and the whole process repeats itself but now with the opposite sense of rotation. Thus A and B rotate "back and forth," each about its own axis, while AB rotates "forth and back" about the center of mass C_{AB} [see Fig. 3(a)]. The period of motion is on the order of several seconds. Depending on how far the spring has been wound up initially, AB can realize up to a whole revolution. If the bar S is initially set properly into rotation, it will slow down, stop, start in the same direction as before, stop again, and so on.

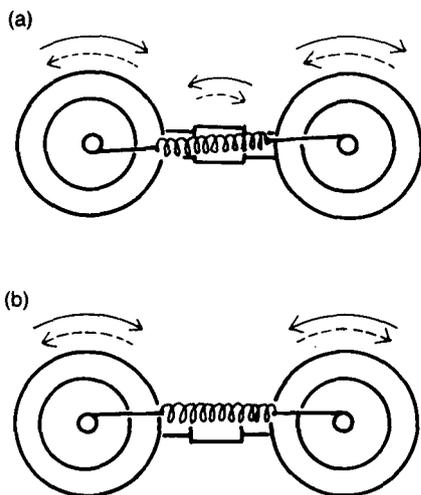This behavior is easily understood in terms of the trans-

(a)

(b)

Fig. 3. (a) Demonstration of nondissipative spin–orbit coupling. The sense of rotation of both wheels is identical and opposite to that of the system as a whole. Both senses of rotation oscillate in sign. (b) Demonstration of nondissipative spin–spin coupling. The wheels rotate back and forth in opposite directions. The system as a whole remains at rest.

fer of angular momentum. After releasing the system, both L_A and L_B increase. In order to keep the total angular momentum $L_A + L_B + L_{AB}$ constant, L_{AB} has to increase by the negative of $L_A + L_B$. A similar phenomenon occurs within an atom with spin–orbit coupling. Of course, in the

atom, the angular momentum does not change by changing the amount of the spin angular momentum of the electron (which is not possible) but rather by changing its direction.

If both wheels are initially turned by the same number of turns in opposite directions and then released, the increase of L_A is the negative of the increase of L_B [Fig. 3(b)]. Then the sum $L_A + L_B$ always remains zero. Both wheels turn "back and forth" but A always in the opposite sense of direction of B and the bar S remains at rest. Angular momentum is exchanged only between A and B, but not with AB. This is a model of spin–spin coupling.

Although in all these demonstrations quantitative measurements can be made, the main message of the experiments is to display angular momentum transfer in a very direct and convincing way.

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